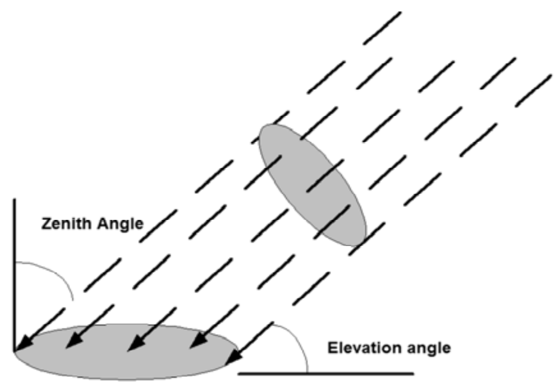
Lecture 7
Solar Radiation, Earth-Sun Geometry and Climatology, part III

- Lambert's cosine law
- Earth-Sun geometry
  - Earth declination, elevation, zenith, hour and azimuth angles
  - seasons
  - sunrise and sunset times
- Radiation Climatology and Functional Relations for 'Weather Generation'

Lambert's cosine law

Definition of zenith and elevation angles and the projection of area normal to incident rays on a flat surface

$$R(i) = R_{\perp} \frac{A_{beam}}{A_{horizontal}}$$



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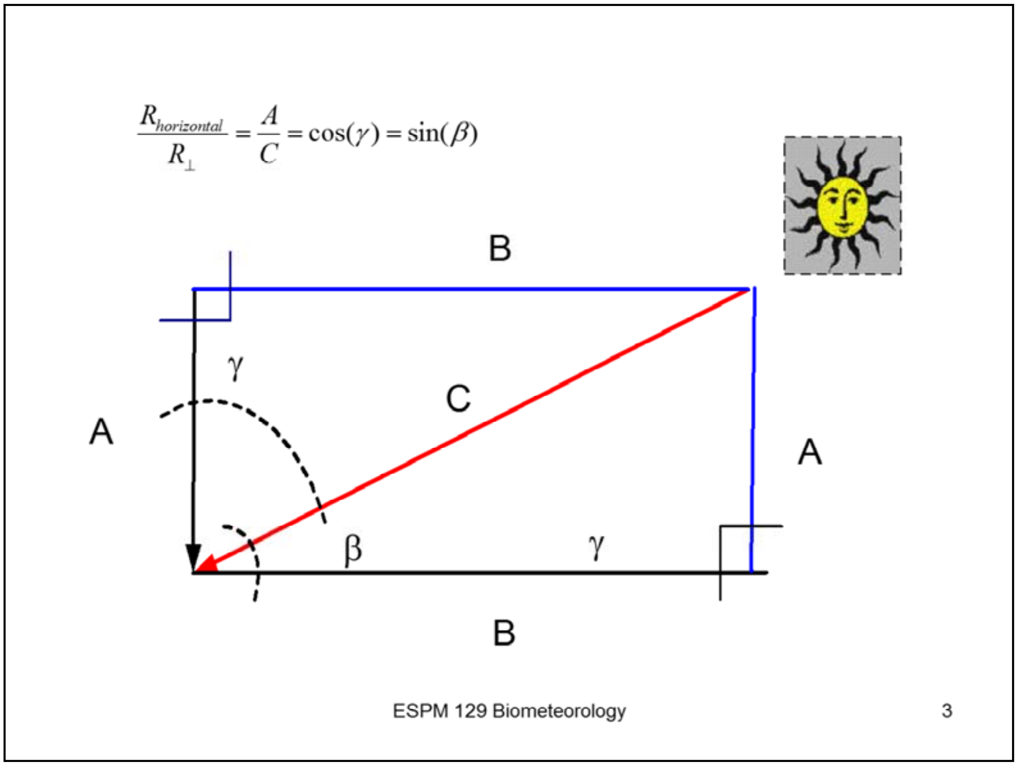
Climate means slope and it is particularly apt for solar radiation. As the sun angle changes with time and location we need to know how this translated to the flux density observed on a horizontal (or inclined) surface. Lambert's cosine law is a start in this journey

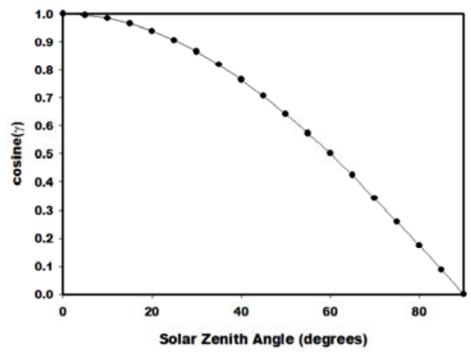


The translation of a vector of sunlight from the sun (C) to that normal to the surface (A) is either a function of the cosine of the zenith angle, gamma, or the sine of the elevation angle, beta

Lambert's Cosine Law

$$R_{horizontal} = R_{\perp} \cos \gamma = R_{\perp} \sin \beta$$

Cosine of solar zenith angle



Values vary non-linearly between 1 and zero

Graphic showing the non linear, cosine response to zenith angle

Spherical Geometry, Computing Sunlight on Horizontal and Inclined Surfaces

Solar Elevation (β) and Zenith (γ) Angles

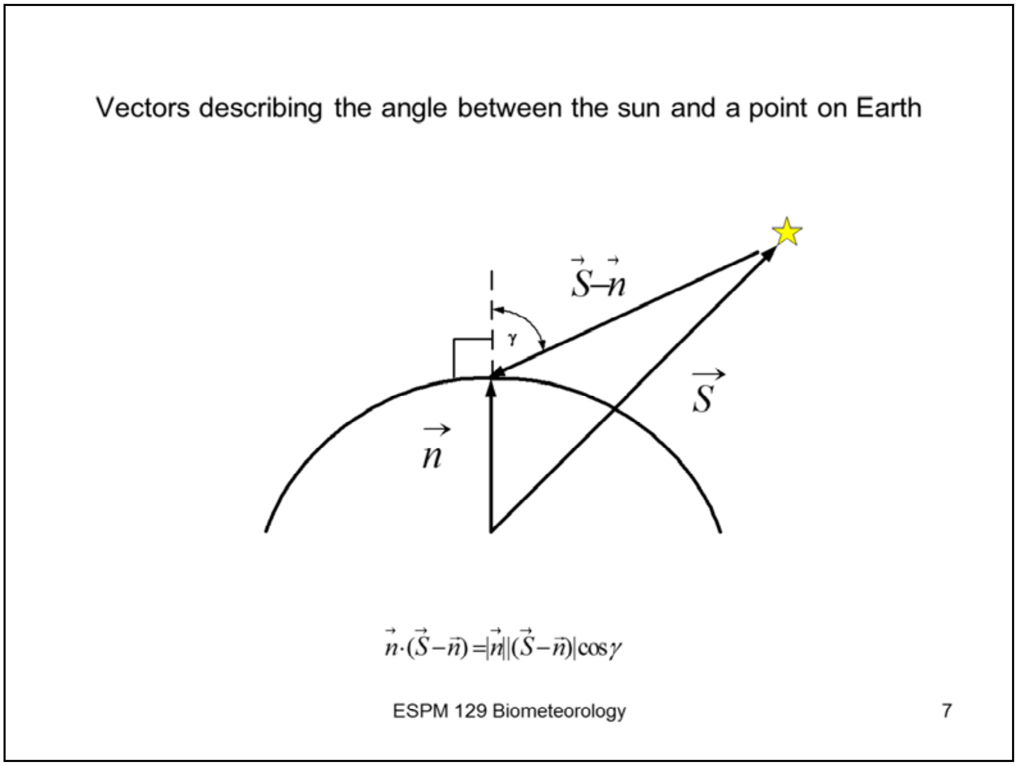
$$\sin(\beta) = \sin(\lambda)\sin(\delta) + \cos(\lambda)\cos(\delta)\cos(h) = \cos(\gamma)$$

λ is latitude, h is the hour angle and δ is declination angle.

Units: Radians!

Next we want to know the project of sunlight on any location on a sphere, like the Earth. Information like this is critical to solving the surface energy budget. But we also use forms of this equation to solve for the energy on inclined solar panels, and to optimize their design, and to interpret the radiation budget on inclined leaves

From spherical geometry we can calculate the sine of elevation angle as a function of the latitude, lamba, the declination angle of the planet, with respect to the sun, delta, and the hour angle (h) representing longitude and rotation angle relative to solar noon.



For those who like to think in vectors, one can visualize the solar zenith angle by the differences in the projections of the sun to the center of the earth and the observer.

Earth rotates on its axis **2π radians** in one day $h = \frac{2\pi t}{24} = \frac{\pi t}{12}$

Hour angle, h, is the fraction of 2π that the earth has turned ***after local solar noon***:

$$h = \frac{\pi}{12}(t - t_0) \quad \text{(radians)}$$

$$t_0 = 12 - l_c - e_t$$

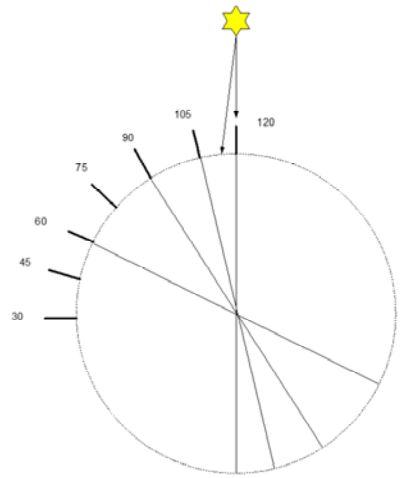
l_c, longitudinal correction
e_t, equation of time

What is hour angle? We can divide the rotation of the earth into 24 sectors, relative to solar noon.. A few other corrections are needed such as we all don’t live on distinct longitudal bands..and there is a correction called the equation of time.

l_c is the longitudinal correction.

- 4 minutes for each degree of longitude east of the standard meridian
- \- 4 minutes for each degree of longitude west of the standard meridian

Longitudinal Correction



4 minutes for each degree of longitude east of the standard meridian

- 4 minutes for each degree of longitude west of the standard meridian

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Visual of longitudinal correction

Equation of Time, hours

$$e_t = \left[\frac{-104.7\sin(f) + 596.2\sin(2f) + 4.3\sin(3f) - 12.7\sin(4f) - 429.3\cos(f) - 2.0\cos(2f) + 19.3\cos(3f)}{3600}\right]$$

$$f = \frac{\pi}{180}\left[279.5 + 0.9856d\right]$$ radians

Campbell and Norman, 1998

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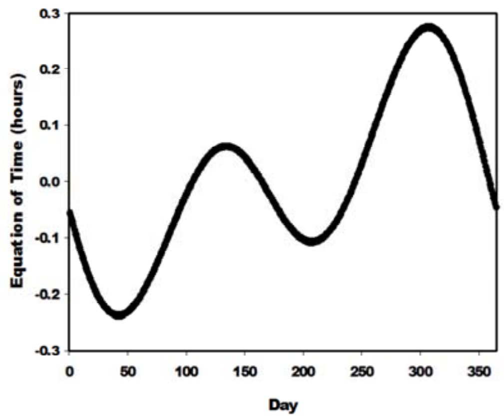
The equation of time is required because the revolution of the earth around the sun is eccentric and axis of the planet is tilted with respect to the orbital plane around the sun, called obliquity

Equation of time



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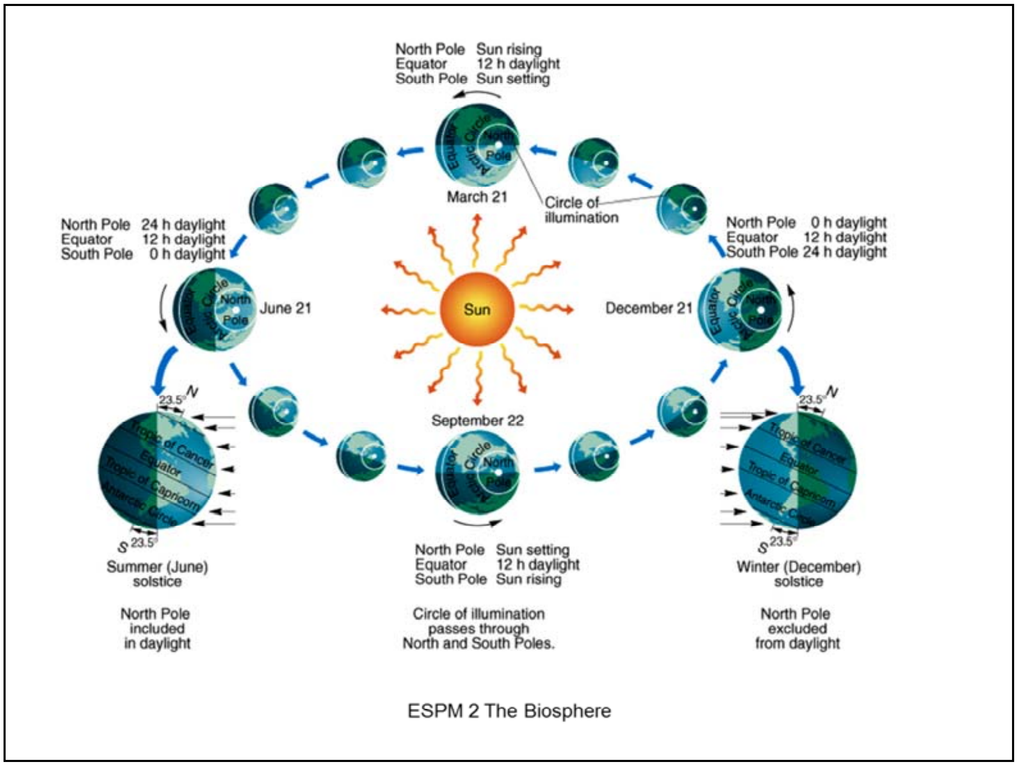
The previous equation computes a correction function which is the sum of two cosine functions

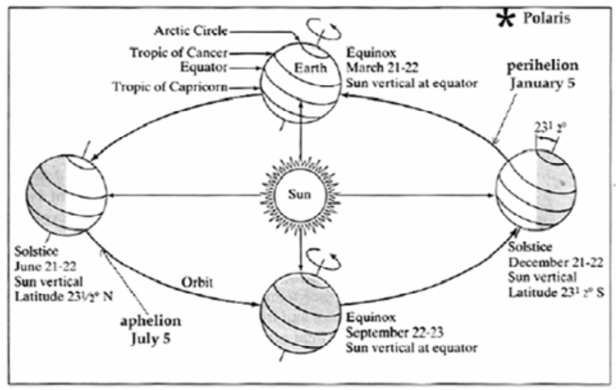
Values of h and cos(h) for several reference values

Solar time (hour)	h (radians)	Cos(h)
6	0.5π	0
12	π	-1
18	1.5π	0

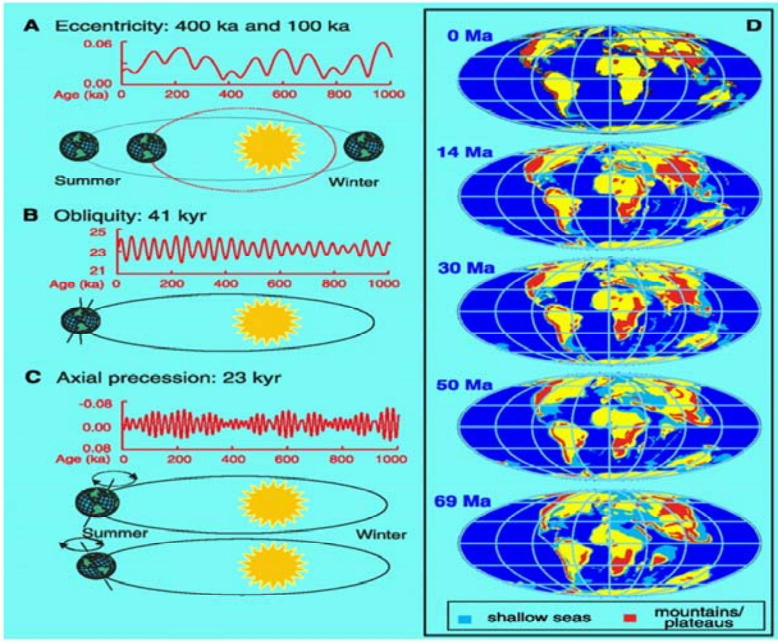


Seasonal change in the relationship between the tilt in the Earth's axis and the Sunlight it receives.

Basic Earth/Sun Geometry



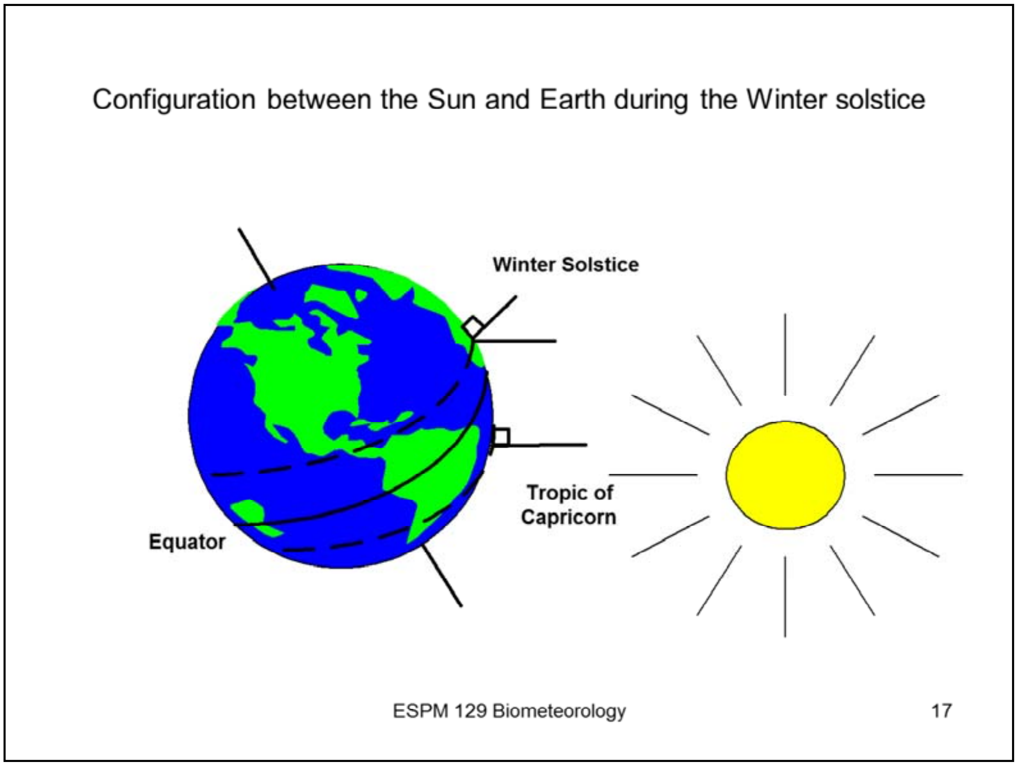
Lutgens/Tarbuck, *Atmosphere*



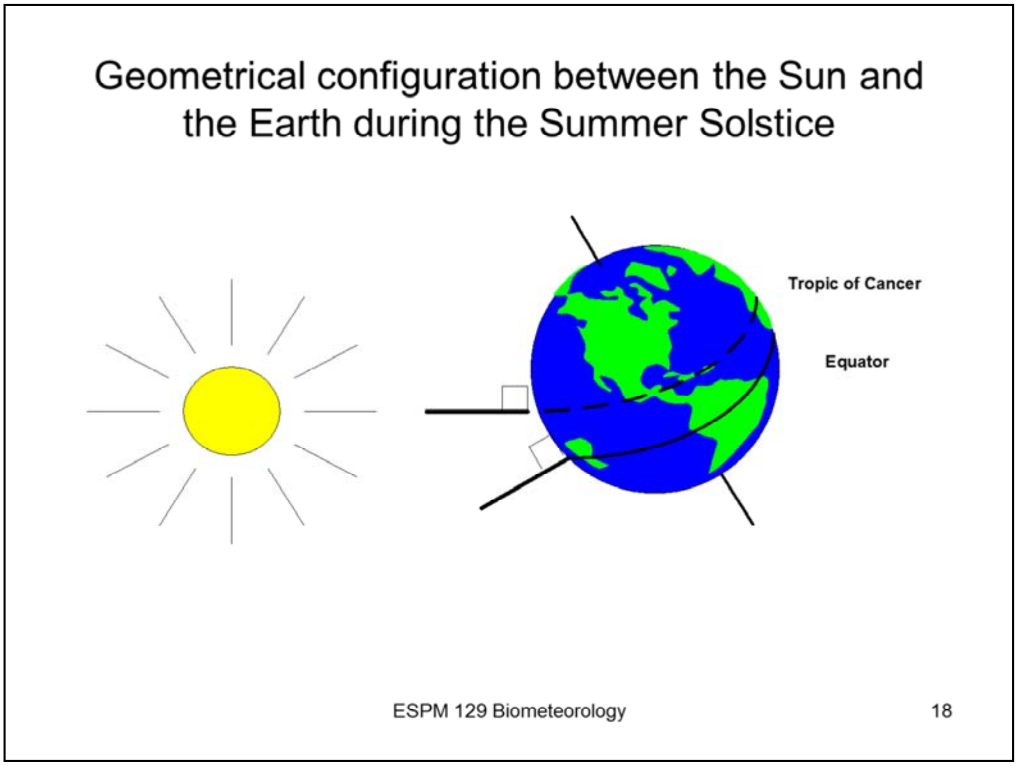
Zachos et al Science 2001

The tilt and orbit of the planet and its revolution around the sun and its axis has not been constant. It oscillates with time and this has lead to major climatic shifts between ice ages and inter glacial periods.

Zachos et al Science 2001

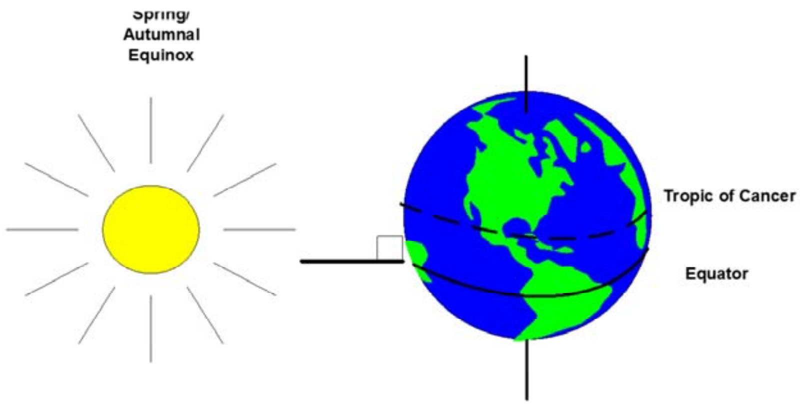


During the winter solstice the sun is directly overhead at noon at the Tropic of Capricorn. The zenith angle is 23.5 degrees at the equator (elevation is 90-23.5) and the zenith angle is 47 degrees at the tropic of cancer (elevation is 90 – 47).



During the summer solstice the sun is directly overhead at noon at the Tropic of Cancer. The zenith angle is 23.5 degrees at the equator (elevation is 90-23.5) and the zenith angle is 47 degrees at the Tropic of Capricorn (elevation is 90 – 47).

Earth and Sun Geometry during the Spring and Autumnal Equinox



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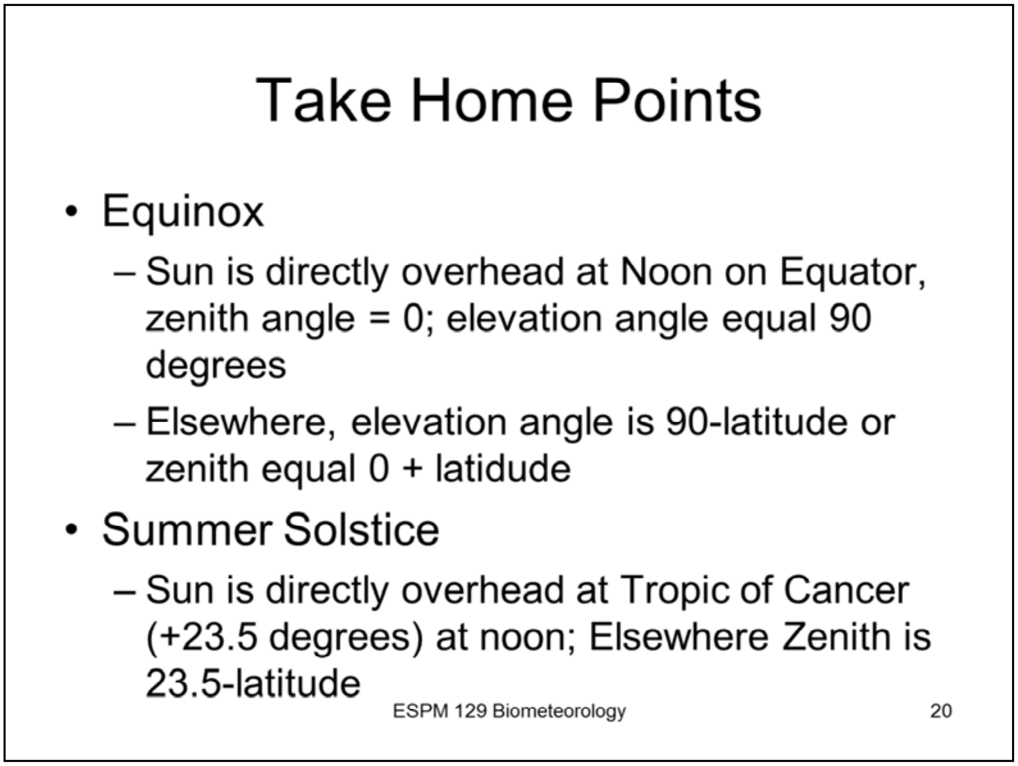
During the equinox the sun is directly overhead at the equator at noon. At other locations the zenith angle is the latitude.



Think about the Antarctic circle, the zenith angle is 23.5- -66.5=90…. The elevation angle is its complement, so it is zero. For test be able to compute some of these simple combinations

Solar Declination Angle

The Solar Declination is the Angle between the Vector of Incoming Solar Rays and the Vector Normal to the Equator

It arises because the Axis of the Earth is Tilted 23.5 Degrees

The Solar Declination Angle Varies Seasonally and Explains the Seasons

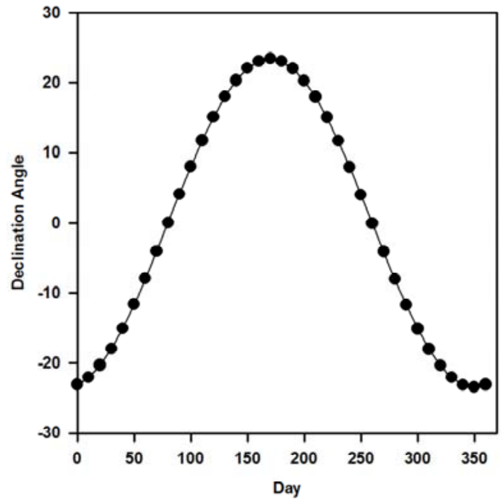
The Solar Declination Angle is Zero during the Spring and Autumn Equinox

The Solar Declination Angle is +23.5 degrees during the Summer Solstice

The Solar Declination Angle is – 23.5 Degrees during the Winter Solstice

Summary of key alignments

Seasonal variation in the Solar Declination Angle



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The seasonal variation in the solar declination angle follows a sine wave as a function of day of year

Solar Declination Angle, radians

$$\delta = -23.45 \frac{\pi}{180} \cdot \cos(\frac{2\pi(D+10)}{365})$$

D, day of year

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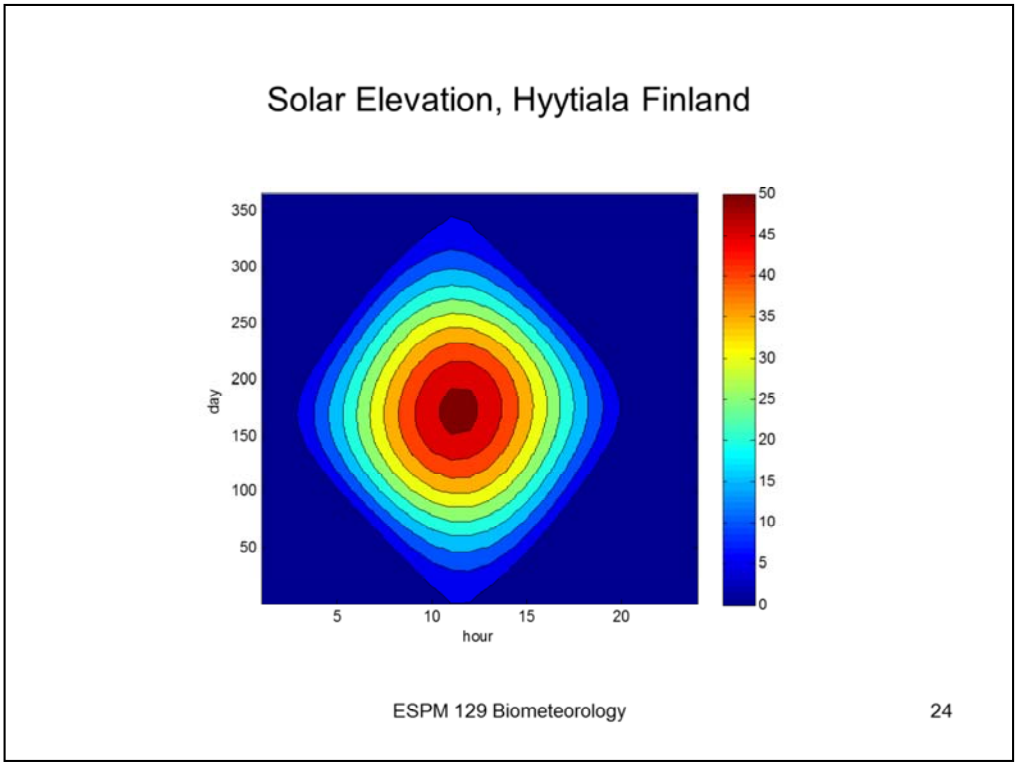
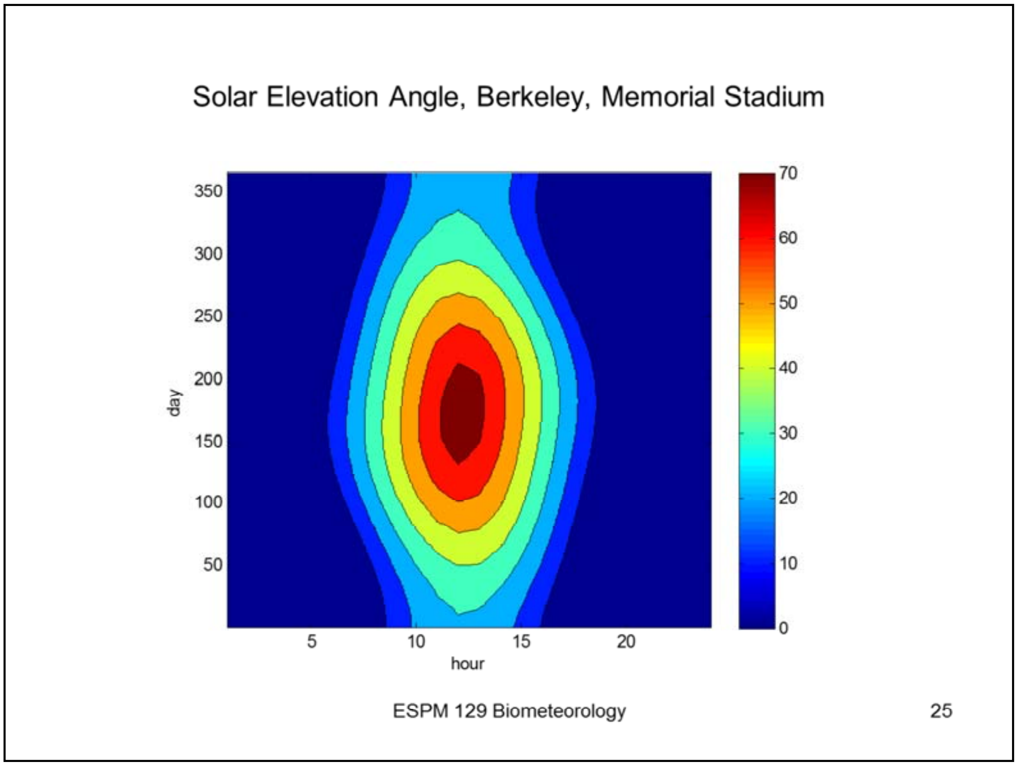
Here is a simple function. Note its units are in terms of radians. There is a 10 day offset due to our calendar, which is not aligned perfectly with the solstices. Don't use Julian day. This is based on a different astronomical calendar.

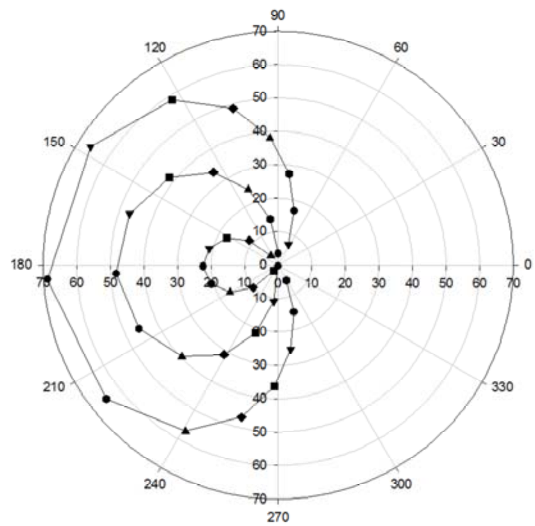


With simple matlab code we can compute the angle of the sun at any location on earth for any day or time. Here is a field site I have visited in Finland. It is way north, almost at 62 degrees. In winter the days are very short, the sun rises late, after 10, and sets early, by 14, with sun angles hovering near the horizon. During summer the days can be quite long and sunrise very early



In Berkeley we live at a more intermediate latitude. Here is the fingerprint map of sun angles.

Solar elevation and azimuth angles at Sisters, OR (44° N) for summer and winter solstices and the equinoxes



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You can also calculate the solar azimuth in the sky.

Time of Sunrise/Sunset

$$\sin(\lambda)\sin(\delta)+\cos(\lambda)\cos(\delta)\cos(h)=0$$

$$\cos(h)=-\tan\lambda\tan\delta$$

$$t_{sunset}=t_{12}+\frac{h}{15}\frac{180}{\pi} \qquad t_{sunrise}=t_{12}-\frac{h}{15}\frac{180}{\pi}$$

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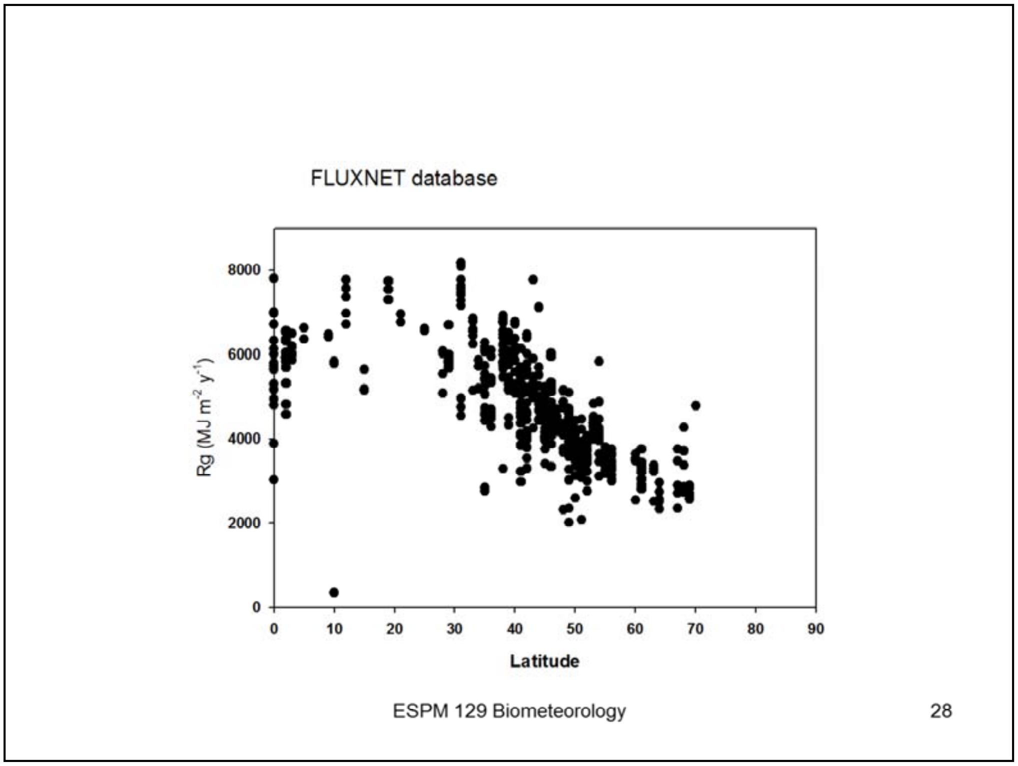
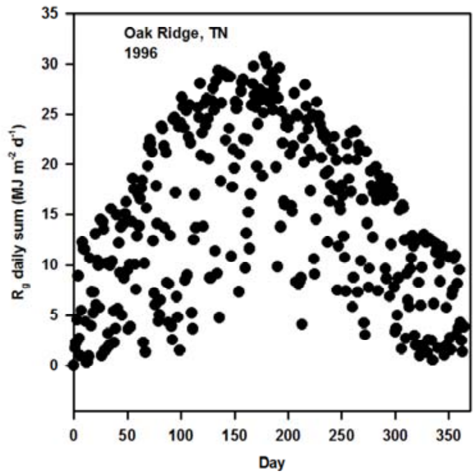
With the equation of spherical geometry for solar elevation, we can manipulate it and compute sunrise and sunset times. Set the equation to zero and solve for hour angle, then make the conversion to clock time and convert from radians to hours. Remember dividing the rotating planet into 24 hours produces a rotation of 15 degrees per hour.



How does solar radiation, integrated over a year, vary with latitude? Where is solar radiation maximal? At the Equator where the sun is highest, or elsewhere? Where and why?

Magnitude and Seasonality in Solar Radiation, Rg



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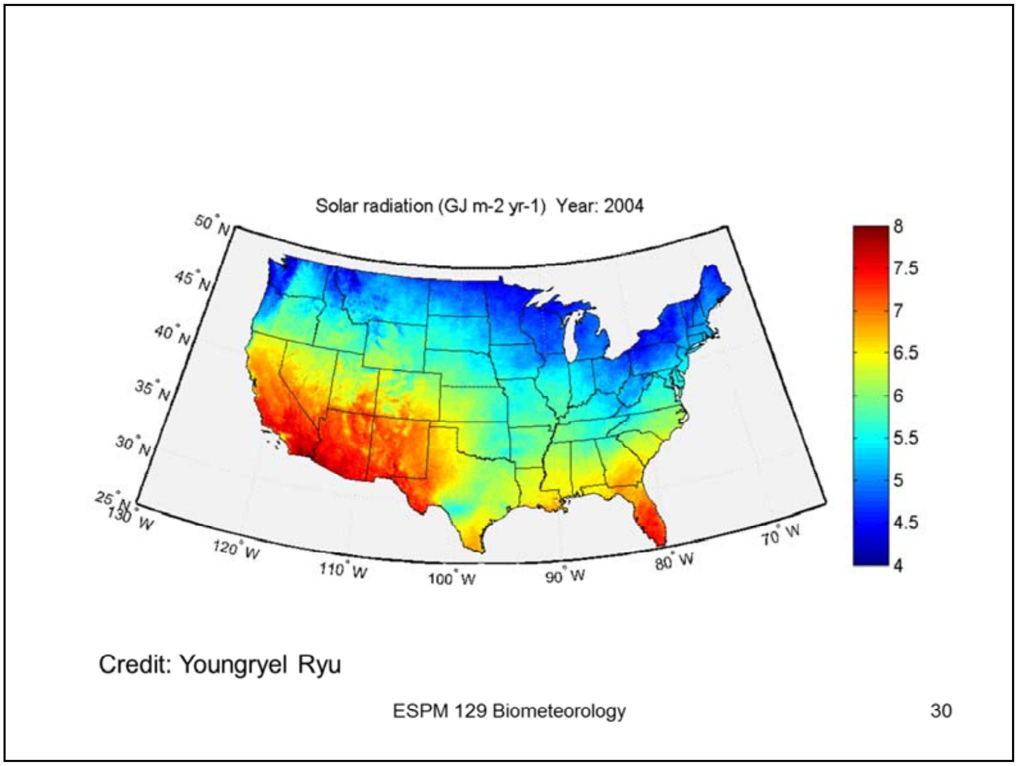
Here is the seasonal course of daily integrated solar radiation at Oak Ridge Tennessee, note the upper envelope for clear days and the effect of clouds.



Using satellite data we were able to create a map, at 1 km resolution, of solar radiation across the US. This is good for siting solar panels, biofuel crops and energetics. Also shows where it is sunniest and cloudiest. Cornell is even cloudier than Harvard.

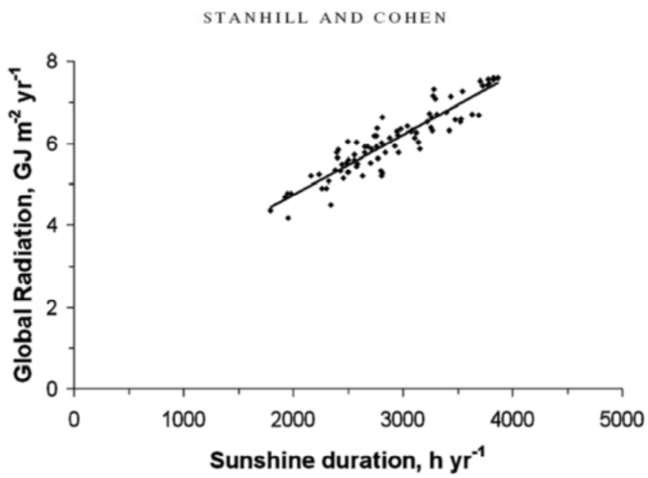


FIG. 1. Relationship of annual totals of global irradiance (GJ m^{-2}) and sunshine duration (h) measured at 26 sites in the United States between 1977 and 1980.

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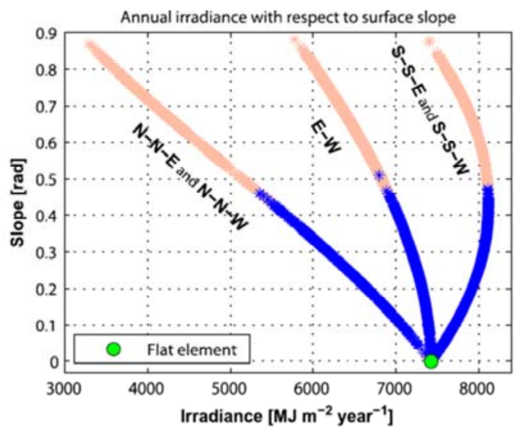
Solar flux density scales with sunshine hours.



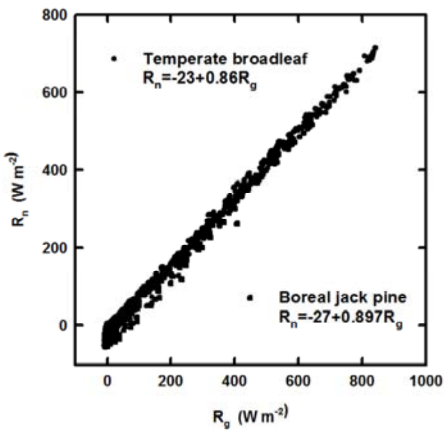
Ivanov et al 2008 WRR

Here we can visualize slope effects

How Does Energy Availability Compare with Energy Use?

- US Energy Use: 105 EJ/year
  - 10^{18}J per EJ
  - US Population: 300 10^{6}
  - 3.5 10^{11} J/capita/year
- US Land Area: 9.8 10^{6} km^2=9.8 10^{12} m^2 = 9.8 10^{8} ha
- Energy Use per unit area: 1.07 10^{7} J m^{-2}
- Potential, Incident Solar Energy: 6.47 10^{9} J m^{-2}
  - Ione, CA
- Assuming 20% efficient solar system
  - 8.11 10^{10} m^2 of Land Area Needed (8.11 10^{5} km^2, the size of South Carolina)

R_n as a f(R_g)



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Net radiation may not be measured as often as solar radiation. So weather generator algorithms can be used based on correlations between one another. Often the correlation is tight, but the slope and intercept can and will vary by functional grouping.

Rn = a + b Rg

intercept (W m-2)	slope	vegetations	source
-73.85	0.787	Zea mays	Davies and Idso (1979)
-29	0.842	boreal forest	Dubuyah
-27	0.897	*Pinus banksiana*	ddb + CV
-23	0.860	deciduous forest	ddb +cv
-66	0.86	*Picea sitchensis*	Jarvis et al. (1976)
-110	0.87	*Pinus taeda*	Jarvis et al. (1976)
-38	0.91	*Pinus sylvestria*	Jarvis et al. (1976)
-63 to -124	0.655 to 0.808	wheat	Denmead (1976)
-91	0.72	grassland	Ripley and Redman (1976)
-60	0.85	Ponderosa pine, OR	Anthoni (unpublished

Here are some regression stats from the literature. The fluxnet database gives one the opportunity to compute these for more locations

Summary

- Lamberts cosine law describes the amount of radiation received on the horizontal relative to the sun's zenith angle.
- The amount of radiation received on an inclined surface is a function of the angle between the sun the that projection normal to the surface.
- The axis of rotation of Earth is tilted 23.5 degrees
- The amount of radiation received is a function of its latitude and the sun's declination angle.
- Seasons arise as the Earth revolves around the sun because the Earth's tilted rotation axis causes the shaded and sunlit faces of Earth to change.
- Radiation fluxes can be estimated empirically with weather variables like temperature and humidity