# Leaf Energy Balance, part 1

- Introduction
  - History of leaf energy balance studies
  - Role of leaf energy balance on Evolution and leaf shape
  - Resistance/conductance networks
- Leaf Energy Balance: Steady-State Linear Theory
- Dry leaf
  - Tl-Ta, leaf-air temperature differences
- Wet and transpiring leaf
  - Tl-Ta, leaf-air temperature differences
  - Evaporation, f(net radiation)
  - Evaporation, f(isothermal radiation)
- Night, dew and frost



We've been building up the skills in this class to reach this stage and assess the energy balance of a leaf. It is critical to compute transpiration, sensible heat flux, surface temperature that drives VOC emissions, respiration and the kinetic rates of photosynthesis. We will start simple with a dry leaf then go to a wet leaf.

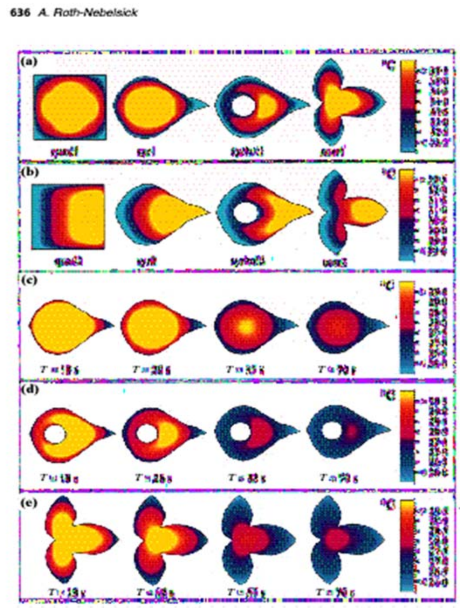
## History of Leaf Temperature

- Curtis (1926)
  - Transpiration as a cooling agent is not important
- Wallace and Clum (1938)
  - Leaf temperature can differ from air temperature in a range between -7 and +4 C
  - Refuted by Curtis (1938) for poor experimentation and faulty logic
- Rascke (1960) and Gates (1965) first theory on leaf energy balance
  - Showed leaves can be cooler than air



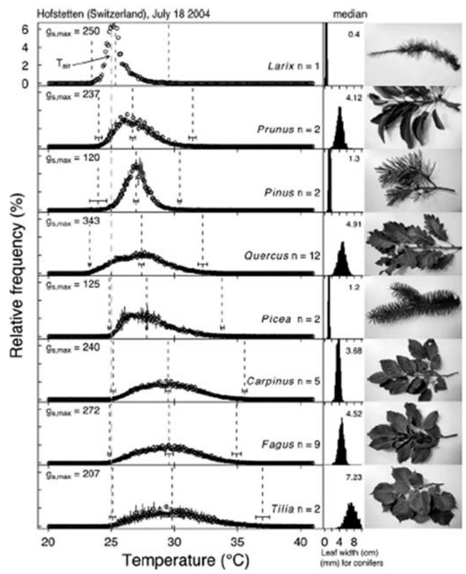
Leaf energy balance has a long history. Some of the past theories were biased on where the authors lived. Curtis lived in the East and could not envision transpirational cooling. Rascke developed his theories while in hot India and it became clear to him from observations and theory

Impact of leaf size on the transient development of heat transfer across a spectrum of leaf shapes



## Leaf Temperature and Shape and Size

S. Leuzinger, C. Körner / Agricultural and Forest Meteorology 146 (2007) 29–37

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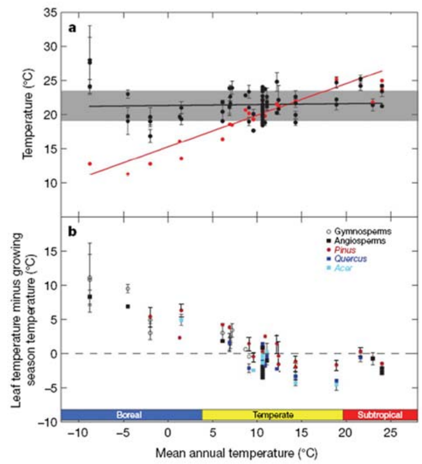
Interesting histogram of leaf temperatures associated with the shape of leaves.. All in a simlar climate in Switzerland.  Bigger leaves are warmer and the small needles of conifers are cooler. After Leuzing and Korner. 2007

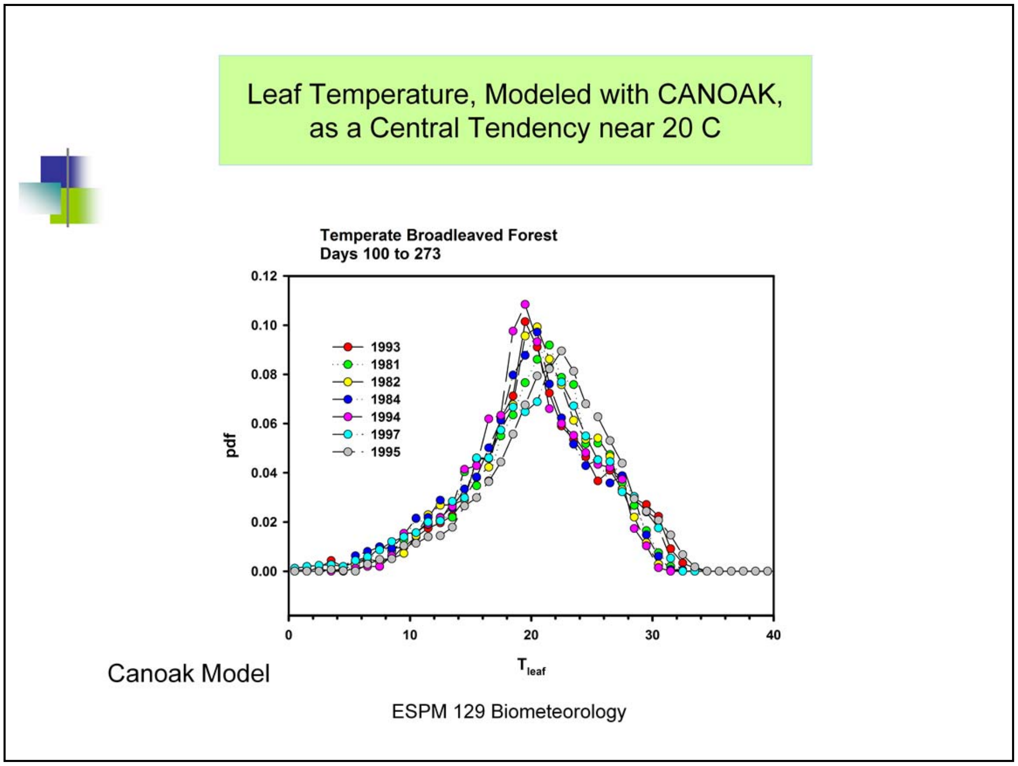
Global Convergence of Seasonal Average Leaf Temperature (????), as inferred by Isotopes




Helliker and Richer 2008, Nature

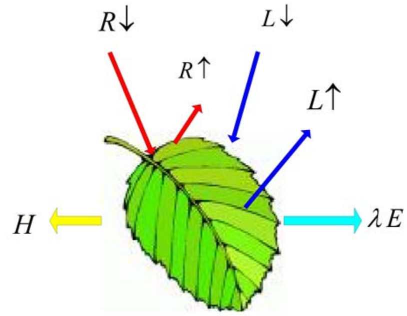


Helliker deduced leaf temperatures from measurements of stable isotopes. They concluded that there was a global convergence on the seasonally average leaf temperature across a global gradient of mean annual temperatures. Why could and should this happen? We argue that the isotope signature is a flux weighted temperature as it is associated with transpiration and photosynthesis.




We tested this idea with a canopy energy balance model and computed the histogram of leaf weighted temperature for a forest growing in TN, mean annual air temperature 13.6 C.  We find the central tendency of this distribution to be near 20 C.  So models are important for testing theories. And Giving some explanation.

## Leaf Energy Balance, top-side




- R: is shortwave solar energy, W $m^{-2}$
- L: is Longwave, terrestrial energy, W $m^{-2}$

□λE: Latent Heat Flux Density, W $m^{-2}$

- H: Sensible Heat Flux Density, W $m^{-2}$

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Let’s look at the fluxes of energy into and out of a leaf.  This brings us back to the lesson in energy balance

## Net Radiation Balance of a Leaf, top-side

$$R_n = R\downarrow - R\uparrow + L\downarrow - L\uparrow = H + \lambda E$$

•R: Shortwave solar energy, W $m^{-2}$

•L: Longwave, terrestrial energy, W $m^{-2}$

□λE: Latent Heat Flux Density , W $m^{-2}$

•H: Sensible Heat Flux Density, W $m^{-2}$



There are important feedbacks to consider because leaf temperature will be a function of the net radiation budget, which is a function of leaf temperature; We solve for Tleaf knowing the functions of H(Tleaf), LE(Tleaf) and Lout(Tleaf)

## Leaf Energy Balance, Dry Leaf

Net radiation balance equals Sensible Heat Exchange

$$R_n = R\downarrow - R\uparrow + L\downarrow - L\uparrow = H$$

- R: is shortwave solar energy, W $m^{-2}$
- L: is Longwave, terrestrial energy, W $m^{-2}$
- H: Sensible Heat Flux Density, W $m^{-2}$

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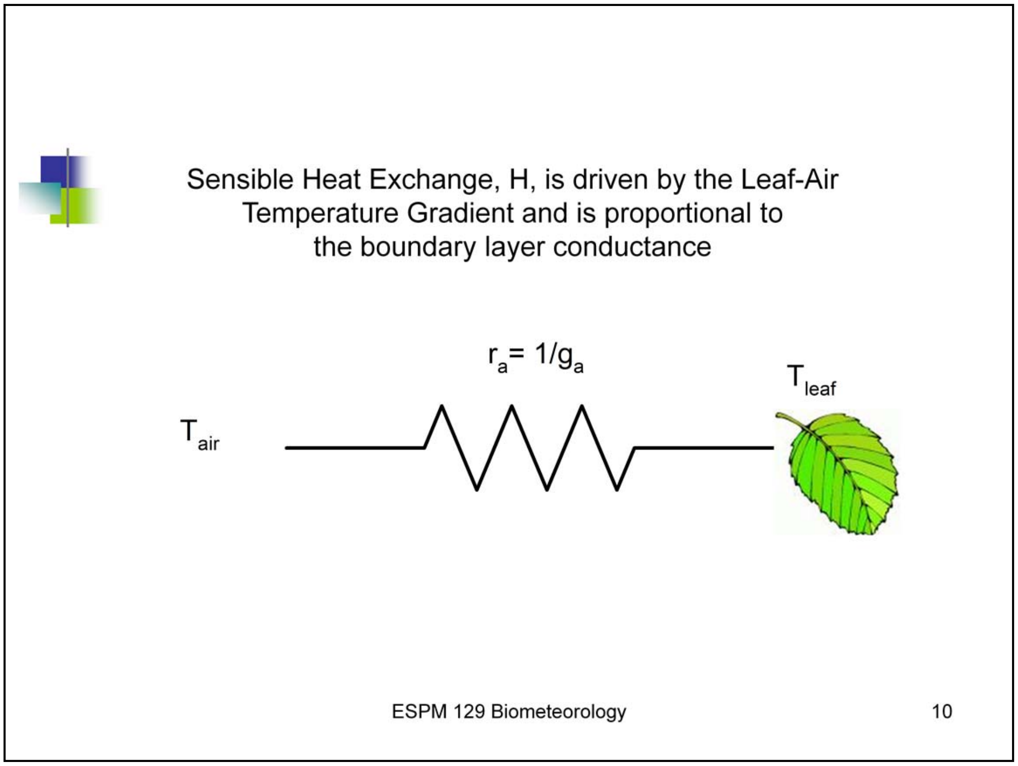
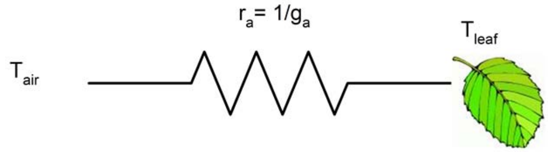
Lets first start with a dry leaf to keep the math and derivation simple.




We apply Ohm’s Law resistance analog to compute H

$$H = \rho_a C_p (T_l - T_a) g_{ah}$$




- $\rho_a$ density of dry air, g $m^{-3}$
- $C_p$, specific heat of air
- $T_l$, leaf temperature
- $T_a$, air temperature
- $g_a$, boundary layer conductance

Algebraic Manipulation produces an Equation for air-leaf temperature difference

$$(T_l - T_a) = \frac{R_n}{g_a \rho_a C_p}$$

But Rn = f(T)!

$$L\uparrow = \varepsilon \sigma T_l^4 + (1-\varepsilon) L\downarrow$$

ε: emissivity

## Re-Visit Radiation Balance

$$R_n = R\downarrow - R\uparrow + L\downarrow - [\varepsilon\sigma T_l^4 + (1-\varepsilon)L\downarrow]$$

$$R_n = R\downarrow - R\uparrow + \varepsilon L\downarrow - \varepsilon\sigma T_l^4$$

Define Q: net in vs outgoing energy streams, exclusive of those dependent on T

$$Q = R\downarrow - R\uparrow + \varepsilon L\downarrow$$

And simplify in terms of leaf reflectivity, ρ

$$R\uparrow = \alpha R\downarrow$$

$$Q = (1-\alpha)R\downarrow + \varepsilon L\downarrow$$

## New Version of Leaf Energy Balance

$$Q = (1-\alpha)R\downarrow + \varepsilon L\downarrow = \varepsilon\sigma T_l^4 + \rho_a C_p g_h (T_l - T_a)$$

But it now has a non-linear 4th order term, $T^4$



Here is the new version of the equation with a fourth order term for leaf temperature. What are we to do to solve this???

## Linearize with 1st order Taylor's Expansion Series

$$f(x) \sim f(x_0) + (x - x_0)\frac{df}{dx}$$

$$\varepsilon\sigma T_l^4 = \varepsilon\sigma T_a^4 + 4\varepsilon\sigma T_a^3 (T_l - T_a)$$

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Linearization is our friend. We can break the fourth order term for Tleaf into a linear equation based on information we know about the boundary conditiona, air temperature, that is raised to the 4th and 3rd powers. This assumption works best for small temperature differences. One should use a second order approximation with greater temperature differences.

## Substitute and Manipulate

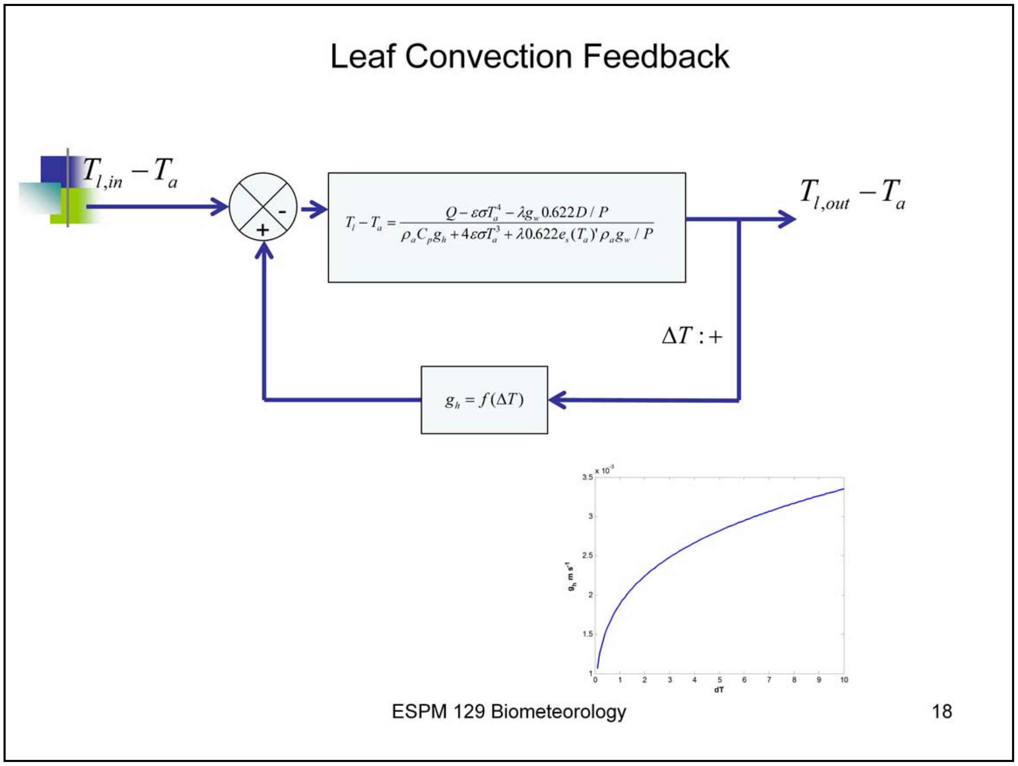
$$Q - \varepsilon\sigma T_a^4 - 4\varepsilon\sigma T_a^3 (T_l - T_a) = \rho_a C_p (T_l - T_a) g_a$$

*Voila'*

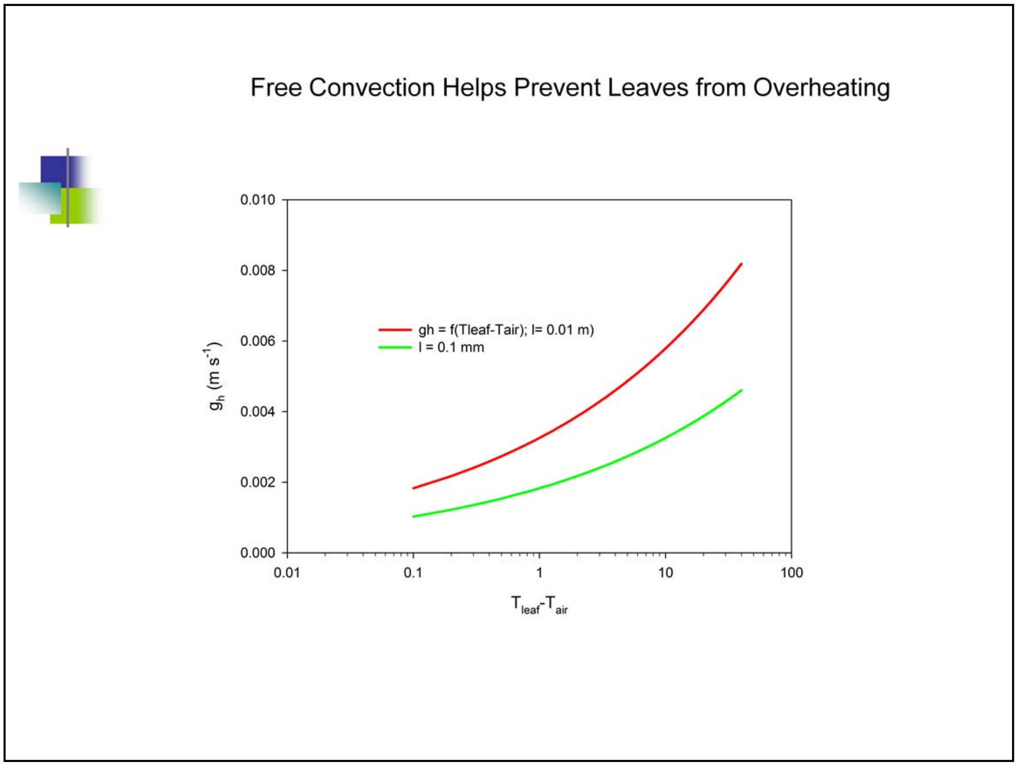
$$T_l - T_a = \frac{Q - \varepsilon\sigma T_a^4}{\rho_a C_p g_a + 4\sigma\varepsilon T_a^3}$$



Now we can solve for leaf temperature, or more specifically, the leaf air temperature difference. Not so hard. It gets a bit more complicated, but as tractable with a wet leaf.




Leaf air temperature differences can drive free convection, as defined by the Grasshof number. So  this will increase gh and reduce delta T through a negative feedback.




Incremental increases in leaf air temperature differences drive larger boundary layer conductances through convection. And they are a function of leaf size

## Leaf Energy Balance, Wet, Transpiring Leaf

$$R_n = H + \lambda E$$

Net Radiation is balanced by the sum of
Sensible and Latent Heat exchange

$$Q = (1-\alpha)R\downarrow + \varepsilon L\downarrow = \varepsilon\sigma T_l^4 + H + \lambda E$$

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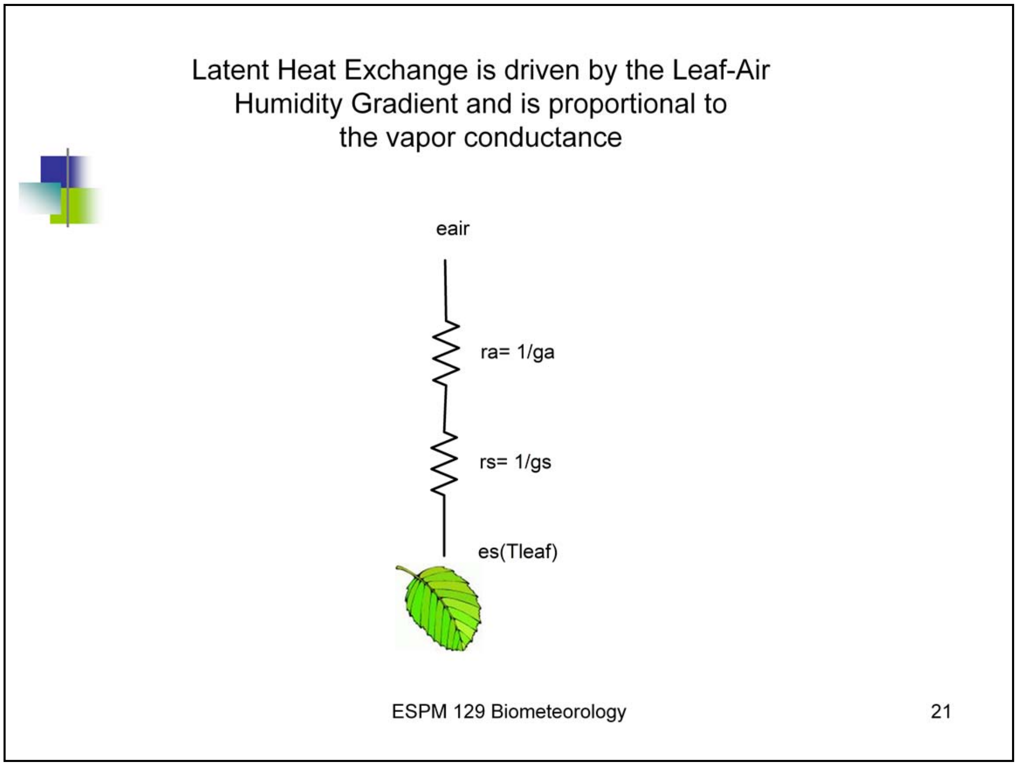
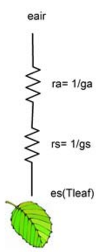
Now we consider latent heat exchange to consider a wet leaf




We consider this network with stomatal and boundary layer resistances

## Ohm's Law Equation for Latent Heat Exchange

$$\lambda E = \frac{(m_v / m_a)\lambda \rho_a g_w (e_s(T_l) - e_a)}{P}$$




$m_v$, molecular weight of water vapor, g mole$^{-1}$
$m_a$, molecular weight of dry air, g mole$^{-1}$
$g_w$,water vapor conductance, m s$^{-1}$
P, pressure, kPa
$e_s$, saturation vapor pressure, kPa
$e_a$, vapor pressure, kPa



Ohm's Law analog for latent heat exchange, expressed in terms of the total conductance to water transfer, gw

## Leaf resistance/conductance for water vapor, $g_w$

$$r_a + r_s = \frac{1}{g_{av}} + \frac{1}{g_s} = \frac{g_{av} + g_s}{g_{av} g_s} = \frac{1}{g_w}$$

- Boundary layer resistance, $r_a$
- Stomatal resistance, $r_s$
- Boundary layer conductance, $g_a$
- Stomatal conductance, $g_s$

r, s/m
g, m/s



For simplification we will consider the total conductance for water transfer that is the sum of resistances, gw.

## Various Forms

$$\lambda E = \frac{(m_v / m_a)\lambda \rho_a g_w (e_s(T_l) - e_a)}{P} =$$
$$\frac{(m_v / m_a)\lambda \rho_a (e_s(T_l) - e_a)}{P} \frac{g_s g_{av}}{g_{av} + g_s} =$$
$$\frac{(m_v / m_a)\lambda \rho_a (e_s(T_l) - e_a)}{P(r_s + r_a)}$$

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Here is Ohm's Law relation for latent heat exchange. Note it is a function of the saturation vapor pressure at leaf temperature. Another non linear function is added.

## Linearize the Saturation Vapor Pressure function

$$e_s(T_l) = e_s(T_a) + e_s'(T_l - T_a)$$

$$e_s(T)' = \frac{de_s(T)}{dT}$$

$$e_s(T_l) = e_x(T_a) + e_s'(T_l - T_a) + \frac{e_s''}{2}(T_l - T_a)^2$$

$$e_s(T)'' = \frac{d^2 e_s(T)}{dT^2}$$



We can linearize this one, es(T), too. Here we need to slope of the saturation vapor pressure curve. Remember when we looked at that?

Linearized Energy Balance of a Wet Leaf

$$Q-\varepsilon\sigma T_a^4-4\varepsilon\sigma T_a^3(T_l-T_a)=$$
$$\rho_a C_p(T_l-T_a)g_a+\frac{0.622\lambda\rho_a g_w(e_s(T_a)-e_a)}{P}+\frac{0.622\lambda\rho_a g_w e_s(T_a)'}{P}(T_l-T_a)$$

$$D=e_s(T_a)-e_a$$

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We can express this in terms of vapor pressure deficit, which we measure. Ultimately we can solve for Tl-Ta

$$T_l - T_a = \frac{Q - \varepsilon\sigma T_a^4 - \lambda g_w 0.622 D / P}{\rho_a C_p g_h + 4\varepsilon\sigma T_a^3 + \lambda 0.622 e_s(T_a)' \rho_a g_w / P}$$

- Factors driving $T_{leaf}$-$T_{air}$ greater
  - More energy, Q
  - Larger leaves, calm conditions, smaller gh
- Factors driving $T_{leaf}$-$T_{air}$ smaller
  - Greater $g_w$, more evaportative cooling
  - Greater humidity deficit
  - Smaller leaves and windy conditions, greater $g_h$



Solution

**Amphistomatous** leaves have stomata on both sides.

$$R_n = 2H + 2\lambda E = (R\downarrow - R\uparrow + L\downarrow - L\uparrow)_{top} + (R\uparrow - R\downarrow + L\uparrow - L\downarrow)_{bottom}$$

**Hypostomatous** leaves have stomata on one side,

Evaporation on one side, Heat exchange on both sides.

$$R_n = 2H + \lambda E = (R\downarrow - R\uparrow + L\downarrow - L\uparrow)_{top} + (R\uparrow - R\downarrow + L\uparrow - L\downarrow)_{bottom}$$

## Leaf-Scale Penman-Monteith Equation for Transpiration

$$\lambda E = \frac{sR_n + D\rho_a C_p g_h}{(s + \gamma \frac{g_h}{g_w})}$$

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Penman Monteith equation is a cornerstone of biometeorology. Here is the function for a leaf.

## Derivation

1: Leaf Energy Balance

$$R_n = H + \lambda E$$

2: Resistance Equations for H and $\lambda$E

$$\lambda E = \frac{(m_v / m_a)\lambda \rho_a g_w (e_s(T_l) - e_a)}{P}$$

$$H = \rho_a C_p (T_l - T_a) g_h$$

3: Linearize $T^4$ and $e_s(T)$

$$e_s(T_l) - e_a = D + e_s'(T_l - T_a) = \frac{\lambda E \cdot P}{\rho_a \lambda (m_v / m_a) g_w}$$

$$\varepsilon \sigma T_l^4 = \varepsilon \sigma T_a^4 + 4 \varepsilon \sigma T_a^3 (T_l - T_a)$$

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Here are the steps for finding this equation

$$e_s(T_l) - e_a = D + e_s'(T_l - T_a) = \frac{\lambda E \cdot P}{\rho_a \lambda (m_v / m_a) g_w}$$

$$\lambda E = \frac{\rho_a (m_v / m_a) \lambda g_w (D + e_s'(T_l - T_a))}{P}$$

$$T_l - T_a = \Delta T = \frac{\lambda E \gamma}{s \rho_a C_p g_w} - \frac{D}{s}$$

$$\lambda E = R_n - \left(\frac{\lambda E \gamma}{s \rho_a C_p g_w} - \frac{D}{s}\right) \rho_a C_p g_h$$

$$\lambda E \left(1 + \frac{\gamma g_h}{s g_w}\right) = R_n + \frac{D \rho_a C_p g_h}{s})$$

$$\lambda E = \frac{s R_n + D \rho_a C_p g_h}{(s + \gamma \frac{g_h}{g_w})}$$

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Voila’

# Intellectual Enrichment

- Download the Energy Balance GUI from the class web site and play with different combinations of climate and leaf features to perturb leaf energy balance

# Summary

- Net radiation balance of a wet leaf is partitioned into sensible and latent heat exchange
- The leaf energy balance equation is derived using four principles and operations:
  - The leaf energy balance
  - Ohm's Law Resistance analogy for transpiration and sensible heat exchange
  - Linearization of the saturation vapor pressure relationship
  - Numerical elimination of leaf-air temperature differences
- Leaves can be hypostomatous or amphistomatous, a fact that affects the derivation of the leaf energy balance equation
- The Isothermal energy balance derivation accounts for differences between leaf and air temperature, the loss of longwave energy and its impact on available energy