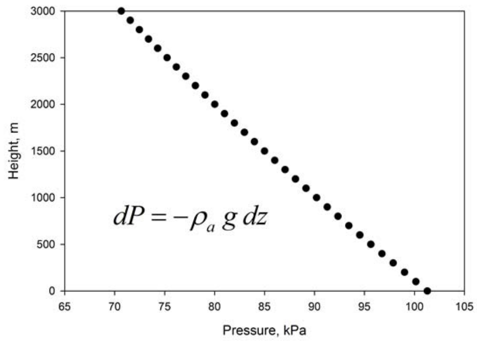
Temperature and Thermodynamics, Part II

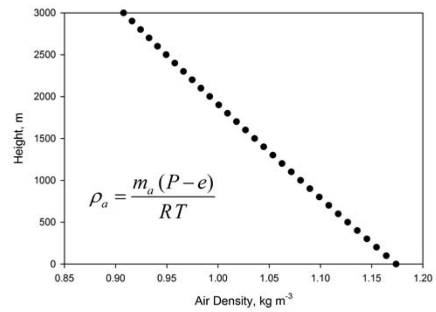
- Topics to be Covered
  - Profiles of Temperature in the Boundary Layer
  - Potential temperature
  - Adiabatic Lapse Rate
  - Thermal Stratification

Why are We Interested in Thermal Stratification of the Atmosphere?

- Compare Temperatures measured at Different Heights and Altitudes
- Creates or Suppresses Turbulence and Diffusion
- Adiabatic Lifting can Cause Cooling
  - It can cause water to change phases
  - Condensation Promotes the Formation of Clouds and Rain
- Adiabatic Compression can Cause Heating
  - Downslope Santa Ana winds, Hot and Dry
- Microclimate Modification
  - Wind machines Break the Nocturnal Inversion Layer and Prevent Frost

Changes in Air Pressure and Density with Height, Ideal Conditions





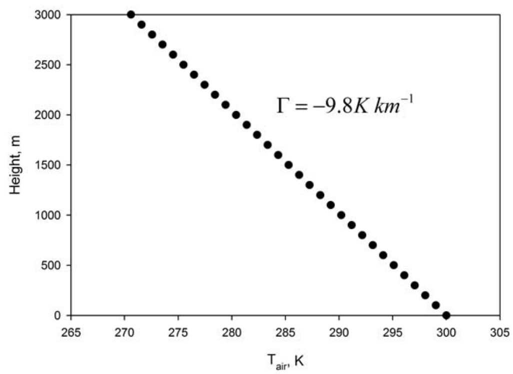
Air Parcel Lifted Expands and Becomes Less Dense
because Surrounding Pressure is Less, Consequently
Its Temperature Drops as the Parcel Expends

ESPM 129 Biometeorology

Pressure overhead is related to the density of air times the acceleration of gravity. Pressure decreases with height as the density decreases.

Temperature Profiles in the Boundary Layer:

Adiabatic Lapse Rate



ESPM 129 Biometeorology

As we lift parcels of air they are surrounded by a less dense atmosphere. So those parcels expand and cool adiabatically, without heat exchange. This leads to the adiabatic lapse rate.

Potential Temperature, θ

The Temperature a Parcel of Air would have if it were Expanded or Compressed, Adiabatically, from its Existing Temperature and Pressure to a Standard, near sea level (P=101.3 kPa):

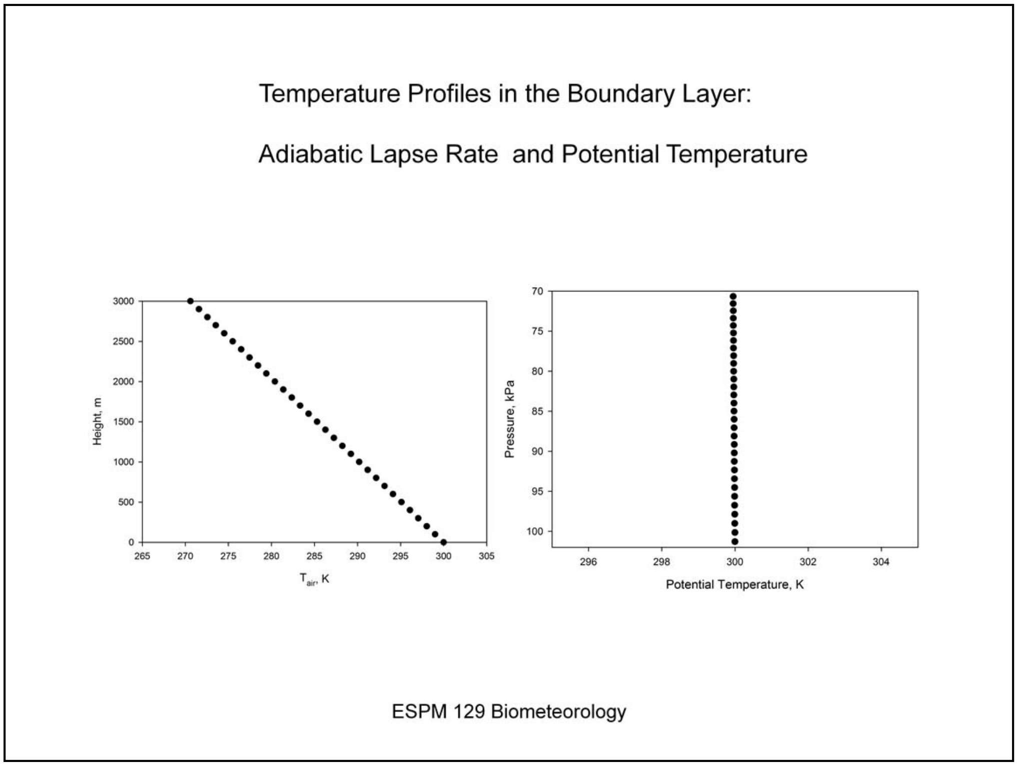
Wallace and Hobbs, 1977

$$\theta = T(\frac{P_0}{P})^{R/(m \cdot c_p)} \qquad \theta = T(\frac{P_0}{P})^{R/C_p}$$

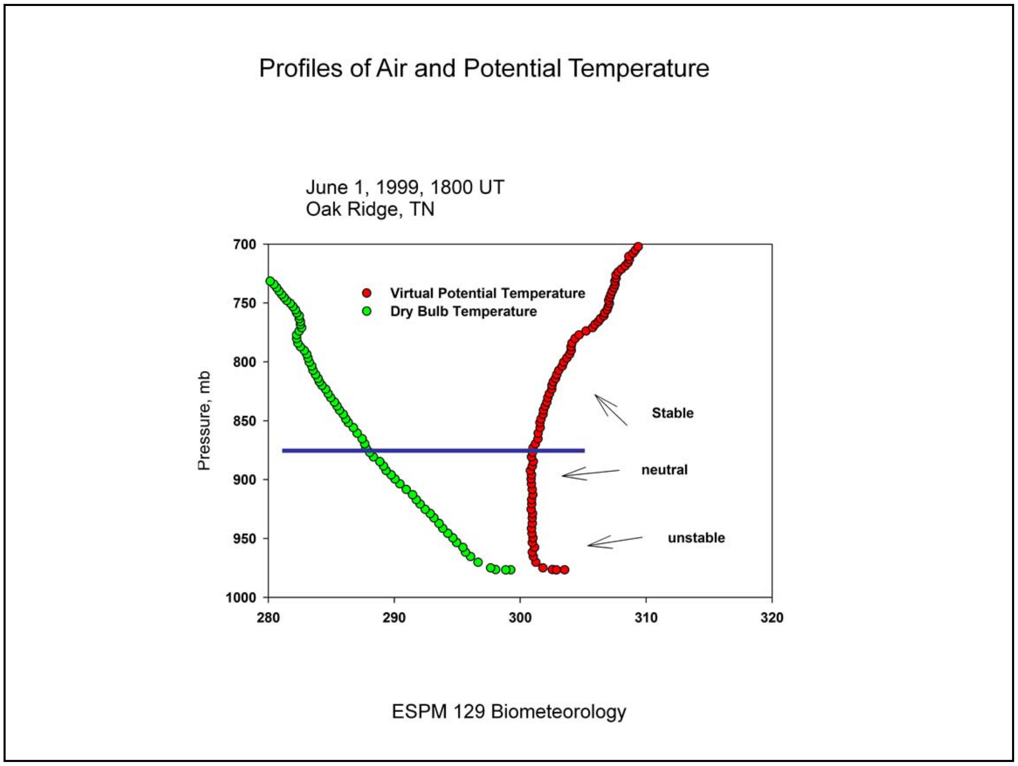
T, Temperature (K)
R, Universal Gas Constant, 8.314
P, Pressure, kPa
P_0, reference pressure, eg 101.3 kPa
C_p, Heat Capacity of air at constant pressure
c_p, specific heat capacity of air at constant pressure

ESPM 129 Biometeorology

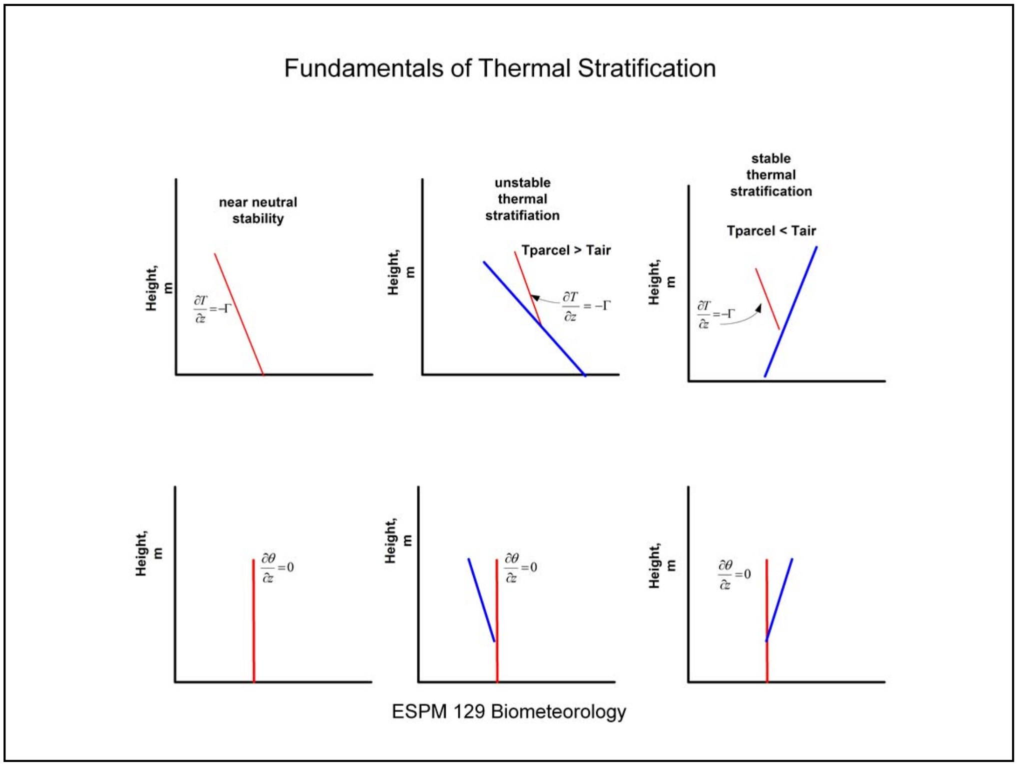
Atmospheric scientists like to work with potential temperature as that is the temperature of a parcel of air lifted or dropped adiabatically, from a local pressure, P, to the reference, P0, typically at sea level.



It is important to distinguish the difference between the temperature profile, which decreases with the lapse rate, and that of the potential temperature, which is constant with height.



Here is a comparison of data in the real planetary boundary layer. You see 3 layers of different thermal stratification. The lowest is unstable as the air is warmer at the surface, than above. The next is well mixed and neutral. Above the boundary layer we have a stable layer where the potential temperature increases with height. Note air temperature still declines but at a rate less than the lapse rate.



Important concepts to know. The distinct differences in profiles of air temperature and potential temperature for near neutral, unstable and stable thermal stratification

Thermal stratification causes the atmosphere to be either buoyant or stable

- The atmosphere is neutrally stratified if: $\frac{\partial \theta_v}{\partial z} = 0 \qquad \frac{\partial T_v}{\partial z} = -\Gamma$
- The atmosphere is unstably stratified if: $\frac{\partial \theta_v}{\partial z} < 0 \qquad \frac{\partial T_v}{\partial z} < -\Gamma$
- The atmosphere is stably stratified if: $\frac{\partial \theta_v}{\partial z} > 0 \qquad \frac{\partial T_v}{\partial z} > -\Gamma$

Upcoming Features

- Describe Adiabatic Processes and Potential Temperature in terms of variables we measure, P and T
- Start with 1st Law of Thermodynamics
- Normalize 1st Law by Mass, yielding 'Specific' Equation
- Substitute Terms with Gas Laws, converting Equation from a function of heat capacity for volume, Cv, to that for pressure, Cp
- Rearrange Equation so T is on one side and P is on the other
- Integrate both sides, yielding equations Ln(T) and Ln(P)
- Take anti-log and solve for potential temperature

Adiabatic Process

An air parcel changes its physical state (either its pressure, volume or temperature) **without heat** being added or withdrawn from the surrounding environment

$$dQ = 0$$

ESPM 129 Biometeorology

A--Diabatic, without heating…A– like A-theist…

Understanding Adiabatic Compression and Expansion,
And its Impact on Temperature Profiles
With the First Law of Thermodynamics

$$dQ = C_V dT + PdV = 0$$

$$C_V dT = -PdV$$

Change in internal Energy   Change in Work

No Heat is Exchanged, But Temperature Can Change

Change in Internal Energy Equals Change in Work Done on the Parcel

ESPM 129 Biometeorology

If dQ is zero then new balance

Understanding Adiabatic Processes in the Atmosphere by considering Parcels of Air per unit Mass (m), **Specific** quantities

$$q = \frac{Q}{m}$$

$$w = \frac{W}{m}$$

$$u = \frac{U}{m}$$

$$c_v = \frac{C_v}{m}$$

$$c_p = \frac{C_p}{m}$$

Change in Volume per unit Mass is
Equivalent to the change of the inverse in density, ρ

$$\frac{dV}{m} = d\left(\frac{1}{\rho}\right) = d\alpha$$

ESPM 129 Biometeorology

When you hear the word specific in science it tends to be associated with a normalization by mass. When you hear about specific gravity of beer or wine, it refers to the density of that fluid divided by the density of water. A mass per mass per unit volume

The Specific Form of the 1st Law of Thermodynamics

$$dq = c_v dT + Pd(\frac{1}{\rho}) = c_v dT + Pd\alpha$$

Algebraic Tricks to Express dq as f(T, P)
and solve for $Pd\alpha$

$$P = \frac{\rho RT}{m} \qquad P\alpha = \frac{RT}{m}$$

$$d(P\alpha) = \alpha dP + Pd\alpha = \frac{1}{m}d(RT) = \frac{1}{m}RdT$$

$$Pd\alpha = -adP + \frac{R}{m}dT$$

ESPM 129 Biometeorology

A little algebraic and calculus hocus pocus and re-arranging terms

Algebraic Tricks to Express dq as f(T, P), things we can measure

$$Pd\alpha = -adP + \frac{R}{m}dT$$

$$dq = c_v dT - Pd\alpha$$

$$dq = (c_v + \frac{R}{m})dT - \alpha dP$$

We can now get a term of dq as a function of cp dT and dP…things we can measure

Change in Specific Heat, dq, is a function of the
Specific Heat Capacity, c_p, times a change in Temperature, minus
A change in pressure, dP….terms we can MEASURE

$$dq = (c_v + \frac{R}{m})dT - \alpha dP$$

$$c_p = c_v + \frac{R}{m}$$

$$dq = c_p dT - \alpha dP$$

Define specific heat capacity at constant P

Isobaric case, dP equals 0

$$dq = (c_v + \frac{R}{m})dT - \alpha dP$$

$$\frac{dq}{dT}|_p = c_v + \frac{R}{m} = c_p$$

Adiabatic Process, dq=0

$$dq = 0 = c_p dT - \alpha dP$$

$$c_p dT = \alpha dP = \frac{dP}{\rho}$$

ESPM 129 Biometeorology

Let's derive potential temperature

$$c_p dT = \alpha dP = \frac{dP}{\rho}$$

$$\rho = \frac{mP}{RT}$$

$$c_p dT = \frac{RTdP}{mP}$$

P: Pressure
V: volume
n: number of moles
R: Universal Gas Constant
m: mass per mole
T: absolute air temperature
ρ: air density

$$c_p \frac{dT}{T} = \frac{R}{m}\frac{dP}{P}$$

ESPM 129 Biometeorology

$$\frac{dT}{T} = \frac{R}{m \cdot c_p} \frac{dP}{P} = \frac{R}{C_p} \frac{dP}{P}$$

$$\int_{\theta}^{T} \frac{dT}{T} = \frac{R}{m \cdot c_p} \int_{P_0}^{P} \frac{dP}{P}$$

$$\ln \frac{T}{\theta} = \frac{R}{m \cdot c_p} \ln \frac{P}{P_0}$$

$$\ln\frac{T}{\theta} = \frac{R}{m \cdot c_p}\ln\frac{P}{P_0}$$

$$\exp(\ln\frac{T}{\theta}) = \exp(\frac{R}{m \cdot c_p}\ln\frac{P}{P_0})$$

$$\frac{T}{\theta} = \frac{P}{P_0}^{\frac{R}{m \cdot c_p}}$$

Potential Temperature

$$\theta = T\left(\frac{P_0}{P}\right)^{R/(m \cdot c_p)}$$

$$\theta = T\left(\frac{P_0}{P}\right)^{R/C_p}$$

Adiabatic lapse rate, How Temperature Changes with Height

$$0 = \rho c_p dT - dP$$

$$\rho c_p dT = dP$$

Hydrostatic Equation $$\frac{dP}{dz} = -\rho g$$

Change in Pressure with height is a function of the density of Air times the acceleration due to gravity, g

ESPM 129 Biometeorology

What is the lapse rate

Dry Adiabatic Lapse Rate

$$\rho c_p \frac{dT}{dz} = \frac{dP}{dz} = -\rho g$$

$$\frac{dT}{dz}\bigg|_{adiabatic} = -\frac{g}{c_p} = \Gamma$$

The dry adiabatic lapse rate, 9.8 K km^{-1}

Moist Adiabatic

water vapor condenses and latent heat of condensation is released.

$$\frac{dT}{dz}\Big|_{moist} = -\frac{g}{c_p + \lambda \frac{de_s}{dT}} = \Gamma_{moist}$$

λ is the latent heat of vaporization
de_s/dT is the slope of the saturation vapor pressure-temperature curve.

ESPM 129 Biometeorology

Latent heat exchange of a moist atmosphere can play a role, too, as with Hurricanes.

Key Points

- Define Thermal Stratification
  - Neutral, Stable and Unstable Conditions
- Define Potential Temperature
- Define Adiabatic Processes
  - Dry and Moist Adiabatic Lapse Rate
- Define First Law of Thermodynamics

Summary

- Potential temperature is the temperature of a parcel of air that is moved Adiabatically from a level in the atmosphere to a reference Pressure.
  - It is defined for conditions when changes heat energy are zero (dQ=0).
- Thermal stratification causes the atmosphere to be either buoyant or stable
  - The atmosphere is neutrally stratified if: $\frac{\partial \theta_v}{\partial z} = 0$
  - The atmosphere is unstably stratified if: $\frac{\partial \theta_v}{\partial z} < 0$
  - The atmosphere is stably stratified if: $\frac{\partial \theta_v}{\partial z} > 0$