

## Lecture 18, Wind and Turbulence, Part 1, Surface Boundary Layer: Theory and Principles

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Topics to be Covered:

### A. Processes

- A. Wind and Turbulence
  - 1. Concepts
    - a. Definition of Turbulence
    - b. Reynold's Number
    - c. Conservation equation for wind
    - d. TKE budget, conceptual

### L18.1 Processes

We experience wind and turbulence in every day life. It affects how airplanes fly, the efficiency of automobile travel and our ability to the predict weather and climate.

Wind is moving air. Wind has **speed** and **direction**. Thereby, wind velocity is a vector quantity. It is distinct from its relative wind speed, which only has magnitude. Using the Cartesian coordinate system we define three wind velocities. The longitudinal wind velocity is  $u$ . This is the vector along the horizontal mean wind ( $x$ ). The lateral wind velocity is  $v$ . This is the cross-wind component ( $y$ ). The vertical ( $z$ ) wind velocity is  $w$ .

The instantaneous wind velocity is comprised of 3 components, the **mean wind velocity**, a **periodic wave velocity** and **random fluctuating** velocities. The mean wind advects material. Waves tend to occur at night under stable thermal stratification. If they are regular they transport very little heat and mass. Fluctuating component of wind is associated with turbulence. This component is very important for it is a mechanism for transporting heat, energy and mass between the biosphere and atmosphere.

The study of fluid flow is **complex**. It contains motions that are **highly organized** (like a whirlwind, tornado or hurricane) and **chaotic** (as in turbulent flow). The study of fluid flow and turbulence is considered by many to remain one of the unsolved problems of modern physics. The famous fluid mechanic, von Karman, is credited with stating:

*‘There are two great unexplained mysteries in our understanding of the universe. One is the nature of a unified generalized theory to explain both gravity and electromagnetism. The other is an understanding of the nature of turbulence. After I die, I expect God to clarify the general field theory to me. I have no such hope for turbulence’.*

With regards to biometeorology, turbulence is responsible for:

- 1) transferring heat, momentum and mass (water vapor, carbon dioxide, biogenic gases, pollutants) between the biosphere and the atmosphere and diffuses pollutants in the atmosphere.
- 2) imposing drag forces on plants, causing them to wave, bend and break
- 3) mixing the air and diffusing air parcels with different properties, thereby forming spatial gradients
- 4) gusts associated with turbulence place loads on the surface, which can erode soils and eject dust, spores and insect eggs into the atmosphere.

#### L18.1.1 Laminar and Turbulent Flows

Fluid flow can either be **laminar** or **turbulent**. A visualization of the transition of fluid flow between **laminar** and **turbulent** states is shown below.

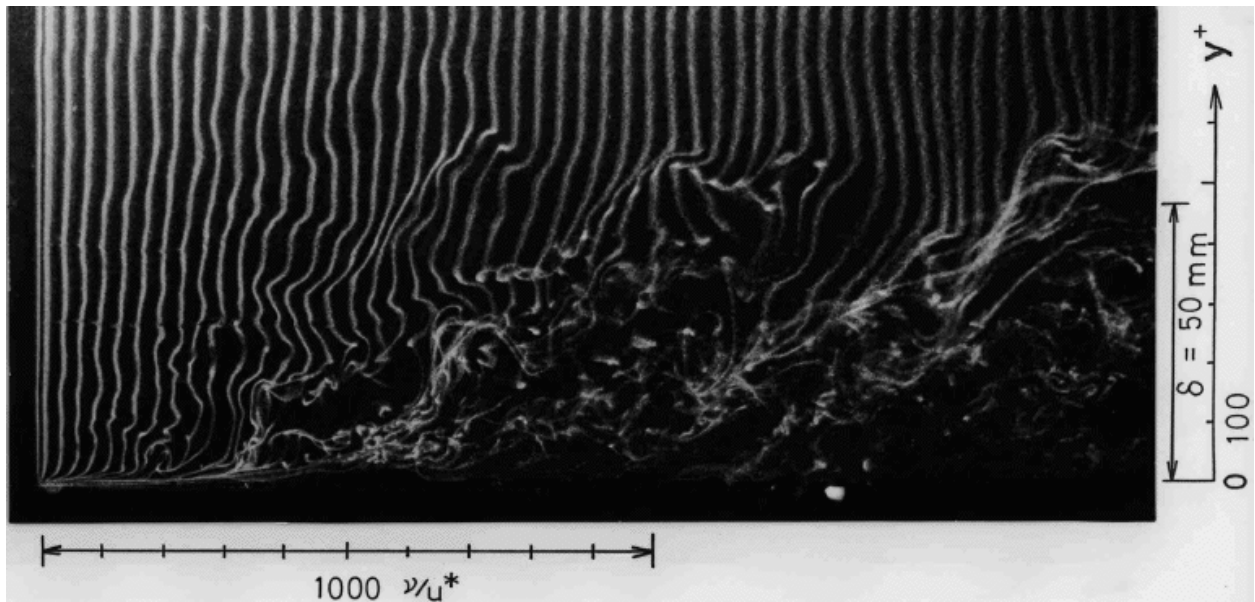


Figure 1 Visualization of turbulence <http://www.thtlab.t.u-tokyo.ac.jp/index.html>

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Concepts distilled from the photograph include a gradual evolution between laminar and turbulent boundary layers as a fluid in a laminar state flows over a surface. First shear occurs at the interface, but the flow is still laminar. Soon, turbulence forms near the surface as the shear becomes unstable, while the overlaying flow is still laminar. If the fluid flow occurs over a surface for a long enough distance, the entire profile can become turbulent. Hence over small boundary, like leaves, fluid flow can have a mix of transition states.

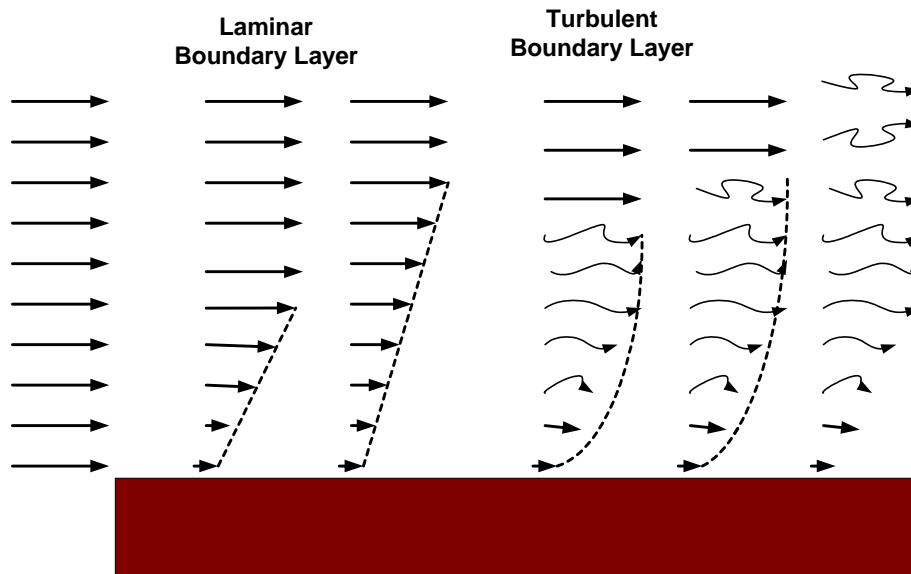


Figure 2 Evolution of boundary layers as wind blows over a surface

Turbulence is the **chaotic** and seemingly random motion of fluid parcels. Turbulence has **mechanical** and **convective** origins. **Shear forces** cause mechanical turbulence while **buoyant instabilities** (due to the intermingling of fluid parcels with different densities) causes convective turbulence.

A distinct property of turbulence in the natural environment is a wide **spectrum of scales** (eddies) that are associated with the fluid flow. The largest scales of turbulence are produced by forces driving the mean fluid flow. Dynamically unstable, these large eddies break down into progressively smaller eddies via an inertial cascade. This cascading breakdown of eddy size continues until the eddies are so small that energy is consumed by work against viscous forces that convert kinetic energy into heat.

A qualitative understanding of the multiple scales of turbulence can be attained through poetry. The famous fluid mechanic, L.F. (Lewis Fry) Richardson is attributed with writing a poem on turbulence, a parody of Jonathan Swift's poem the flea<sup>1</sup>:

<sup>1</sup> So, naturalists observe, a flea  
Has smaller fleas that on him prey;  
And these have smaller still to bite 'em;  
And so proceed ad infinitum.

*Great whirls have little whirls  
That feed on their velocity  
And little whirls have lesser whirls  
And so on to viscosity*

Another poetic illustration of a property of turbulence can be acquired with a rhyme used by air pollution engineers:

*dilution is the solution to pollution*

As biological beings we probably could not exist as we do without the efficient transfer of mass and energy via turbulence, as needed to sustain our rates of metabolism. Just think as a kid when you stuck your head under the sheets, didn't get hard to breathe after a while??

**Atmospheric** turbulence differs from turbulence generated in a **laboratory** or in pipe flow. In the atmosphere, convective turbulence coexists with mechanical turbulence.

On the global scale, the rotation of the earth affects how we view geophysical fluid dynamic problems. Solar energy is the original source of winds on Earth. Temperature and pressure gradients, due to differential heating of land and sea cause air to move. The kinetic energy due to wind movement, however, is rather small compared to the solar energy budget. Oort (1964) computes that  $3.1 \text{ W m}^{-2}$  of solar energy ( $345 \text{ W m}^{-2}$ ) produces kinetic energy to generate winds.

### *L18.1.2 Properties of Turbulence*

Turbulent flow has a number of unique properties. Turbulence is a complex phenomenon defined by the chaotic and seemingly random motion of fluid parcels, on one hand, and coherent features on the other. In principle, turbulence is: 1) three dimensional; 2) non-Gaussian, 3) non-linear, 4) dissipative, 5) contains a spectrum of length scales and 6) diffusive (Finnigan, 2000; Frisch, Orszag, 1990; Wyngaard, 1992). In the following we examine each of these attributes.

1. ***Turbulence is non-linear.*** The dynamic properties of fluid velocity are defined by a set of partial differential equations, the Navier-Stokes Equations. These equations predict that the change in wind velocity for a discrete time step will depend on where the wind is coming from, multiplied by how the wind at that location differs from your point of reference ( $du/dt \sim u du/dx$ ). Another characteristic of this non-linearity is that predictions are very sensitive to initial conditions. This fundamental feature causes turbulence to be stochastic, chaotic and sensitive to initial conditions (Lorenz, 2006).

2. **Turbulence is non-Gaussian.** The seemingly random and intermittent nature of turbulence causes its probability density distribution not to be characterized by the bell-shaped curve. Instead it is skewed and kurtotic.
3. **Turbulence is three-dimensional.** The fluid motions are rotational and anisotropic, This feature causes vortex stretching, which allows velocity gradients to be established in all directions.
4. **Turbulence is diffusive.** This enables the efficient mixing of two parcels of gases.
5. **Turbulence is dissipative.** This effect causes turbulence to be a continuum phenomenon. In other words, its **energy of motion** is degraded to **internal energy** or heat of the fluid by viscosity. One consequence of this property is that energy must be continually **supplied** to maintain fluid motion.
6. **Turbulence consists of multiple length scales.** The dissipative nature of turbulence causes turbulent energy to cascade through a spectrum of eddies. The largest scale eddies have lengths on the order of a kilometer or three. These eddies are of the scale of the depth of the planetary boundary layer, which is approximately the height of the base of fair weather convective clouds. These large eddies are responsible for transport of mass and energy. The smallest eddies are defined by the Kolmogorov microscale. It is the scale at which eddies dissipate into heat. This smallest scale defined as from the ratio of kinematic viscosity ( $\nu$ ) and the rate of dissipation ( $\epsilon$ ):

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

The size of the smallest eddies are on the order of  $10^{-3}$  m. The dissipation rate scales with  $u^3/l$ .

**Viscosity** is a function of the molecular velocity and its mean free path, so it is dependent upon temperature and pressure. An understanding of boundary layers and an examination of momentum transfer, as described by Newton's Law of Viscosity, is needed to understand the concept of viscosity:

*the shear force per unit area is proportional to the negative of the local velocity gradient,*  $\frac{F}{A} = P = -\mu \frac{V}{Z}$  ( $\text{kg m}^{-1} \text{s}^{-1}$ )

The constant of proportionality is the dynamic viscosity ( $\mu$ ). Viscous fluids like honey or motor oil require more force to maintain a certain velocity gradient than a lower viscous fluid like water.

**Kinematic viscosity** is defined as the dynamic viscosity ( $\text{kg m}^{-1} \text{s}^{-1} = \text{Pa s}$ ) normalized by the density of the fluid:

$$\nu = \frac{\mu}{\rho} \text{ (m}^2 \text{ s}^{-1}\text{)}$$

Fluid mechanics use the Reynolds number to define whether the flow is turbulent or laminar. It is a dimensionless quantity named after the 19<sup>th</sup> Century scientist, Sir Osborne Reynolds. Conceptually, Re is defined as the **ratio between inertial and viscous forces**. (inertial forces are associated with the mean motion of the fluid and the viscous forces are associated with frictional shear stress.). Mathematically, the Reynolds number is defined as:

$$\text{Re} = \frac{d \cdot u}{\nu}$$

$d$  is a characteristic length scale,  $u$  is the wind velocity and  $\nu$  is the kinematic viscosity of the fluid. The threshold for the onset of turbulence is about 2000. Most atmospheric flows are turbulent ( $\text{Re} \gg 2000$ ) as  $\nu$  is on the order of  $10^{-5} \text{ m}^2 \text{ s}^{-1}$ ,  $d$  is of meters and  $u$  is on the order of  $0.1$  to  $10 \text{ m s}^{-1}$ . Re for a  $3 \text{ m s}^{-1}$  wind velocity over a  $3 \text{ m}$  corn field would be 90000, for example. In contrast, Re for wind blowing  $0.25 \text{ m s}^{-1}$  over a  $10 \text{ cm}$  leaf is 250. Consequently, we can also state that turbulence is associated with large Reynolds numbers. In other words, the characteristic length scale of the flow is much larger than the thickness of the laminar boundary layer (Blackadar, 1997).

### *L18.1.3 Equation of motion*

Micrometeorologists and fluid mechanics use theory and experimentation to study turbulence. Theoretical development involves the simulation of turbulence with analytical and numerical models. Experimentation involves field experiments, tank and wind tunnel studies.

The **equation of motion** describes how fluid velocity at a point in space affects the velocity at other points in space. The concept is based on three ideas, **locality**, **conservation** and **symmetry**. Fluid flow is local when the parcels in motion are affected only by its neighbors. Fluid flow is conservative because fluid parcels are never lost, they are only transferred from one area to another. Fluid flow can be symmetric when it is isotropic and rotationally invariant. Recent use of cellular automata (Wolfram, 2002) have exploited these ideas using rules to simulate many of the properties of turbulence.

The **equation of motion** defines how wind velocity accelerates or decelerates. For a defined volume it is derived by examining the balance between the **net rate of momentum accumulation** and the momentum transfer in and out of a controlled volume), which is equal to the **sum of forces acting on the system** (pressure, force per unit area, friction). Conceptually we can express this concept as:

$$\rho \frac{\Delta u}{\Delta t} = \frac{\Delta(F/A)}{\Delta x} = \frac{\Delta P}{\Delta x} + F_{friction}$$

When evaluating turbulence, fluid mechanics examine instantaneous and averaged relations. To understand the equation for mean fluid flow, we must introduce some new concepts, such as partial derivatives and Reynolds averaging rules.

Lets start with an examination of the total time derivative of horizontal velocity. This velocity is a function of t, x, y and z.

$$\frac{du(t, x, y, z)}{dt} = \frac{\partial u}{\partial t} + \frac{dx}{dt} \frac{\partial u}{\partial x} + \frac{dy}{dt} \frac{\partial u}{\partial y} + \frac{dz}{dt} \frac{\partial u}{\partial z}$$

We know, for example that  $u=dx/dt$ ,  $v=dy/dt$  and  $w=dz/dt$ , so we can redefine  $du/dt$

$$\frac{du(t, x, y, z)}{dt} = \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}$$

This new expression of the time derivative of velocity is extremely important for it shows how **non-linearities**, hence **Chaos**, in fluid flows arise. This total derivative is a function of the local derivative and advection terms that are non-linear (e.g. terms with u times u).

For instantaneous wind in the surface boundary layer the budget equation is defined in terms of **pressure** and **viscous shear** forces:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \approx -\frac{1}{\rho} \frac{\partial p}{\partial x} - \frac{\mu}{\rho} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right]$$

The second derivatives arise ( $\frac{\mu}{\rho} [\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}]$ ) because they represent the flux

divergence of the shear force, e.g.  $\frac{\partial \tau}{\partial z} = v \frac{\partial(\partial u / \partial z)}{\partial z} = v \frac{\partial^2 u}{\partial z^2}$

For mean wind flow in the **turbulent, surface boundary layer**, the budget equation can be expressed as in terms of **mean pressure gradients** and **turbulent shear stress**:

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{w} \frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x} - \frac{\partial \overline{w'u'}}{\partial z}$$

Overbars denote time averaging, primes denote fluctuations from the mean,  $\rho$  is density and  $p$  is pressure. This equation assumes that the **Coriolis** and **viscous forces** are negligible for turbulent atmospheric flows with high Reynolds numbers. For mean turbulence the diffusive fluxes are insignificant compare to the turbulent fluxes, which is why the diffusive flux drops out. From dimensional arguments viscosity has a magnitude of about  $10^{-4} \text{ m}^2 \text{ s}^{-1}$ . In comparison, turbulent diffusivities are on the order of  $1 \text{ m}^2 \text{ s}^{-1}$ , scaling with height and a concept called friction velocity.

Under steady state conditions and horizontal homogeneity, the flux divergence of the Reynolds shear stress is zero, so **momentum transfer to the surface is constant with height in the surface boundary layer**.

### L18.1.2 Turbulent Kinetic Energy Budget

Kinetic energy is a function of mass times velocity squared,  $\frac{1}{2} mv^2$ . We are interested in the turbulent kinetic energy budget to quantify how turbulence varies with time ( $\frac{d \frac{1}{2} \overline{q^2}}{dt}$ ). TKE is produced by mechanical (or shear) and buoyancy. It is destroyed by dissipation. TKE may also be transported into (a gain) or out (a loss) of a region.

**Mechanical** turbulence is generated by the **frictional** retardation of the wind by the ground and plants. Consequently there must be a downward flux of momentum from the air flow to the surface. Mean shear (or mechanical turbulence) is the sole source of turbulence during neutral and stable conditions.

$$-\overline{w'u'} \frac{\partial \bar{u}}{\partial z} = \varepsilon$$

Convective turbulence is caused by buoyancy forces that result from the heating of the surface. Hot air is less dense the cool air, so it is often stated by warm air rises. Another view would be the gravity pulls denser air to the surface, which displaces the warm and less dense air upward, in order to meet continuity requirements.

In a simplified manner where there is no transport and the system is at steady state, then the **production of turbulent kinetic energy** created by **shear and buoyant** production must equal the rate at which energy is **dissipated into heat by viscous processes**.

$$-\overline{w'u'} \frac{\partial \bar{u}}{\partial z} + \frac{g}{\theta_v} \overline{w'\theta_v} = \varepsilon$$

An expanded view of the TKE budget has third order transport terms, which are significant in plant canopies, as we will discuss in later lectures.

$$\frac{\partial \overline{\frac{1}{2}q'^2}}{\partial t} = -\overline{w'u'} \frac{\partial \bar{u}}{\partial z} + \frac{g}{\theta_v} \overline{w'\theta_v} - \frac{\partial}{\partial z} (\overline{\frac{1}{2}q'^2 w'} + \overline{\frac{1}{2}w'p'}) - \varepsilon$$

### References:

- Arya, S. P. 1988. Introduction to Micrometeorology. Academic Press.
- Blackadar, A.K. 1997. Turbulence and Diffusion in the Atmosphere, Springer
- Garratt, J.R. 1992. The Atmospheric Boundary Layer. Cambridge Univ Press. 316 pp.
- Hogstrom, U. 1996. Review of some basic characteristics of the atmospheric surface layer. *Boundary-Layer Meteorology*. 78, 2215-246
- Hogstrom, U. 1988. Non-dimensional wind and temperature profiles in the atmospheric surface layer, a re-evaluation. *Boundary Layer Meteorology*. 42, 55-78.
- Kaimal, J.C. and J.J. Finnigan. 1994. Atmospheric Boundary Layer Flows: Their Structure and Measurement. Oxford Press.
- Mahrt, L. 1999. Stratified atmospheric boundary layers. *Boundary Layer Meteorology*. 90, 375-396.
- Mahrt, L. 2000. Surface heterogeneity and vertical structure of the boundary layer. *Boundary Layer Meteorology*. 96, 33-62
- Panofsky, H.A. and J.A. Dutton. 1984. Atmospheric Turbulence. Wiley and Sons, 397 pp.
- Raupach, M.R., R.A. Antonia and S. Rajagopalan. 1991. Rough wall turbulent boundary layers. *Applied Mechanics Reviews*. 44, 1-25.
- Shaw, R. 1995. Lecture Notes, Advanced Short Course on Biometeorology and Micrometeorology.
- Stull, R.B. 1988. Introduction to Boundary Layer Meteorology. Reidel Publishing. Dordrecht, The Netherlands.
- Thom, A (1975). Momentum, Mass and Heat Exchange of Plant Communities. In: *Vegetation and the Atmosphere*, vol 1. J.L Monteith, ed. Academic Press.

Van Gardingen, P. and J. Grace. 1991. *Advances in Botanical Research*. 18: 189-253.

Wyngaard, J.C. 1992. Atmospheric Turbulence Annual. *Review of Fluid Mechanics* 24: 205-233

Finnigan J (2000) Turbulence in Plant Canopies. *Annu. Rev. Fluid Mech.* **32**, 519-571.

Frisch U, Orszag SA (1990) Turbulence - Challenges for Theory and Experiment.

*Physics Today* **43**, 24-32.

Lorenz EN (2006) REFLECTIONS ON THE CONCEPTION, BIRTH, AND CHILDHOOD OF NUMERICAL WEATHER PREDICTION. *Annual Review of Earth and Planetary Sciences* **34**, 37-45.

Wolfram S (2002) *A New Kind of Science* Wolfram Media, Champaign, IL.

Wyngaard JC (1992) Atmospheric-Turbulence. *Annual Review of Fluid Mechanics* **24**, 205-233.