

## Solar Radiation Transfer through Vegetation, part I

- Beam Radiation transfer through Ideal canopies
  - Beer-Bouguer's Law
  - Poisson Probability Distribution
  - The G function, the Leaf-Sun Direction Cosine
  - Gap Distributions
- Beam Radiation transfer through non-ideal canopies
  - Penumbra
- Diffuse Radiation



9/14/2016

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In this lecture we will study how incident solar radiation interacts with vegetation

## Primary Roles of Sunlight in Biomet

- Photosynthesis
- Stomatal conductance
- Transpiration
- Leaf Energy Balance
  - leaf temperature
    - Respiration
    - VOC volatilization
    - Photosynthesis
    - Saturation vapor pressure at leaf surface
    - Stable Isotope

Why should we care about light? Here is a short list of key biometeorological processes that depend upon the flux density of sunlight

## Secondary Roles of Sunlight

- Plant growth
- Seedling regeneration
- Vertical structure and crown shape of forest stands
- Phenology
- Leaf morphology
- Uptake and emission of trace gases that participate in biogeochemical cycling and atmospheric chemistry.

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There are also indirect roles, too.

## Light transmission through a Vegetation



- Beam Radiation penetrating through gaps in the foliage in the direction of the sun
- Diffuse radiation penetrating through gaps in the foliage in the direction of the sky hemisphere
- Complementary radiation generating by the reflection and transmission of light by leaves

Pt. Reyes National Seashore, Allomere Falls Trail, August 2002

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First, have a qualitative understanding of the paths and routes of light transfer through vegetation. There are 3 key flows of photons.

Radiative transfer through vegetation is a function of:

- incident radiation;
- the optical properties of the leaves and stems;
- the optical properties of the underlying ground surface or litter;
- the architecture of the stand (which includes, leaf area index, leaf angle distribution and the spatial dispersion of leaves, e.g. random, clumped or regular).

This slide gets back to some of the basics we have started to study, and why. Leaf angles, leaf area, sun angles, all play a role in the capture and transmission of photons through foliage space.

## Sources of Heterogeneity in Sunlight

- clumping and gapping of foliage,
- gaps in canopy crowns due to treefall or cultivation practices,
- spatial variations in leaf orientation angles,
- penumbra
- leaf flutter
- clouds
- topography
- seasonal trends in plant phenology
- seasonal and diurnal movement of the sun
- directional and non-isotropic and wavelength dependent scattering of light.

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Light fields in a vegetated canopy can be highly variable. Why? What are the sources of such variability?

**Gap Fraction, Probability of Beam Penetration,  $P_0$**

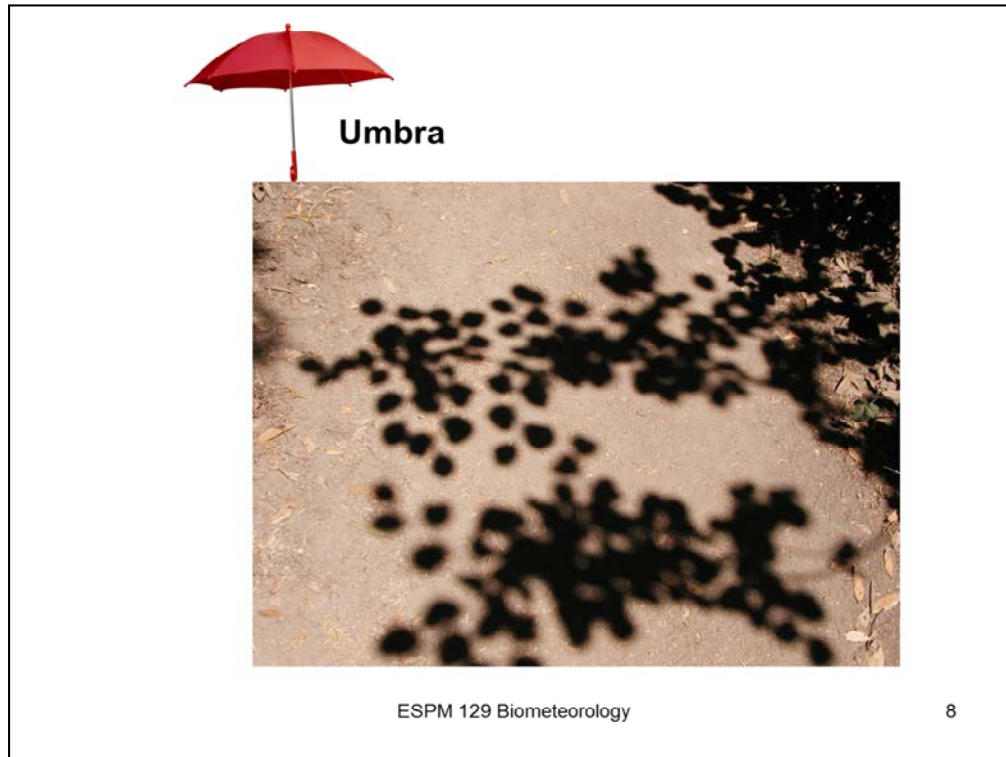


Patterns of Sunflecks, Umbra and Penumbra

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To understand light transfer, let's start with a statistical approach and focus on gap fraction, e.g. the probability of beam penetration. In between gaps you see shadows. But some are more distinct than others. Some are full shade, umbra, like umbrella. Others are partial shade, called penumbra.

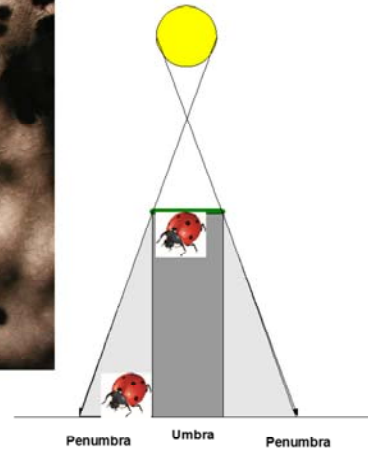


Umbra forms because the projected size of the leaf is greater than the projected size of the sun, relative to an 'observer', in this case the ground.



## Umbra and Penumbra

Solar Radius, viewed from Earth:  
0.533 Degrees



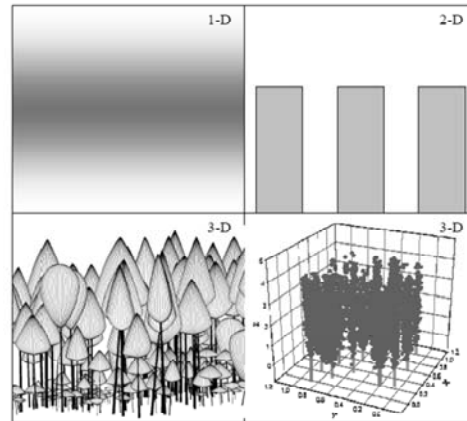
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This cartoon shows how penumbra and umbra form. The lady bug under the disc, or leaf, is in full shade. From her vantage point she cannot see the sun's disc. If she is further away, the disc does not cover the solar disc from her vantage point. Here she enters a region of partial shade.

## Canopy Representation

Cescatti A. and Ninemes U. - Light harvesting: from leaf to landscape -



While we have discussed several types of canopy presentation in the early lecture, here we will focus on the simple case that a canopy is a turbid medium. Leaves are displayed randomly in space, but they may have a preferred leaf angle inclination distribution.

- Probability of Beam Penetration,  $P_0$ , through  $n$  layers of leaves with an incremental amount of leaf area,  $a$ , relative to the total amount of area,  $A$

$$P_0 = \left(1 - \frac{a}{A}\right) \cdot \left(1 - \frac{a}{A}\right) \cdot \left(1 - \frac{a}{A}\right) \dots$$

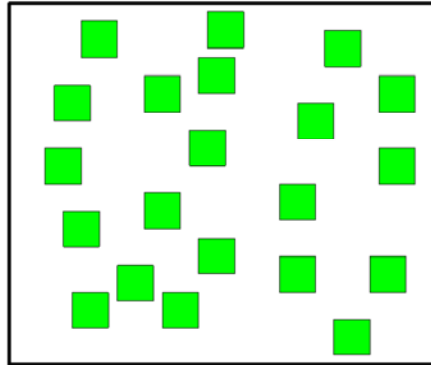
$$P_0 = \left(1 - \frac{a}{A}\right)^n$$

First we want to compute the probability of zero, 0, contacts. The probability of gap through a layer is one minus the ratio of the total area leaves,  $a$ , in a layer, divided by the total area,  $A$ . As one travels into the canopy from the top into successive layers, the problem is multiplicative.

Probability of Zero Contacts,  $P_0$

$A = 40 \text{ by } 48 = 1920$

$a = 16 \times 21 = 336$

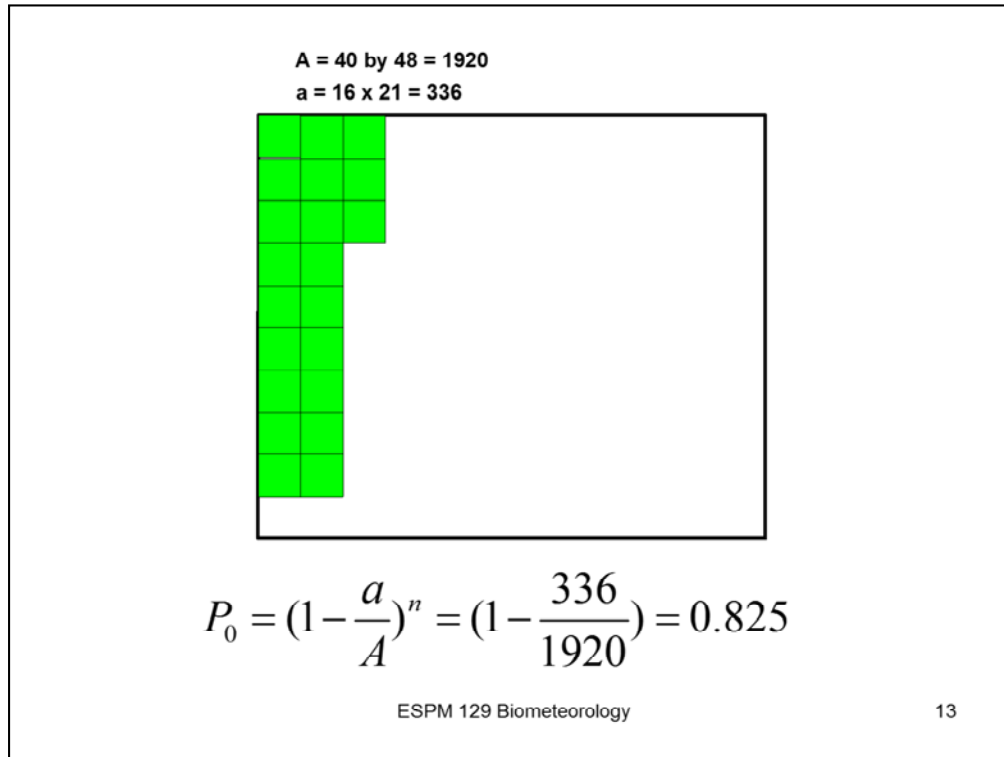


$$P_0 = \left(1 - \frac{a}{A}\right)^n = \left(1 - \frac{16}{1920}\right)^{21} = 0.838$$

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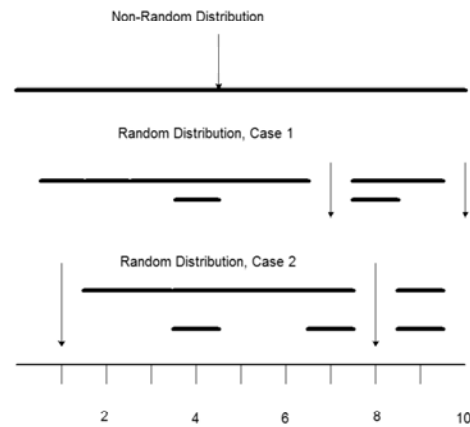
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Lets look at this as a game with 21 tries. At each one we place a square randomly on the table. Its area is 16. The size of the domain is 40 b 48. Work the sums and the probability of gap is 0.838



Lets collect the squares, add them up and compare with the total area. Here we get a very similar value, 0.825.

Conceptual visualization of light transfer through a randomly spaced medium



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This cartoon helps us visualize how we can stack more on one meter square of leaves per meter square of ground.



## Beer's Law, v1.0

Limit n goes to infinity

$$P_0 = \left(1 - \frac{a}{A}\right)^n$$

$$P_0 = \exp(-Na/A)$$

$L = \text{leaf area index} = aN/A$

$$P_0 = \exp(-L)$$

$I$  is the flux density of sunlight for a given wavelength

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Point here is take the limit of  $P_0 = 1 - a/A$  to infinity and we find  $P_0$  is an exponential function of  $L$ , leaf area index. This case assumes the beam of light is directly overhead and leaves are flat.

## Beer's Law, v2.0



Change in Light flux density,  $I$ , through a path  $x$  is a function of its current intensity,  $I$ , and its extinction coefficient,  $k$ , and leaf area density,  $a$ .

$$\frac{dI}{dx} = -k \cdot a \cdot I$$

Note the negative sign, indicating  $I$  decreases with attenuation

We can also use a simple differential equation to derive how and why Beers Law is an exponential function of leaf area index. The change in light flux density,  $I$ , with distance,  $x$ , is a function of its current state,  $I$ , times the leaf area density and an extinction coefficient.



### Beer's Law, cont.

Re-arrange derivatives  $\frac{dI}{I} = -k \cdot a \cdot dx$

Integrate both sides  $\int \frac{dI}{I} = \ln\left(\frac{I(x)}{I(0)}\right) =$

$\int \frac{dx}{x} = \ln(x) =$   $\int -k \cdot a \cdot dx = -k \cdot a \cdot x$

Simplify by taking exp of both sides  $\exp\left(\ln\left(\frac{I(x)}{I(0)}\right)\right) = \exp(-k \cdot a \cdot x)$

$$\frac{I(x)}{I_0} = \exp(-k \cdot a \cdot x)$$

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Take the integral and you end up with Beer's Law

Beer's Law, cont.

$$I(L) = I_0 \exp(-k \cdot L)$$

Leaf area density,  $a$ ,  
times pathlength,  $x$ , equals  
Leaf area index,  $L$

How Much Leaf Area intercepts 99% of incident light?

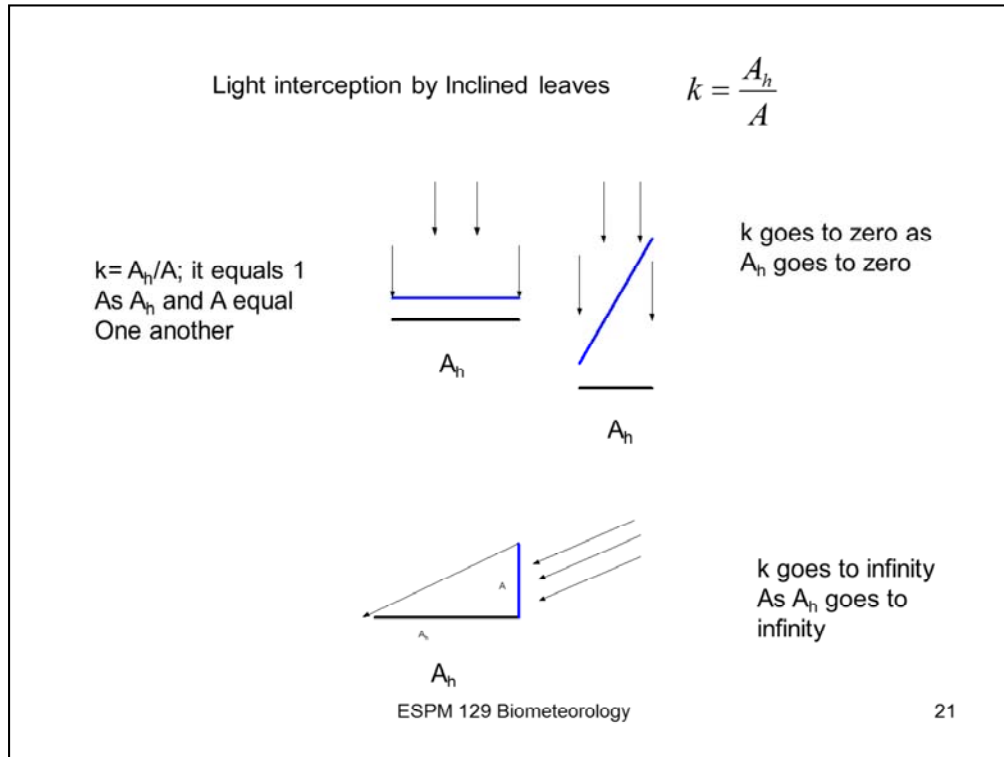
$$\ln\left(\frac{I(L)}{I_0}\right) = \ln(0.01) = -kL$$

If  $k = 1.00$ , then  $L = 4.6$

The extinction coefficient,  $k$ , equals the fraction of hemi-surface leaf area ( $A$ ) that is **projected onto the horizontal** ( $A_h$ ), from a particular zenith angle.

$$k = \frac{A_h}{A}$$

Now let's deconstruct the extinction coefficient,  $K$ . In principle it is the ratio of the projection of a leaf on the horizontal,  $A_h$ , relative to the area of that leaf,  $A$ .



These are important limits I want you to know. Under what situations is  $k$  big, little, equal to one. You also need to realize that  $k$  is a function of the orientation of the leaf relative to the light source. So  $k$  is not constant.

Define K in terms of Sun and Leaf Inclination Angles

$$\frac{A_{horiz}}{A_{leaf}} = \frac{\cos \alpha_{leaf \perp sun}}{\sin \beta_{sun}} = K = \frac{G}{\sin \beta_{sun}} = \frac{G}{\cos \theta_{sun}}$$

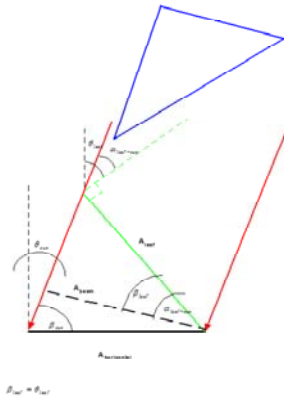
$\alpha$ , angle between leaf normal  
And solar zenith,  $\theta$

$\alpha_{leaf \perp sun}$

$\beta_{sun}$ , solar elevation angle

k, extinction coefficient

G, G-function or mean direction cosine



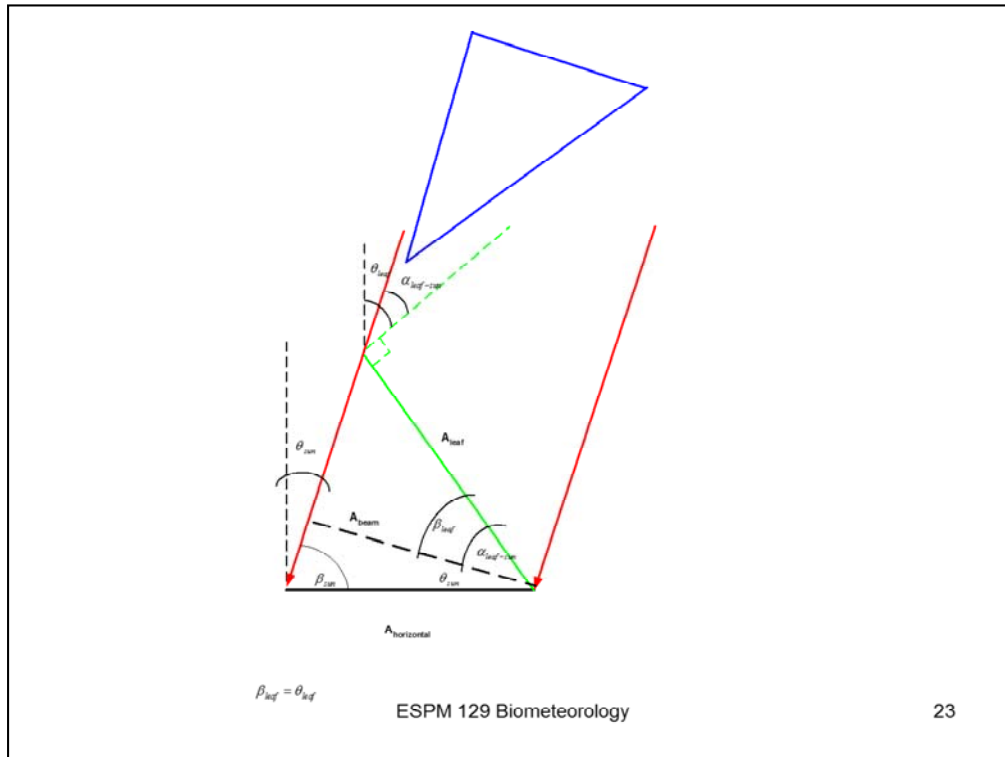
$\cos \alpha_{leaf \perp sun}$

$\beta_{sun} = \theta_{sun}$

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To apply these ideas for more general canopies, we prefer to deconstruct k even further. Classic books by authors like Ross show k is related to the ratio of the cosine of the angle between the leaf normal and the solar beam, divided by the cosine of the solar zenith angle.



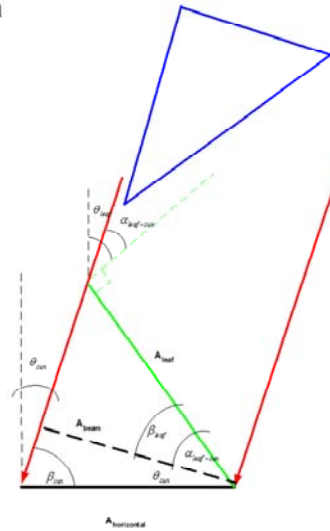
Here are the set of angles..lets step forward

1. Project the Area of a Leaf,  $A_{leaf}$ , onto the Area Normal to the Solar Beam,  $A_{beam}$

$$\frac{A_{beam}}{A_{leaf}} = \cos \alpha_{leaf \perp sun}$$

2. Project the Area of The Sun's Beam ( $A_{beam}$ ) onto the horizontal ( $A_{horiz}$ ).

$$\frac{A_{beam}}{A_{horiz}} = \cos \theta_{sun} = \sin \beta_{sun}$$



$$\beta_{sun} = \theta_{sun}$$



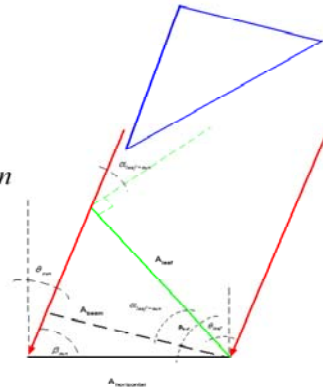
# Algebraic Manipulation

$$A_{beam} = A_{leaf} \cos \alpha_{leaf \perp sun}$$

$$A_{beam} = A_{horiz} \cos \theta_{sun}$$

$$A_{leaf} \cos \alpha_{leaf \perp sun} = A_{horiz} \sin \beta_{sun}$$

$$\frac{A_{horiz}}{A_{leaf}} = \frac{\cos \alpha_{leaf \perp sun}}{\cos \theta_{sun}} = \frac{\cos \alpha_{leaf \perp sun}}{\sin \beta_{sun}}$$



*Voilà'*

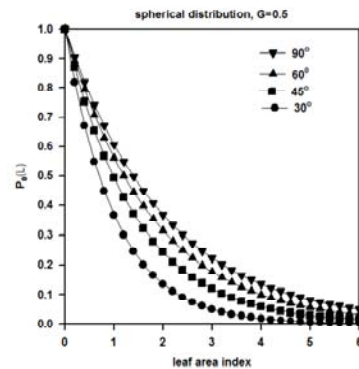


$$\kappa = \frac{A_{horiz}}{A_{leaf}} = \frac{\cos \alpha_{leaf \perp sun}}{\cos \theta_{sun}} = \frac{G}{\cos \theta_{sun}} = \frac{G}{\sin \beta_{sun}}$$

So I want to show you these equations are Not magic. I also want to give you confidence that you can tackle problems like this with simple thinking, a bit of algebra and trigonometry.

Probability of Beam Penetration or  
the Poisson Probability of Zero Contacts,  $P_0$

$$P_0 = \exp\left(-\frac{LG}{\cos \theta_{sun}}\right) = \exp\left(-\frac{LG}{\sin \beta_{sun}}\right) = \exp(-kL)$$



L: leaf area index  
G: direction cosine between sun  
and leaf normal  
 $\beta$ : solar elevation angle  
 $\theta$ : Solar zenith Angle  
k: extinction coefficient

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This expression is an exponential function of leaf area index and an extinction coefficient.

G function for Plant Canopies

$$G(\theta, \varphi) = \int_0^{2\pi} \int_0^{\pi/2} g'(\theta_l) g''(\varphi_l) |\cos(\vec{n} \cdot \vec{n}_l)| d\theta_l d\varphi_l$$

$$\cos(\vec{n} \cdot \vec{n}_l) = \cos \alpha = \cos \theta_{sun} \cos \theta_{leaf} + \sin \theta_{sun} \sin \theta_{leaf} \cos(\phi_{sun} - \phi_{leaf})$$

Note Similarity to Equation used to calculate the Sun Angle on Earth!

$\cos(\phi_{sun} - \phi_{leaf})$  Leaf Azimuth angle, relative to Sun,  
Equivalent to the cosine of the hour angle

$\theta_{sun}$  Solar Elevation angle, Relative to Zenith

$\theta_{leaf}$  Leaf Inclination angle, Relative to Zenith

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In real canopies there is a distribution of leaves, so we have to apply statistical weighting of the cosine between the leaf normal and the sun angle. Note we invoke the equation of light on a spherical surface again, but apply it to leaf angles instead of latitude and longitude (hour angles).

# G function for Plant Canopies

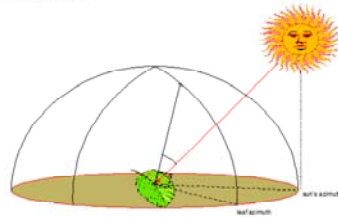
$$G(\theta, \varphi) = \int_0^{2\pi} \int_0^{\pi/2} g'(\theta_l) g''(\varphi_l) |\cos(\vec{n} \cdot \vec{n}_l)| d\theta_l d\varphi_l$$

$$\int_0^{2\pi} g''(\varphi_l) d\varphi_l = 1$$

Weight by Azimuth Angle Probability Distr

$$\int_0^{\pi/2} g'(\theta_l) d\theta_l = 1$$

Weight by Leaf Elevation Angle Dist



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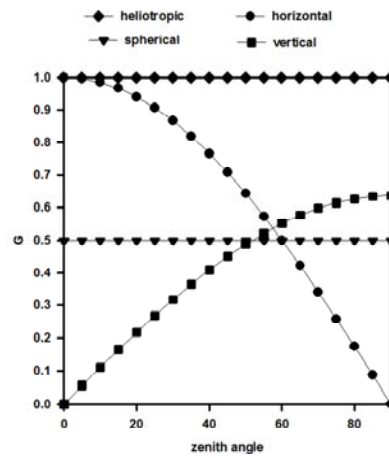
### General Functions for Different Leaf Orientations

Leaf Angle Distribution	G, direction cosine	K, extinction coefficient
Horizontal	$\cos(\theta)$	1
Vertical	$2/\pi \sin(\theta)$	$2 \tan(\theta/\pi)$
Conical	$\cos(\theta) \cos(\theta_0)$	$\cos(\theta)$
<b>Spherical or random</b>	0.5	$1/(2 \cos(\theta))$
Heliotropic	1	$1/\cos(\theta)$
Ellipsoidal	*	**

$$\frac{A_{horiz}}{A_{leaf}} = \frac{\cos \alpha}{\sin \beta_{sun}} = \kappa = \frac{G}{\sin \beta_{sun}} = \frac{G}{\cos \theta_{sun}}$$

What is the good news? Despite all these complications of Geometry, G can be as simple as 0.5 for canopies with a spherical leaf angle distribution. It is a pretty good assumption if you don't know other properties.

## G functions for different leaf inclination angle distributions



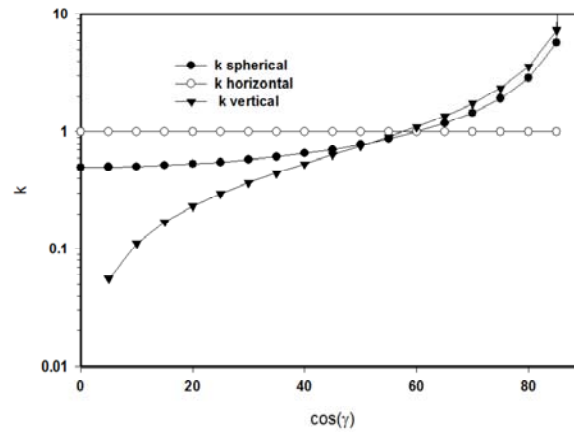
All G functions Converge on  $G=0.5$  at zenith angle equal 1 Radian!

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The other interesting lesson is that all these leaf angle distributions converge when the zenith angle is 1 radian.

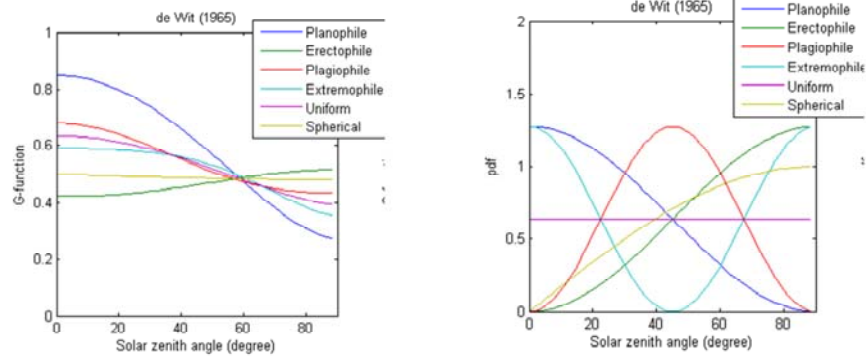
## Extinction coefficients with different solar zenith angles



$K$  converges to 1 when zenith angle equals 1 Radian,  $180/\pi$



## Family of G-Functions

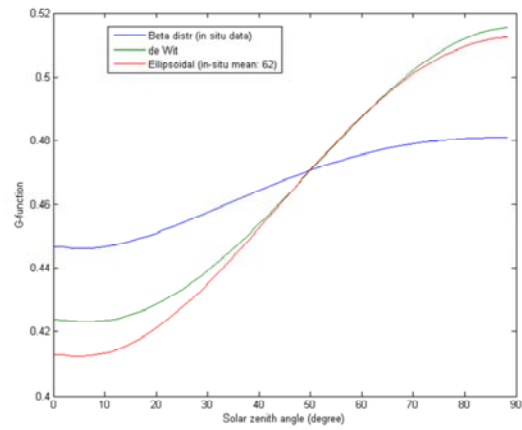


Computed Y. Ryu

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## G-Function for Oak Savanna

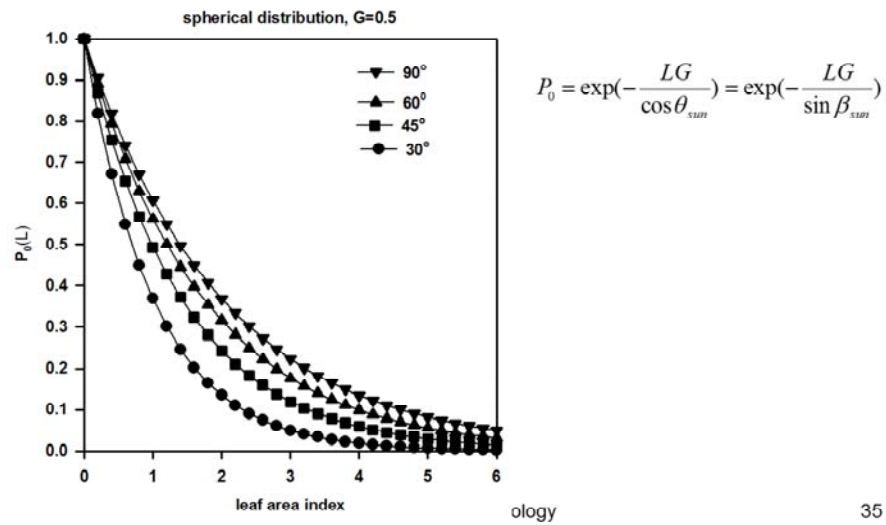


Data of Ryu, Sonnentag, Vargas, 2008

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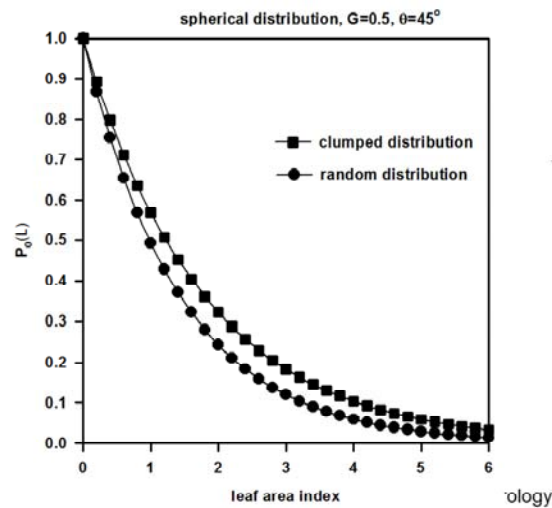
# Sun Angles and the probability of beam penetration, $P_0$



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Now we have theory in hand, we can inspect the attenuation of sunlight with different sun angles

Probability of beam penetration with clumped and randomly distributed foliage



$$P_0 = \exp\left(-\frac{L G \Omega}{\sin \beta}\right)$$

$\Omega$ , clumping coef

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One more complication arises. A group of us studying light transmission are finding that leaves are clumped and this enhances beam transmission. So we have to work and study clumping more..

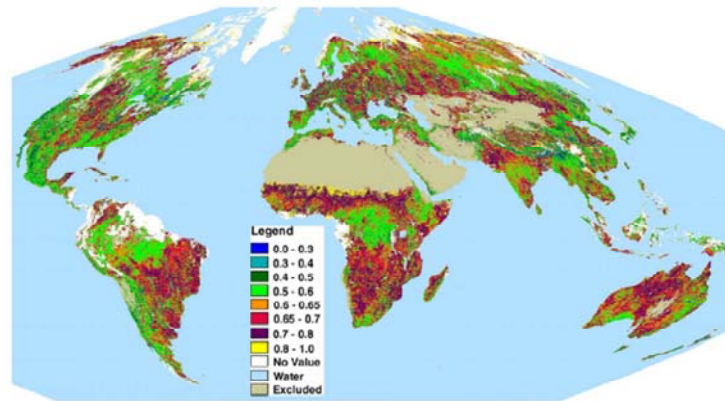


Fig. 6. Global vegetation clumping index map derived from POLDER 1 data using the normalized difference between interpolated hotspot and darkspot NIR reflectance and applied to vegetated land cover. Vegetation clumping increases with decreasing values of the index.

Chen's group attempted to produce a global map of clumping factors from satellite data. May not be perfect and may have scale issues, but it is a start on this conversation.

### Clumping Factors

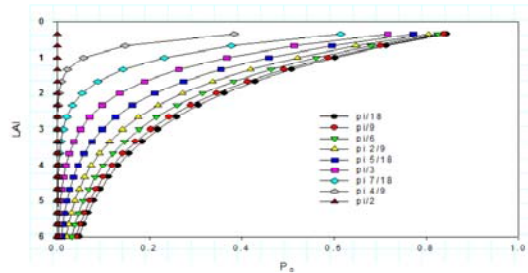
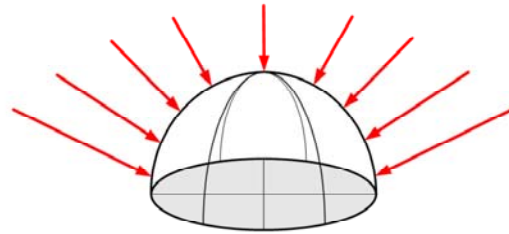
Class	Class names	Min	Max	Mean
1	Tree Cover, broadleaf, evergreen	0.59	0.68	0.63
2	Tree Cover, broadleaf, deciduous, closed	0.59	0.79	0.69
3	Tree Cover, broadleaf, deciduous, open	0.62	0.78	0.70
4	Tree Cover, needleleaf, evergreen	0.55	0.68	0.62
5	Tree Cover, needleleaf, deciduous	0.60	0.77	0.68
6	Tree Cover, mixed leaf type	0.58	0.79	0.69
7	Tree Cover, regularly flooded, fresh water	0.61	0.69	0.65
8	Tree Cover, regularly flooded, saline water	0.65	0.79	0.72
9	Mosaic: Tree Cover / Other natural vegetation	0.64	0.82	0.72
10	Tree Cover, burnt	0.65	0.86	0.75
11	Shrub Cover, closed-open, evergreen	0.62	0.80	0.71
12	Shrub Cover, closed-open, deciduous	0.62	0.80	0.71
13	Herbaceous Cover, closed-open	0.64	0.83	0.74
14	Sparse herbaceous or sparse shrub cover	0.67	0.84	0.75
15	Reg. flooded shrub and/or herbaceous cover	0.68	0.85	0.77
16	Cultivated and managed areas	0.63	0.83	0.73
17	Mosaic: Cropland / Tree Cover / Natural veg	0.64	0.76	0.70
18	Mosaic: Cropland / Shrub and/or grass cover	0.65	0.81	0.73
19	Bare Areas	0.75	0.99	0.87

Chen et al 2005 RSE

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### Light Penetration is Different from Each Sky Sector



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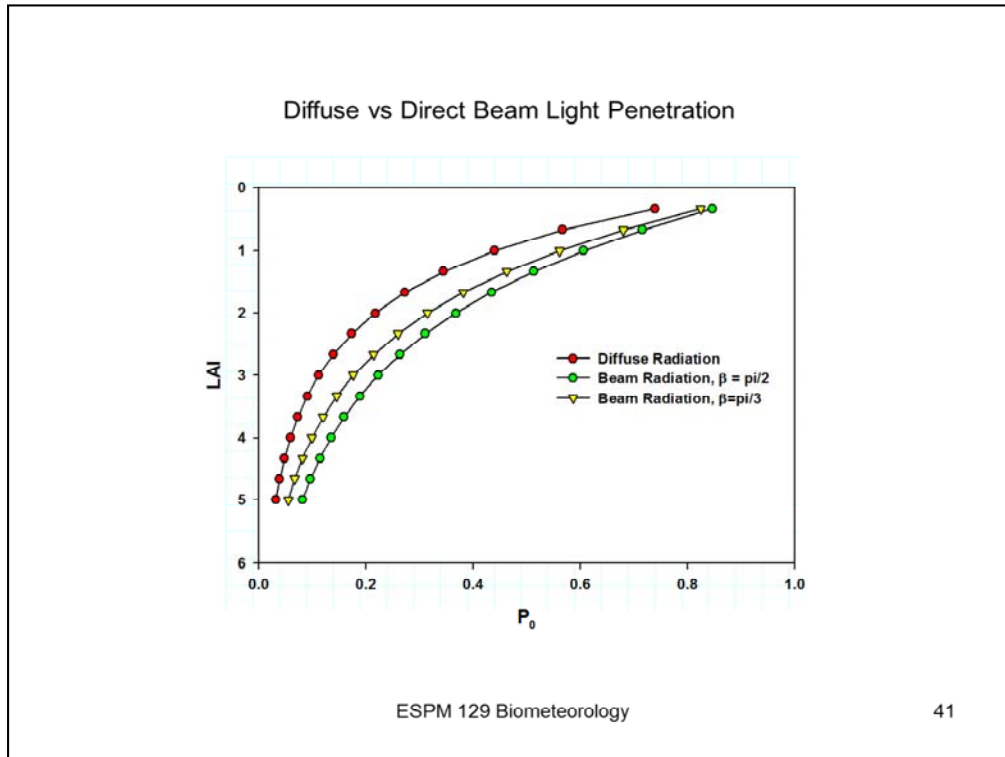
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How does diffuse light penetrate through a canopy? Different angles have different degrees of penetration.

## Hemispherical Diffuse light in a canopy

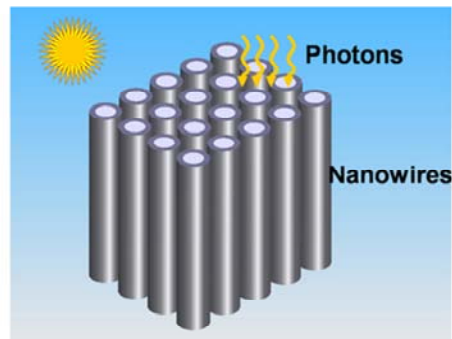
$$P_{diffuse} = 2 \int_0^{\pi/2} P_0 \cos \theta \sin \theta \cdot d\theta$$





In many instances a canopy is more effective in capturing diffuse light.

### Ideas for Better Solar Panels



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If we want to develop better solar panels we should learn and apply lessons from nature. Plants are the best solar collectors and they have several hundred years of evolution to do it well and better. They don't establish canopies with LAI of one, so why should solar panels. Some engineers are starting to use nano wires to make surfaces that capture photons better.

## Summary

- The flux density of light energy received at a particular location inside a plant canopy consists of **beam** and **diffuse solar radiation** that **penetrates through gaps** in the canopy. It also contains **complementary radiation** that is generated by the **interception** and the consequent, (wavelength-dependent) **transmission** through leaves and **reflection** by leaves and soil.
- Sources of spatial variation of light in a canopy include: clumping and gapping of foliage; gaps in canopy crowns due to treefall or cultivation practices; spatial variations in leaf orientation angles; penumbra; leaf flutter; clouds, and directional and non-isotropic and wavelength dependent scattering of light and topography;
- Sources of temporal variation of radiation include: topography; seasonal trends in plant phenology; seasonal and diurnal movement of the sun.
- The extinction coefficient,  $k$ , is defined as the ratio between the G function (the cosine between the solar zenith and the mean leaf normal angle) and the sine of the solar elevation ( $k=G/\sin\beta$ )
- Clumping of leaves enhances the probability of beam transmission



# Inverting LAI from Radiation Transfer Measurements

$$P_0 = \exp(-kL)$$

$$\overline{\ln(P_0)} \neq \ln(\overline{P_0})$$

$$L = 2.996$$

$$[P_0]=0.05$$

$$[\ln(P_0)]=-2.996$$

$$\ln[P_0]=-2.996$$

$$L^*=2.996$$

$$L = 1.497$$

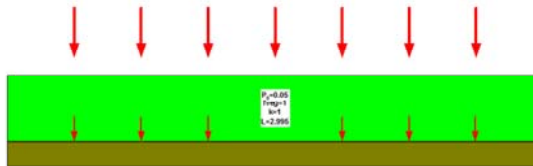
$$[P_0]=0.525$$

$$[\ln(P_0)]=-1.497$$

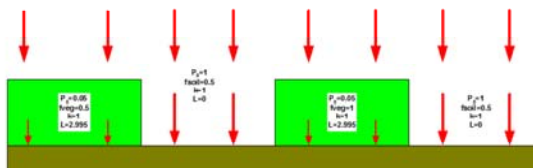
$$\ln[P_0]=-0.644$$

$$L^*=0.644$$

$$L = -\frac{\overline{\ln(P_0)}}{k} = -\frac{\ln(\overline{P_0})}{k}$$



$$L = -\frac{\overline{\ln(P_0)}}{k} \neq -\frac{\ln(\overline{P_0})}{k}$$

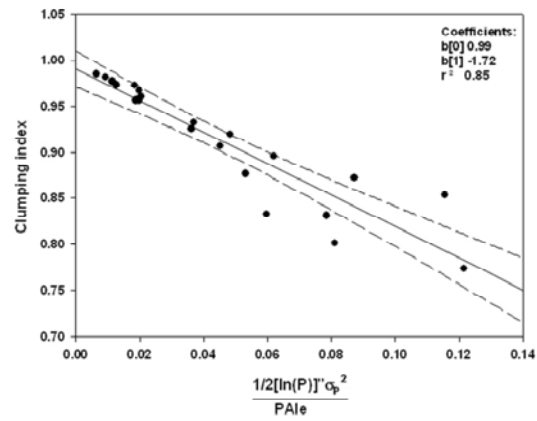


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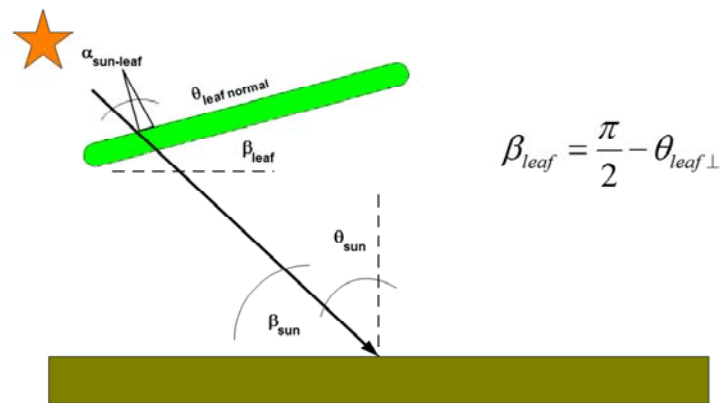
## Heterogeneous Canopies, Clumping and Radiative Transfer

$$\overline{\ln(P_0)} = \ln(\overline{P_0}) + \frac{1}{2} \ln(\overline{P_0})'' \sigma_{P_0}^2$$



Youngryel Ryu, unpublished

## Leaf-Sun Angle Definitions



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This cartoon shows the key angles of interest and their compliments.