Lecture 10 Solar Radiation Transfer Through Vegetation, Part 1: Theory

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Lecture Topics

1. Beam Radiation transfer through Ideal canopies
    - A. Beer-Bouguer's Law
    - B. Poisson Probability Distribution
    - C. The G function, the Leaf-Sun Direction Cosine
    - D. Gap Distributions

2. Beam Radiation transfer through non-ideal canopies
    - A. Penumbra

3. Diffuse Radiation

L10.1 Introduction.

Solar energy is the prime source of energy for work performed by plants and the ecosystem. Assessing the radiation field in the vicinity of leaves and the soil is complicated by its heterogeneity in time and space. When tramping through the woods or a field an observant hiker will notice a plethora of bright and dark light patterns throughout the vegetation and on the ground. If one decides to site on a rock under the forest in a shaft of light, one will notice how this shaft will move and soon you may be sitting in shade again. The shape, lifetime, and amount of radiation in these sun and shade light patches are of great importance to plants and the local environment and it the focus of this lecture.

Light energy directly drives many fundamental plant and biophysical processes (photosynthesis, stomatal conductance, transpiration, leaf temperature, respiration).

Light energy also directly/indirectly influences many secondary plant processes. A list of these processes includes:

1) plant growth

2) seedling regeneration
3) vertical structure and crown shape of forest stands
4) leaf morphology
5) uptake and emission of trace gases that participate in biogeochemical cycling and atmospheric chemistry.

The physics of photon transport is explained fundamentally by Maxwell's equation, which describes electromagnetic wave theory. In the natural environment, wave information is of secondary importance, so theories derived by astrophysicists to describe photon transport (Chandreksekar, 1960; Ross, 1980) are used. Other uses of photon transport theory include neutron scattering, laser beam propagation, metallurgy. These theories are developed on the assumption that the system is plane-parallel, an optically isotropic and turbid medium.

The qualitative nature of the light environment within a plant canopy can be described with ease. The flux density of light energy received at a particular location inside a plant canopy consists of **beam** and **diffuse solar radiation** that **penetrates through gaps** in the canopy. It also contains **complementary radiation** that is generated by the **interception** and the consequent, (wavelength-dependent) **transmission** through leaves and **reflection** by leaves and soil (Lemeur, Blad, 1974; Myneni *et al.*, 1989; Ross, 1980) ;. On the other hand, it is very difficult to quantify the light environment in a plant canopy because the light environment exhibits much spatial and temporal variability. This variability is associated with structural and environmental heterogeneity on a variety of space and time scales. Key factors causing heterogeneity in the canopy light environment include:

1) clumping and gapping of foliage,
2) gaps in canopy crowns due to treefall or cultivation practices,
3) spatial variations in leaf orientation angles,
4) penumbra
5) leaf flutter
6) clouds
7) topography
8) seasonal trends in plant phenology
9) seasonal and diurnal movement of the sun
10) directional and non-isotropic and wavelength dependent scattering of light.

Figure 1 Light transmission through a forest canopy. Pt Reyes National Seashore, Allomere Falls trail, August 2002. Notice light coming from the sun and from the sky. Leaves both intercept and transmit light, as differentiated by the dark and light green patches.

Determining the existence and extent of the cited factors is a critical component of any theoretical or experimental study on radiative transfer in heterogeneous plant canopies.

Radiative transfer through plant stands is a function of:

1) incident radiation;
2) the optical properties of the leaves and stems;
3) the optical properties of the underlying ground surface or litter;
4) the architecture of the stand (which includes, leaf area index, leaf angle distribution and the spatial dispersion of leaves, e.g. random, clumped or regular).

In biometeorology we are concerned with problems involving remote sensing, leaf energy balances and photosynthesis. For the computation of photosynthesis, one can assume leaves to be dark, as scattering is negligible.

The simplest and most ideal case is the one-dimensional vertical canopy. It is considered to be horizontally homogeneous, plane-parallel and to have isotropic leaves. The detection and simulation of reflected radiation requires elaborate theories that have azimuthal and elevational dependences on scattering. In this situation, the reflected signal is a small faction of the incident signal and is directional dependent, so small errors in simulating this secondary radiation cannot be tolerated.

How radiation varies with vertical depth through vegetation and can be defined conceptually and quantified with a differential equation. The change in radiation intensity with depth into the canopy is due to the differences in gains and losses. It is a function of the **direction of the source**, the **depth in the canopy**, the **attenuation by the canopy** (k) and the **scattering** by leaf and soil elements of photons from direction Ω' into a unit solid angle of direction Ω.

In this lecture we will discuss the theories used to predict light transmission through and reflection from plant canopies. In the following lecture we will present data from field studies.

Abstracting the Canopy

There are several ways to define canopy architecture, for the purpose of computing photon transfer through vegetation. The simplest and most ideal case assumes that the vegetation is horizontally homogeneous and consists of layers of plane-parallel leaves, who spatial distribution is random and varying vertically (Figure 1a). More complicated representations of the canopy consider two dimensional arrays of foliage (Figure 1b), three-dimensional abstractions based on ideal geometrical shapes (Figure 1c) or actual data of leaf and stem placement within a stand (Figure 1d). The simplest probability models, discussed next assume a canopy is a plane-parallel turbid medium. More sophisticated models are needed for more complex canopy abstractions.

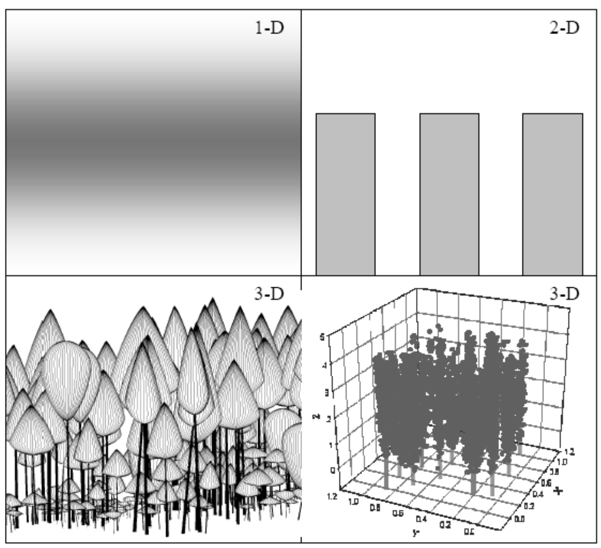


Figure 2 Different canopy abstractions for radiative transfer calculations (Cescatti, Niinemets, 2004)

L10.2 Beam Radiation Transfer through Ideal Canopies

One of the fundamental questions that is asked by a biometeorologist walking through a forest is what portion of sunlight incident at the top of a canopy reaches a given level? The inverse question pertains to "*what fraction of the ground is cast in shadow*?"

Probability statistics and differential analysis are useful tools for evaluating the transmission of light beams through foliage envelopes (Lemeur, Blad, 1974; Myneni *et al.*, 1989; Nilson, 1971). We illustrate the probability of beam penetration through a plant canopy using statistical arguments. Probability density functions for beam transmission, or interception, are classically derived by considering a plant canopy as a horizontally homogeneous turbid medium and by dividing the canopy into a number of statistically independent layers (N).

First let's consider a case with leaves that are spread uniformly, so they form a monolayer. In this case a canopy with a leaf area of one intercepts all of the light incoming from overhead.

Now lets see what happens if we distribute the leaves randomly in space. To do so, let's use a random number generator and create a set of random numbers between 1 and 10. We can then place leaves in space that correspond with their position number. Doing so produces zones with distinct gaps and with multiple leaves. If the leaves are randomly spaced, a leaf area index of one will not intercept all of the incoming sunlight.

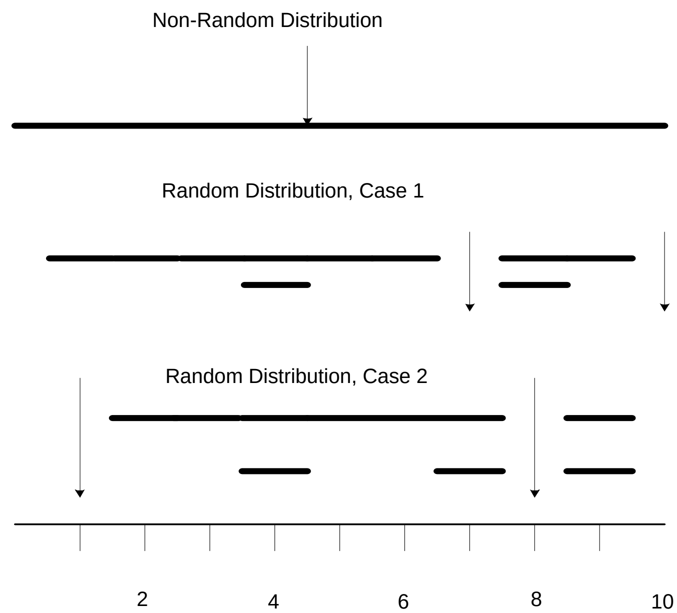


Figure 3Conceptual visualization of light transfer through a randomly spaced medium

We will show later how much leaf area is needed to intercept at least 99% of the light, when the leaves are arrayed randomly in space.

The optical thickness of a layer is defined by the ratio between the cumulative leaf area index (L) and N. The probability that a ray of light passes through a foliage layer, without interception, is a function of the layer's leaf area that is projected in the direction perpendicular to the incoming ray (Monsi, Saeki, 2005; Nilson, 1971)
Let's consider the simplest case, random and horizontal leaves spread out in a volume with a width of one unit. The area of leaves obscuring the downward directed sunbeams is **a** over some larger area **A**. So the ratio **a/A** is the probability that a beam of light passing through

a layer of foliage will intercept a leaf. The ratio **a/A** is also the shadow ratio. In converse, the probability that light will pass a layer is (1-a/A). The probability that a ray of light will pass through two layers is (1-a/A)(1-a/A). The probability that it will pass through multiple, n, layers is:

$$P_0 = (1 - \frac{a}{A})^n$$

Taking the limit as *n* goes to infinity yields:

$$I(L) = \exp(-{}^{Na}\!/_{A})$$

The equation can be expressed in terms of leaf area index, *L*, because *L =aN/A*

$$I(L) = \exp(-L)$$

This equation is valid only for sunlight coming from directly overhead and for horizontal leaves, as there is no geometric correction of inclined leaves or light source; effectively *k* equals one.

Next, let's derive an analytical relation for predicting the attenuation of light transmission using differential calculus. Light interception occurs along a photons path. The change in light flux density across a certain distance, x, is a function of the amount of leaves in the volume (a, the leaf area density, area of leaves per unit volume), the flux density of light at that position, *I*, and a proportionality constant, *k*, that describes light extinction.

$$\frac{dI}{dx} = -k \cdot a \cdot I$$

The negative sign is introduced because attenuation of light reduces the flux density. The key point to be made here is that the derivative of I with x is a linear function of I. Now we can manipulate this simple equation and compute the relative change in *I.* It is a function of the pathlength (*dx*), a proportionality constant (*k*) and the amount of attenuating material. In the case of a canopy its leaf area density (*a*); in the free atmosphere it's the concentration of trace gases or particles

$$\frac{dI}{I} = -k \cdot a \cdot dx$$

Out next step is to manipulate this equation once more to derive an analytical equation stating how I varies with x. Integrating the previous equation with respect to x yields:

$$\int \frac{dI}{I} = \ln(\frac{I(x)}{I(0)}) = -k \cdot a \cdot x$$

Further manipulation yields **Beer's Law**, the famous exponential relation for light attenuation:

$$I(x) = I_0 \, exp(-k \cdot a \cdot x)$$

I_0 is the flux density of light at the top of the canopy. For the case that the beam is directed downward from the zenith and leaf area index equals *a* times *x,* assuming *a* is constant, as is the case with a turbid medium:

$$I(L) = I_0 \exp(-k \cdot L)$$

In real canopies, we must consider the case of leaves with different angles and a non-vertical sun. The radiative transfer model assumes that foliage is randomly distributed in space and the sun is a point source. In this case the probability of beam penetration is calculated using a Poisson distribution:

$$I(L) = I_0 \exp(-k \cdot L)$$

From this equation we can compute that a leaf area index of 4.6 is needed to intercept 99% of incoming light, if k is assumed to equal 1.

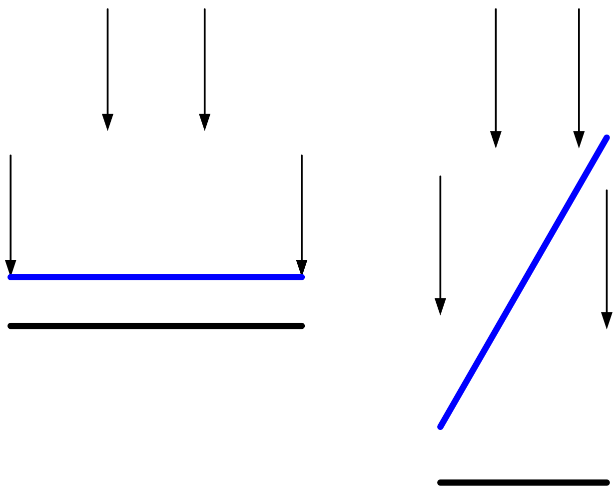
$$\ln(\frac{I(L)}{I_0}) = \ln(0.01) = -kL$$

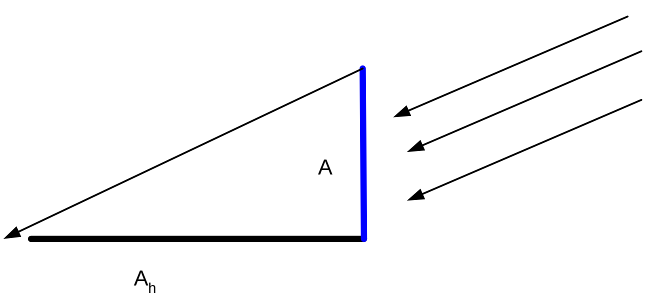
This effect clearly shows how the random positioning of leaves alters how light is intercepted by a canopy, as compared by a leaf.

Defining Extinction Coefficients

In order to compute the probability of beam penetration in foliage space, we need to assess how much foliage is intercepting the solar beam. A canopy may have a leaf area of one, but it will intercept few photons if the leaves are erect and the sun is overhead. The same feature holds for a planophile canopy at sunrise and sunset. In contrast, a planophile canopy will have a much higher probability of intercepting photons when the sun is overhead.

The extinction coefficient can be considered as the fraction of hemi-surface leaf area that is **projected onto the horizontal** *(A_h)*, from a particular zenith angle. In other words, we can equate k to the ratio between the **shadow cast by a leaf on the horizontal** and the area of the leaf (A_h/A) (Campbell, Norman, 1998; Monteith, Unsworth, 1990).



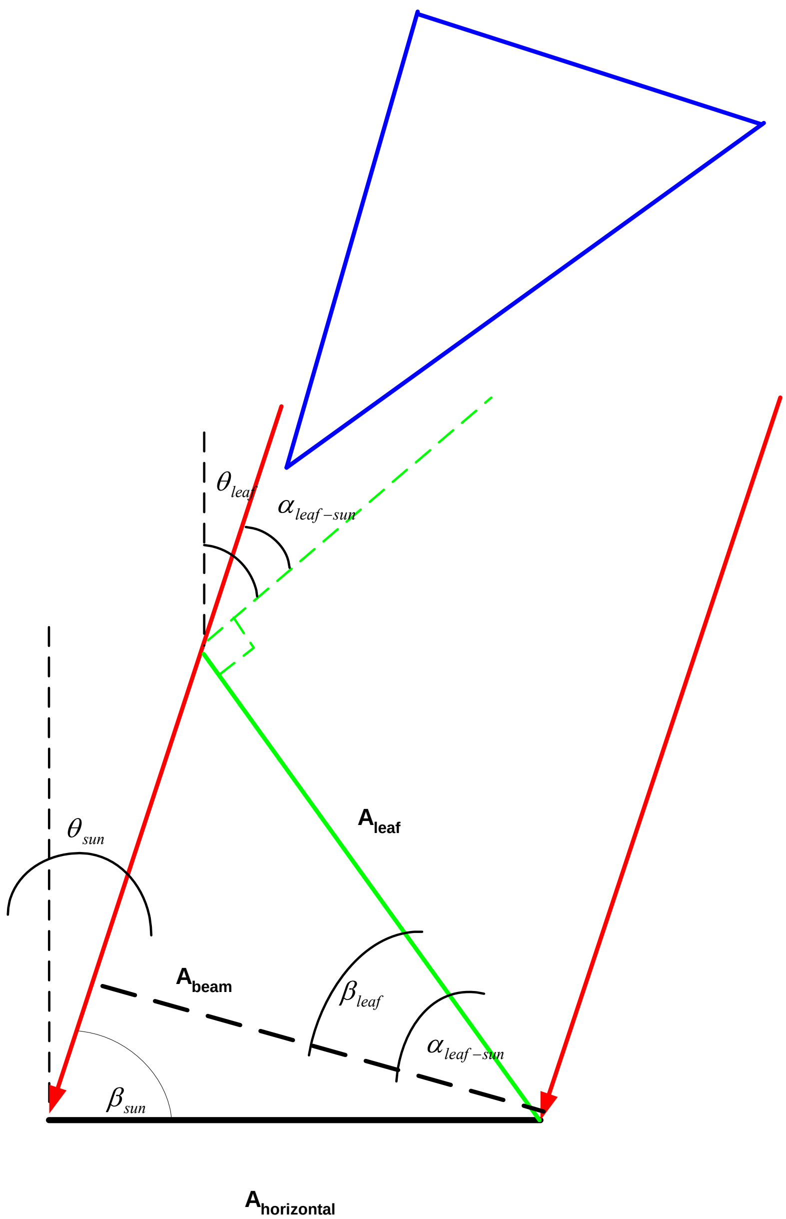


Let's step back and examine the shadow area cast by a leaf and see how it relates to attenuation or extinction of light.

From simple trigonometry we can first project the area of a leaf normal to the sun's beam (A_b) onto the horizontal (A_h).

$$\frac{A_b}{A_h} = \sin\beta = \cos\theta$$

where β is the solar elevation angle and θ is the solar zenith angle.



$\beta_{leaf} = \theta_{leaf}$

Figure 4 Projection of leaf at some inclination angle, relative to the Sun, onto a horizontal surface. The blue triangle can be 'rotation to form the triangle bounded by Aleaf and Abeam and show how the angle between the sun and the leaf normal is translated onto that triangle.

Next we define the projection of the area of a leaf onto the area normal to the solar beam.

$$\frac{A_{beam}}{A_{leaf}} = \cos\alpha$$

where α is the angle between the leaf normal and the plane perpendicular to the sun.

Several algebraic rules can be defined now:

$$A_{beam} = A_{leaf}\cos\alpha$$
$$A_{leaf} = \frac{A_{beam}}{\cos\alpha}$$

We can now project the leaf area onto the horizontal

$$\frac{A_{beam}}{A_{horiz}} = \sin\beta_{sun} = \frac{A_{leaf}\cos\alpha}{A_{horiz}} = \cos\theta_{sun}$$

$$\frac{A_{leaf}}{A_{horiz}} = \frac{\sin\beta_{sun}}{\cos\alpha} = \frac{\cos\theta_{sun}}{\cos\alpha}$$

$$\frac{A_{horiz}}{A_{leaf}} = \frac{\cos\alpha}{\sin\beta_{sun}} = \kappa = \frac{G}{\sin\beta_{sun}} = \frac{G}{\cos\theta_{sun}}$$

The above cases were for ideal leaves with a specific orientation. In real canopies many leaves exist with a statistical distribution of elevation and azimuth angles. The leaf orientation function is derived either using solid angle geometry (Myneni *et al.*, 1989; Ross, 1980) or by using independent two-dimensional probability density functions for elevation (g'(θ)) and azimuthal (g"(ϕ)) distributions of leaves (Lemeur, 1973).

Starting with the two-dimensional approach we define the two probability distributions for leaf angle (θ) and azimuthal (φ)orientation (Lemeur, 1973):

$$\int_0^{2\pi} g''(\varphi_l)\,d\varphi_l = 1$$

$$\int_0^{\pi/2} g'(\theta_l)\,d\theta_l = 1$$

To compute the ratio of the mean projected area of a leaf on the plane normal to the sun and the leaf we evaluate the G function:

$$G(\theta,\varphi) = \int_0^{2\pi}\int_0^{\pi/2} g'(\theta_l)g''(\varphi_l)|\cos(\vec{n}\cdot\vec{n}_l)|\,d\theta_l d\varphi_l$$

The direction cosine between the direction of the sun and the leaf normal, *cos(nn*$_L$*)* is defined from spherical geometry as:

$$\cos\alpha = \cos(\vec{n}\vec{n}_l) = \cos\theta_{sun}\cos\theta_{leaf} + \sin\theta_{sun}\sin\theta_{leaf}\cos(\phi_{sun} - \phi_{leaf})$$

This equation is similar to the one used to compute the solar elevation angle but here the cosine of the hour angle is replaced by the cosine of the difference between the azimuth angles of the sun and leaf.

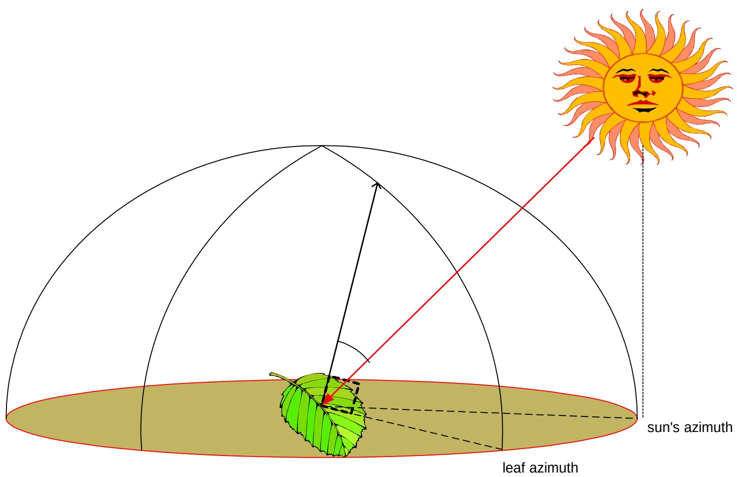


Figure 5 The relation between leaf normals and the sun

The G function behaves differently for different leaf orientations (Campbell, Norman, 1998).

Table 1 Analytical equations defining the direction cosine, G, and the extinction coefficient, k. after [*Anderson*, 1966; *Campbell and Norman*, 1998; *Monteith and Unsworth*, 1990]

Leaf Angle Distribution	G, direction cosine	K, extinction coefficient
Horizontal	$\cos(\theta)$	1
Vertical	$\tilde{2}\pi \sin(\theta)$	$2 \tan(\tilde{\theta} / \pi)$
Conical	$\cos(\theta).\cos(\theta_L)$	$\cos(\theta_L)$

Spherical or random	0.5	1/(2 cos(θ)
Heliotropic	1	1/ cos(θ)
Ellipsoidal	*	**

It is noteworthy that the G value for all distributions approaches 0.5 when the solar zenith angle is one radian. This unique attribute was used by Lang (1987)to estimate leaf area index from transects of light measurements under canopies.

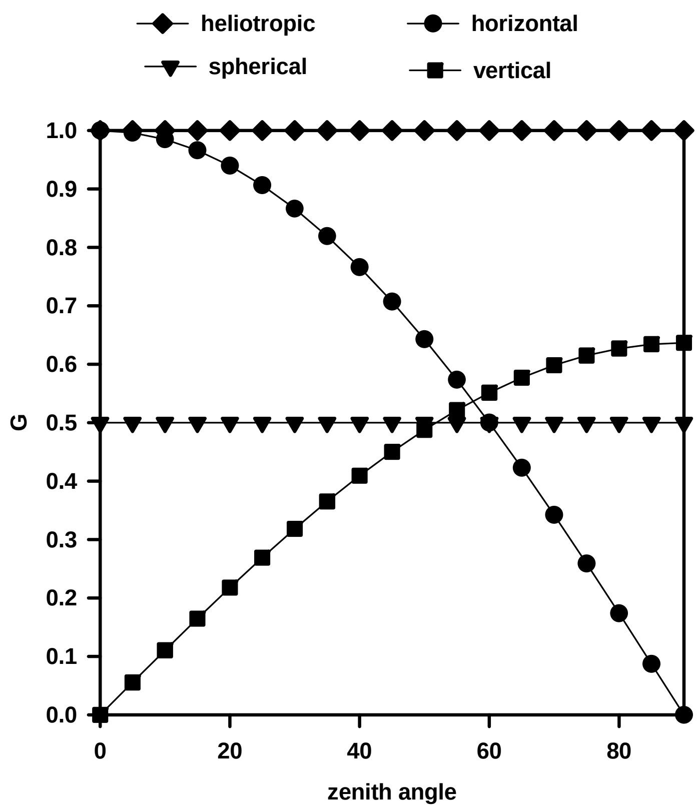


Figure 6 G functions for different leaf inclination angle distributions

From these functions, several important limits relating to canopy architecture and extinction coefficients can be deduced.

- *k* equals one when the leaves are horizontal and it is independent of the solar zenith angle.

- *k* goes to zero when the sun is overhead and leaves are erect
- *k* goes to infinity when leaves are erect and the sun is directly overhead.

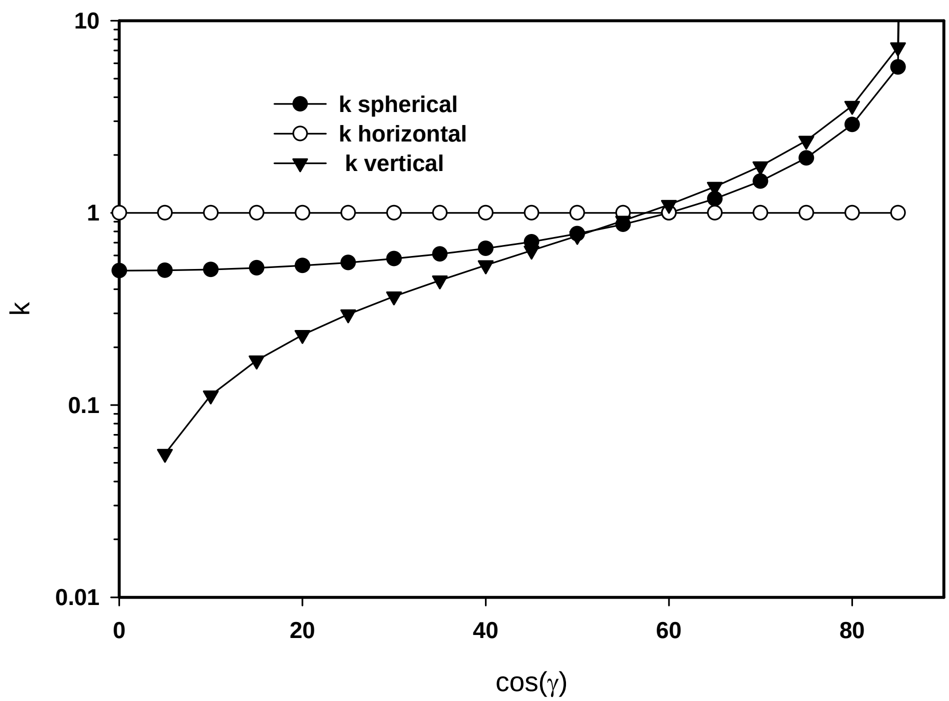


Figure 7 Extinction coefficients with different solar zenith angles

We are now ready to combine information about the leaf inclination angle distribution and the sun's zenith angle to re-define the probability of beam penetration through foliage. The change in the probability of a gap is a function of the interception of photons by increment of projected leaf area, *dL*

$$dP_0 = -P_0 \frac{G}{\cos\theta_{sun}} dL$$

Equation 1

Integrating with respect to leaf area, L, yields:

$$P_0 = \exp(-\frac{LG}{\cos\theta_{sun}}) = \exp(-\frac{LG}{\sin\beta_{sun}})$$

Equation 2

where L is leaf area index, θ is the solar elevation angle and G is the foliage orientation function. Technically, this equation is derived for monochromatic radiation and a turbid medium G represents the direction cosine between the sun and the mean leaf normal.

How the probability of beam penetration varies with different solar elevation angles is shown in Figure 5, as a function of cumulative leaf area index for a canopy that has leaves with a spherical inclination angle distribution. For a given depth in the canopy, with a cumulative leaf area index L, the probability of beam penetration increases. For a canopy with L equals three, for instance, P_0 increases from about 0.05 to 0.24 as solar elevation angle increases from 30 to 90 degrees.

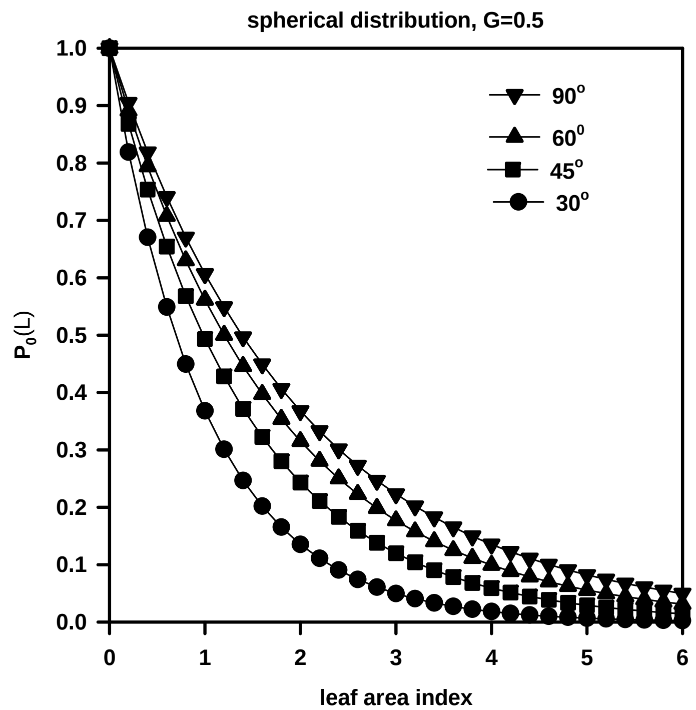


Figure 8 The relation between leaf normals and the sun and the probability of beam penetration, P_0.

With further inspection of the probability of beam penetration we can deduce several important limits.

- I/I_o or P_0 approaches one as the leaf area index or the extinction coefficient goes to zero. The most notable case is for erect leaves when the solar zenith angle approaches zero and is high in the sky.
- I/I_o or P_0 approaches zero as the leaf area index or the extinction coefficient goes to infinity. The most notable case is for very low sun elevation angles (zenith angle approaching 90 degrees) for both spherical and vertical leaves.

Beam Penetration through Non-Ideal Canopies

Many native stands of vegetation do not have leaves that are randomly distributed in space. Instead, many forest stands have clumped foliage. This feature is true for broadleaf forests [*Baldocchi et al.*, 1985; *Neumann et al.*, 1989] and conifer forests: [*Chen*, 1996; *Oker-Blom et al.*, 1991]. Clumping is difficult to quantify, but in these circumstances, the Poisson probability density function is inadequate for computing probabilities of photon transmission through vegetation. In cases with clumped foliage, the Markov model provides a better estimate of the probability of beam penetration [*Nilson*, 1971]:

$$P_0 = exp(-\frac{L\,G\Omega}{sin\ \beta})$$

Equation 3

The new variable, Ω, is a clumping factor. The Markov model assumes that adjacent layers are dependent upon each other. Hence, the future probabilistic behavior depends on the present status of the system. This model allows either one or no contacts and invokes a conditional probability, depending on whether or not a contact occurred [*Nilson*, 1971]. The Markov factor equals one if leaves are randomly distributed, is less than one if they are clumped and is greater than one if leaves are distributed in a regular pattern. Hence, the Markov model becomes identical to the Poisson distribution when leaves are randomly dispersed. A comparison between the probability of beam penetration computed for canopies with clumped and random dispersed foliage is shown in Figure 6. At each level of cumulative leaf area index, more light is transferred through the vegetation with clumped foliage.

It is critical to employ the Markov model to produce better estimates of leaf area index, when inverting measurements of beam penetration in canopies with clumped foliage [*Chason et al.*, 1991; *Chen et al.*, 2005] and if the vegetation is dispersed horizontally one may need to consider its three dimensional distribution [*Cescatti*, 1997]. Of course treatment of this additional level of detail introduces an additional unknown. But we now have the ability to apply the Markov probability theory in larger scale weather, climate and ecosystem models through the efforts of Chen et al. [2005] and Nikolov and Zeller [2006]; they have evaluated clumping indices for ecosystems spanning the globe using remote sensing.

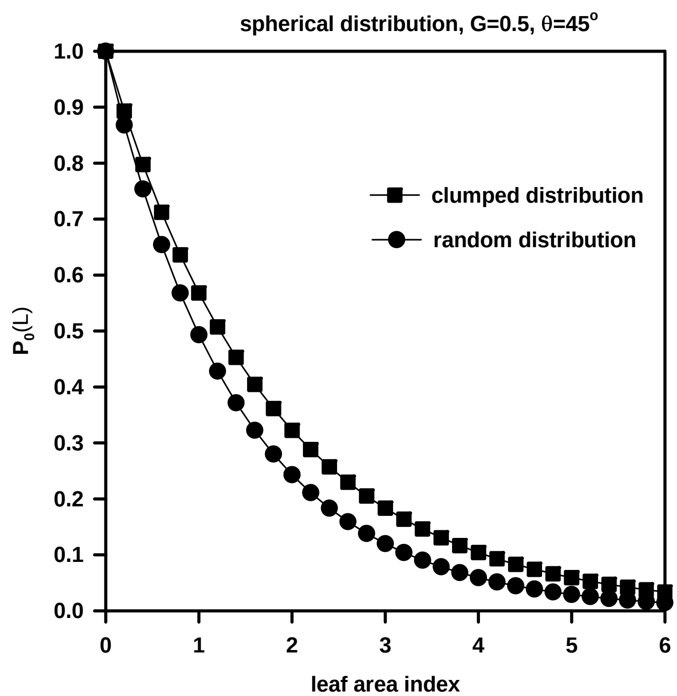


Figure 9 Probability of beam penetration with clumped and randomly distributed foliage. G is assumed to equal 0.5, for a spherical leaf angle distribution, and the solar elevation angle is 45 degrees.

Chen et al. (2005) recently evaluated clumping indices using remote sensing

Table 3
Average statistics for the clumping index values calculated by for the period November 1996 to June 1997

Class	Class names	Min	Max	Mean	d - NL	d - EQ	d - SL
1	Tree Cover, broadleaf, evergreen	0.59	0.68	0.63	-0.006	-0.004	0.024
2	Tree Cover, broadleaf, deciduous, closed	0.59	0.79	0.69	-0.019	-0.001	0.021
3	Tree Cover, broadleaf, deciduous, open	0.62	0.78	0.70	-0.005	0.007	0.025
4	Tree Cover, needleleaf, evergreen	0.55	0.68	0.62	-0.012	-0.017	0.009
5	Tree Cover, needleleaf, deciduous	0.60	0.77	0.68	-0.033	N/A	N/A
6	Tree Cover, mixed leaf type	0.58	0.79	0.69	-0.024	-0.018	0.011
7	Tree Cover, regularly flooded, fresh water	0.61	0.69	0.65	N/A	-0.002	N/A
8	Tree Cover, regularly flooded, saline water	0.65	0.79	0.72	N/A	-0.006	N/A
9	Mosaic: Tree Cover / Other natural vegetation	0.64	0.82	0.72	-0.013	0.008	N/A
10	Tree Cover, burnt	0.65	0.86	0.75	-0.036	N/A	N/A
11	Shrub Cover, closed-open, evergreen	0.62	0.80	0.71	-0.020	-0.010	0.024
12	Shrub Cover, closed-open, deciduous	0.62	0.80	0.71	-0.016	0.009	0.022
13	Herbaceous Cover, closed-open	0.64	0.83	0.74	-0.016	0.003	0.026
14	Sparse herbaceous or sparse shrub cover	0.67	0.84	0.75	-0.019	0.008	0.024
15	Reg. flooded shrub and/or herbaceous cover	0.68	0.85	0.77	-0.026	0.004	0.024
16	Cultivated and managed areas	0.63	0.83	0.73	-0.018	-0.006	0.026
17	Mosaic: Cropland / Tree Cover / Natural veg	0.64	0.76	0.70	-0.011	-0.004	0.024
18	Mosaic: Cropland / Shrub and/or grass cover	0.65	0.81	0.73	-0.018	0.001	0.026
19	Bare Areas	0.75	0.99	0.87	-0.032	-0.03	0.027

The minimum, maximum and mean values are global averages from all valid retrievals and the change per month (d) derived from a regression of the index
values for all pixels where at least one value per season was retrieved is given for the northern latitudes (NL), the equatorial region (EQ), and the southern latitudes (SL). Shading indicates that the class is included in the final product and FN/A_ indicates lack of samples.

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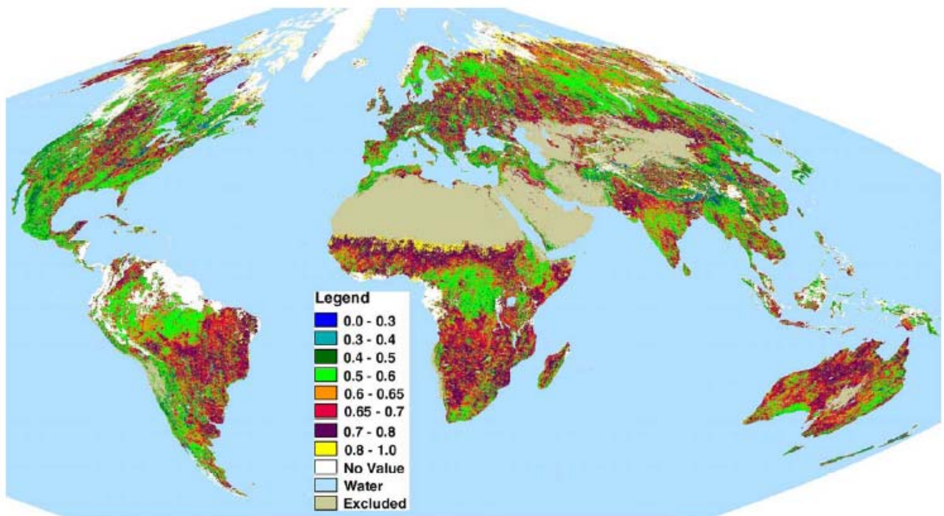


Fig. 6. Global vegetation clumping index map derived from POLDER 1 data using the normalized difference between interpolated hotspot and darkspot NIR reflectance and applied to vegetated land cover. Vegetation clumping increases with decreasing values of the index.

Alternatively, the probability of beam penetration through clumped foliage has also been assessed with the negative binomial probability model [*Nilson*, 1971], which has been shown to match measurements of light transfer through native vegetation [*Baldocchi et al.*, 1985] and row crops [*Sinoquet and Bonhomme*, 1991]

$$P_0 = (1 + \frac{G}{\cos\theta}\frac{L}{N})^{-N}$$

Equation 4

In Equation 22, N is defined as the change in leaf area, relative to the leaf area (ΔL/L). The negative binomial model assumes more than one photon interaction in a layer.

Light transmission through canopies that are heterogeneous (open) in the horizontal dimension consists of photon paths with and without volumes of vegetation (Figure 1). In this situation, light transfer can be modeled by considering discrete arrays of vegetation envelopes [*Cescatti and Niinemets*, 2004]. Foliage envelopes have been abstracted as:

1. hedges with either rectangular or triangular cross sections [*Jackson and Palmer*, 1979; *Sinoquet and Bonhomme*, 1992]
2. nested arrays of ellipsoids [*Norman and Welles*, 1983; *Wang and Jarvis*, 1990]
3. cones
4. cylinders

The simplest modelling approaches consider the probability of beam transmission as a function of the transmission between turbid blocks or rows of plants, defined by the vegetation spacing and geometry (P_f) and the transmission through the vegetated bodies is therefore computed for the two separate spaces:

$$P_o = P_f + (1 - P_f)\exp(-\kappa L_f)$$

Equation 5

Additional complexity can be adopted by defining the foliage space with more complex geometrical shapes [*Cescatti*, 1997].

Monte Carlo models are methods capable of evaluating radiation transfer through complex three-dimensional canopies [*Myneni et al.*, 1989]. Monte Carlo models use stochastic theory to follow the transfer of an ensemble of photons; typically, the transfer of 10^6 photons is followed to obtained reliable results. Monte Carlo approaches are powerful for they do not depend on simplifying assumptions, which are required to solve analytical models for light transfer in heterogeneous canopies. Consequently, they can be made as complicated as necessary, as long as information on the distribution and dispersion of foliage and its optical properties is available; for example these models can calculate the light environment within heterogeneous canopies with arbitrary leaf orientations and spatial dispersions and can consider scattering and penumbra.

Probability of Diffuse Radiation Penetration

The probability of diffuse radiation penetration is computed by integrating the probability of beam penetration over the sky's hemisphere and the sky brightness ($\Gamma(\theta)$)

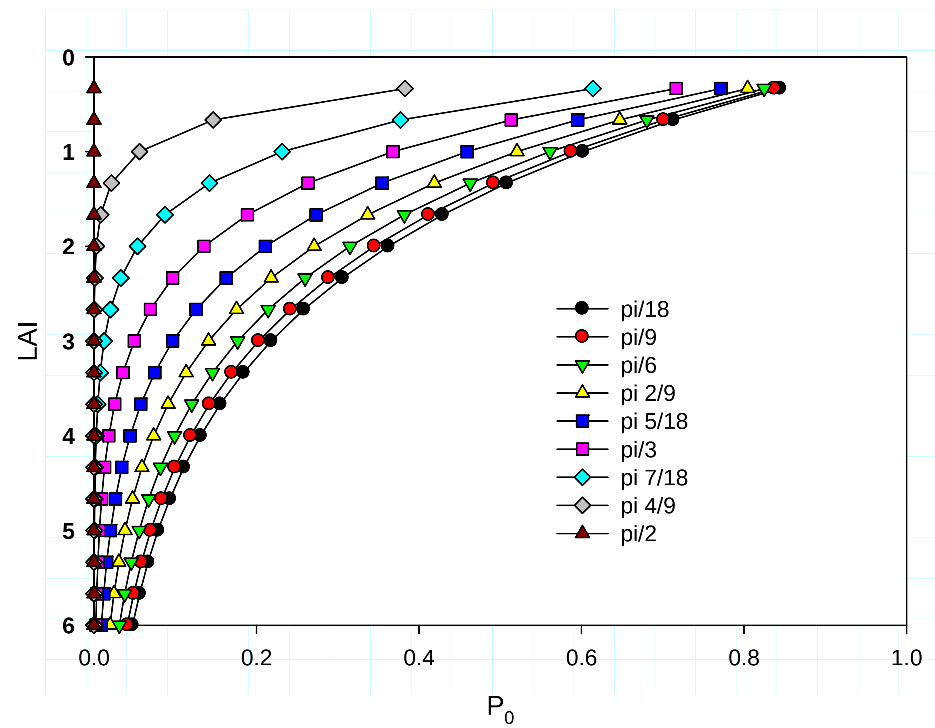
$$P_{diffuse} = 2\int_0^{\pi/2} \Gamma(\theta) P_0 \cos\theta \sin\theta \cdot d\theta$$

Equation 6

For isotropic, uniform skies, the sky brightness factor is one, so this relation can reduce to:

$$P_{diffuse} = 2\int_0^{\pi/2} P_0 \, cos\,\theta \, sin\,\theta \cdot d\theta$$

Equation 7



Penumbra

Two sources of shade exist in a canopy. One is dark shade, umbra, and the other is partial shade, penumbra [*Denholm*, 1981; *Myneni et al.*, 1989; *Norman et al.*, 1971]. Penumbra arises because the sun is not a point source. It has a finite radius, as view from Earth, which is 0.533 degrees. At a given height above the surface the apparent area of the sun is $\pi\ r_{sun}^2$. The apparent radius of the sun is:

$$r_{sun} = h \tan \frac{\alpha_{sun}}{2} = 0.0047h$$

Equation 8

Partial shade will be cast when an object does not completely cover the area of the sun. An example of penumbra is the fuzzy edges of the shadow of leaves, cast on the ground. This will depend on the size of the occluding object and how far it is above the ground.
The apparent area of the sun is:

$$A = \pi \cdot r_{sun}{}^2 = \pi(0.0047h)^2$$

Equation 9

If the apparent size of a leaf, as viewed from some spot underneath exceeds the area of the solar disc, pure umbra (shadow) will be cast. If it is smaller than the sun partial or penubra shade will be cast. Of course, placing a disk closer to the viewer will extend a wider angle and increase the chance of removing penumbra. It is like placing your hand extended in front of our face and looking at a light bulb. As you bring your hand closure you will soon block the bulb from view.

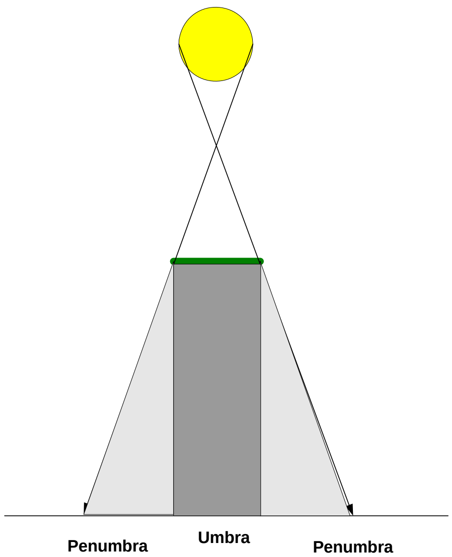


Figure 10 Conceptual view of penumbra and umbra for a disk blocking the sun above an observer.

Spectra

The preferential transmission and scattering of light in a canopy will cause the spectra to differ from that of the sun. It is important not to confuse the use of sensors that measure quanta and energy when the spectra varies. For example, one company makes an inexpensive pyranometer based on a diode system. If this sensor is place upside down to measure reflectance, or placed under a canopy its calibration will be altered.

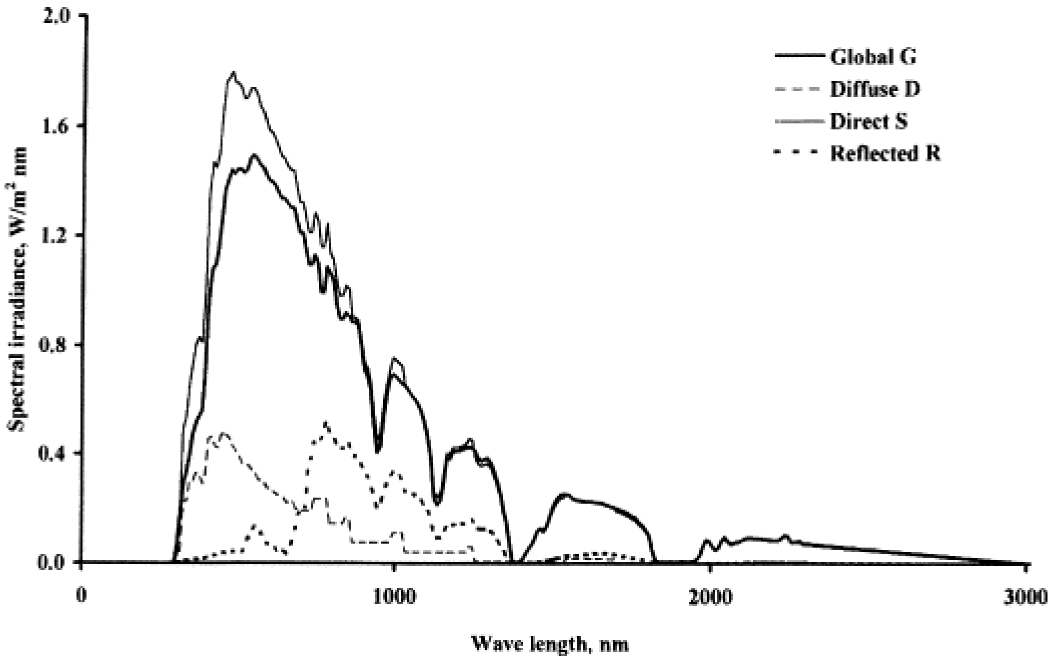


Figure 11 Ross and Sulev, 2000.

SUMMARY

- The flux density of light energy received at a particular location inside a plant canopy consists of **beam** and **diffuse solar radiation** that **penetrates through gaps** in the canopy. It also contains **complementary radiation** that is generated by the **interception** and the consequent, (wavelength-dependent) **transmission** through leaves and **reflection** by leaves and soil.
- Sources of spatial variation of light in a canopy include: clumping and gapping of foliage; gaps in canopy crowns due to treefall or cultivation practices; spatial variations in leaf orientation angles; penumbra; leaf flutter; clouds, and directional and non-isotropic and wavelength dependent scattering of light and topography;
- Sources of temporal variation of radiation include: topography; seasonal trends in plant phenology; seasonal and diurnal movement of the sun.
- the probability of beam penetration for a turbid medium can be calculated using a Poisson probability density function that is dependent on leaf area index:

$$I(L) = I_0 \exp(-k \cdot L)$$

- The extinction coefficient, *k,* is defined as the ratio between the G function (the cosine between the solar zenith and the mean leaf normal angle) and the sine of the solar elevation (*k=G/sinβ)*
- All leaf inclination angles have a G function value approaching 0.5 when the solar zenith angle is near one radian.

- Clumping of leaves enhances the probability of beam transmission
- The fraction of sunlit leaves is computed by the derivative of the probability of beam penetration with respect to leaf area.

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Appendix

Mathematical expression of the radiative transfer equation:

$$-\cos\gamma \frac{dI(z,\Omega)}{dz} + k(z,\Omega)I(z,\Omega) = \int_{4\pi} d\Omega' \sigma_s(z,\Omega' \to \Omega) I(z,\Omega')$$

The Estonia School defines orientations and the G function in terms of solid geometry. Their definition of the leaf orientation function is slightly different:

$$\frac{1}{2\pi}\int_{\Omega_L} g(\theta_l,\varphi_l)d\Omega_L = \frac{1}{2\pi}\int_0^{2\pi} d\varphi_l \int_0^{\pi/2} g(\theta_l,\varphi_l)\sin\theta_l d\theta_l = 1$$

where $d\Omega_L = \sin\theta_L d\theta_L d\phi_L$

g denotes the probability density function of mean angle θ_l, the angle between the vertical and the leaf normal, and ϕ_L, the azimuth of the leaf. To compute the formal value of G function, the mean direction cosine between the sun and the leaf normal, for a statistical distribution of leaf orientations one must use information derived from the orientation function, weighting it by sin(θ). The definition of the G-Function is defined thus defined mathematically as:

$$G(\theta,\varphi) = \frac{1}{2\pi}\int_0^{2\pi} d\varphi_l \int_0^{\pi/2} g(\theta_l,\varphi_l)|\cos(\vec{n}\cdot\vec{n}_l)|\sin\theta_l d\theta_l$$

If there are independent probability functions for elevation and azimuth angles, then one defines:

$$\frac{1}{2\pi} g(\theta,\phi) = g'(\theta)\frac{g''(\phi)}{2\pi}$$

if there is no azimuthal preference then

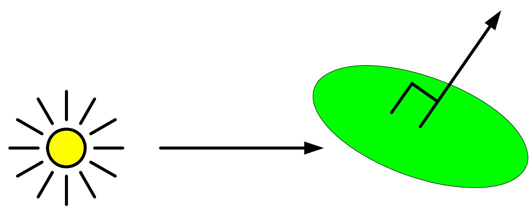
$$g'(\theta) = g(\theta)/\sin(\theta)$$

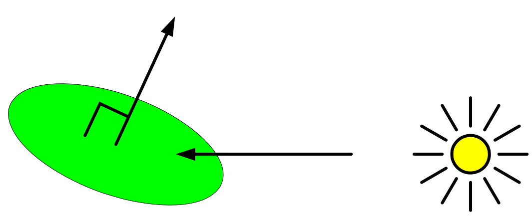
1. When sun is parallel to leaf, $\cos(nn_l) = 0$, the solar zenith angle is perpendicular with respect to the leaf normal

$$0 = \cos\theta_L \cos\theta_S + \sin\theta_L \sin\theta_S \cos(\phi_L - \phi_S)$$

$$\cos(\phi_L - \phi_S) = -\frac{\cos\theta_L}{\sin\theta_L}\frac{\cos\theta_S}{\sin\theta_S} = -\cot\theta_L \cot\theta_S$$

If the sun elevation angle is greater than the leaf elevation angle, then sunlight is on the upper side of the leaf. If the sun elevation angle is less than the leaf elevation angle, sunlight may be on the top or bottom of the leaf, depending on the leaf azimuth angle.





However, Ross reports that plant measurements show that very few leaves are illuminated by the sun on the lower side, so for practical purposes it is assumed that all leaves are hit by direct photons on the upper side.