Biometeorology, ESPM 129

Lecture 26, Leaf Boundary Layers and their Resistances and Mass and Momentum Exchange, Part 1

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Topics to be Covered

A. Concepts of laminar and turbulent boundary layers
B. Induced by laminar or Turbulent Flow
C. Dimensionless Numbers for computing mass and heat transfer coefficients
D. Reynolds Number

L 26.1 Introduction

The layer of air adjacent to leaves is called the **leaf boundary layer**. This boundary layer is extremely important for the functioning of life, as it is a critical path for the transfer of trace gases, momentum and energy between the atmosphere and biosphere (Schuepp 1993). Furthermore, it is a path that cannot be circumvented by metabolizing organisms.

The thickness of the boundary layer and the resistance it imposes on mass, energy and momentum transfer has many implications on biophysical processes that may be of interest to us. A thick leaf boundary layer will retard the transfer of mass, heat and energy, causing values of temperature, humidity and CO_2 concentration at the leaf surface to differ markedly with their corresponding value in the free atmosphere. The diffusion of gases through the boundary layer causes isotope fractionation. So knowledge on leaf boundary layer diffusion is needed if one expects to use isotopes as a tool to study the difference sources and sinks of carbon, oxygen, nitrogen and hydrogen isotopes in plant canopies. An understanding of the properties and behavior of the leaf boundary layer is needed for physiological purposes, besides the assessment of flux densities. Plants and leaves respond to stimuli that are experienced at their leaf surface, not some distance away in the free streaming air (Grantz and Meinzer 1990). If we expect to predict how leaves respond to their environment, we must compute the environment at that surface. This will require an assessment of leaf boundary layers.

The boundary layer of leaves has an array of attributes. Air moving in the leaf boundary layer can **be laminar**, **turbulent** or a **mix**. Wind speed near a leaf is reduced markedly, as compared to the velocity in the free air, due to surface friction. At some level near the surface turbulence becomes suppressed, so mass and heat transfer eventually occurs by molecular diffusion. Finally, **Fick's Law of Diffusion** is applicable in the **laminar layer** closest to the leaf.

Most theories that are used to quantify leaf boundary layers are highly idealized. The majority of the theories are derived from engineering theory. These methods use dimensionless numbers for mass and heat transfer over ideal shapes (flat plates, spheres, cylinders) to generate engineering theory for assessing heat and mass transfer resistances. Subsequently, Fick's Law of Diffusion and Resistance-Analog theories are commonly invoked to evaluate fluxes of heat and trace gases between leaves and the air.

As a word of caution, leaves in the natural environment exhibit much variablity in size, shape and orientation. Consequently, heat, momentum and mass transfer resistances are affected by leaf orientation and leaf size. On the application of theories, we will observe that leaf-size induced changes in resistance affects leaf temperature and diffusion of water, heat and CO_2 between leaf surface and atmosphere.

L24.2. Laminar and Turbulent Flow

To begin understanding mass and energy transfer between surfaces and the atmosphere we must start with concepts on the motion of viscous fluids, with cases involving laminar and turbulent flow. For it is the motion of the fluid that enables material and energy to travel from one location to another.

As fluids flow over surfaces the velocity of the fluid decreases as one approaches the surface. This is a consequence of friction between the surface and fluid and by the viscous forces within the fluid itself. The zone between the surface and the free stream fluid flow is the boundary layer, it is a zone where the boundary is exerting an influence on the properties of the fluid flow. Fluid flow in the boundary layer is either laminar or turbulent. In laminar flow, the flow is well behaved and the velocity streamlines are parallel to one another. In turbulent flow the streamlines are chaotic and seemingly random.

Whether or not laminar or turbulent flow is occurring is determined by a balance between the inertial forces (due to the moving fluid) and the viscous forces., which tend to stabilize the flow and make it laminar. Pouring honey out of a jar is such an example. This fluid is highly viscous and its flow tends to be laminar. The Reynold's number is a dimensionless number that is used by fluid mechanics to indicate if fluid flow is turbulence or laminar. Reynolds Number is defined as the ratio of inertial forces to viscous forces

$$\mathrm{Re} = \frac{lu\rho}{\mu} = \frac{lu}{\upsilon}$$

When viscous forces are large and predominate, flow is able to remain laminar, as when one pours syrup from a jar. For ideal conditions and flat plates a critical Re defines the onset of turbulence ranges between 10^4 to 10^5. A typical critical value is Re > 20000.

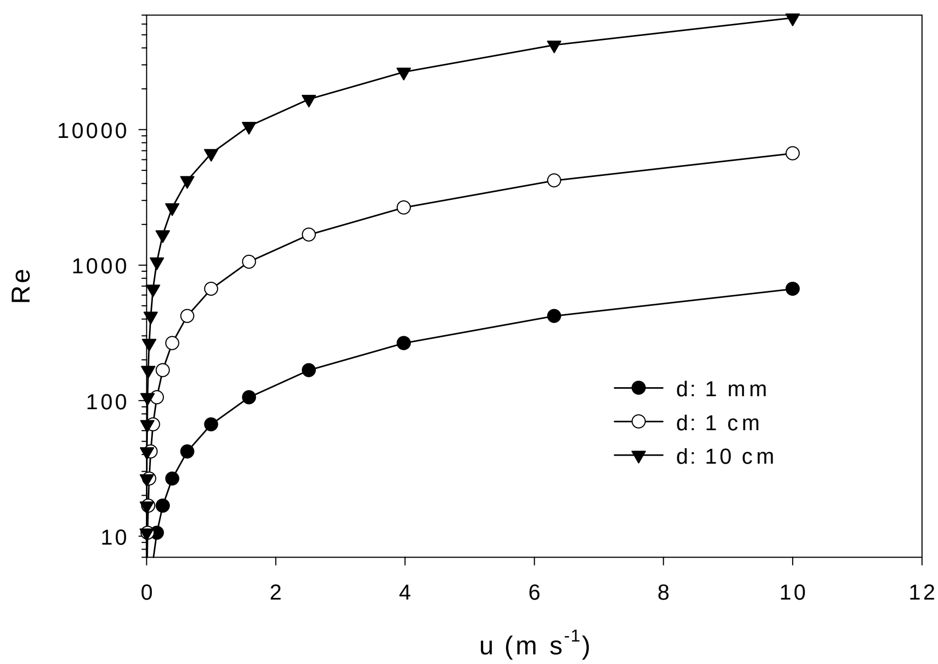

Figure 1 Reynolds number as a function of wind speed and length scale

Surface irregularities of real leaves can cause the critical Re value for the onset of turbulence to be less than 10^4. Over a natural leaf, a laminar flow, will start out laminar, when it encounters the edge of a leaf, but with distance it will start breaking and form a turbulent boundary layer. Grace and Wilson (Grace and Wilson 1976) made wind measurements over a populus leaf exposed to 1 m s^{-1} wind. The boundary layer was laminar only for a short distance along the adaxial orientation. In this case the natural undulations of the leaf caused the transition between laminar and turbulent flow to occur with Re between 400 and 3000.

L24.3 Momentum Transfer and Boundary Layers

An understanding of leaf boundary layers requires an examination of momentum transfer, as described by Newton's Law of Viscosity:

> *the shear force per unit area is proportional to the negative of the local velocity gradient.*

To visualize the process, let's consider a fluid between two plates and set lower plate in motion. As time proceeds the fluid adjacent to the plate gains speed. And fluid at various adjacent layers close to the moving plate start moving too, but at a lower speed. At steady state a linear profile of fluid velocity is established that is bound between the speed of the bulk fluid (in this case zero, for a stagnant volume) and the speed of the moving plate. A constant force (F) is required to maintain the motion of the plate and to keep the fluid velocity profile intact. Force per unit area is proportional to the velocity decrease in the distance Z. The constant of proportionality is the dynamic viscosity (μ).

$$\frac{F}{A} = \mu \frac{V}{Z}$$

The dynamic viscosity has units of kg m^{-1} s^{-1}. A related term, kinematic viscosity, is defined by normalizing dynamic viscosity by the density of the fluid

$$\upsilon = \frac{\mu}{\rho} \ (m^2\ s^{-1})$$

Viscosity is a function of the molecular velocity and its mean free path, so it is dependent upon temperature and pressure.

In laminar flow, momentum is transferred to the surface layer by layer. The motion at the layer adjacent to the surface is retarded by friction. This frictional force is equal and opposite in direction of the direction of fluid motion. Fluid with higher momentum imparts some momentum to the adjacent and slower moving layer. Thereby, the shear force per unit area is proportional to the negative of the velocity gradient.

$$\tau = -\mu \frac{du}{dz} \ (kg\ m^{-1}\ s^{-2})$$

We note that momentum in the horizontal direction (x) is transmitted through the fluid in the z direction. Gases and liquids that behave in this manner are called 'Newtonian' Fluids. For laminar flow the streamlines are straight lines.

The thickness of the boundary layer can be predicted as (Bird, Stewart and Lightfoot, 1960):

$$\delta(x) = 4.64\sqrt{\frac{\nu x}{u_\infty}}$$

In another analysis, Monteith and Unsworth (Monteith and Unsworth 1990) report a different relation for the boundary layer depth.

$$\delta(x) = 172l\sqrt{Re}$$

The boundary layer thickness 5 cm from the upwind edge of a flat leaf would be range from 0.0127 m (12.7 mm) to 0.00127 (1.27 mm) m as u increases from 0.1 to 10 m s-1.

The velocity distribution is:

$$\frac{u}{u_\infty} = \frac{3}{2}\left(\frac{z}{\delta(x)}\right) - \frac{1}{2}\left(\frac{z}{\delta(x)}\right)^3$$

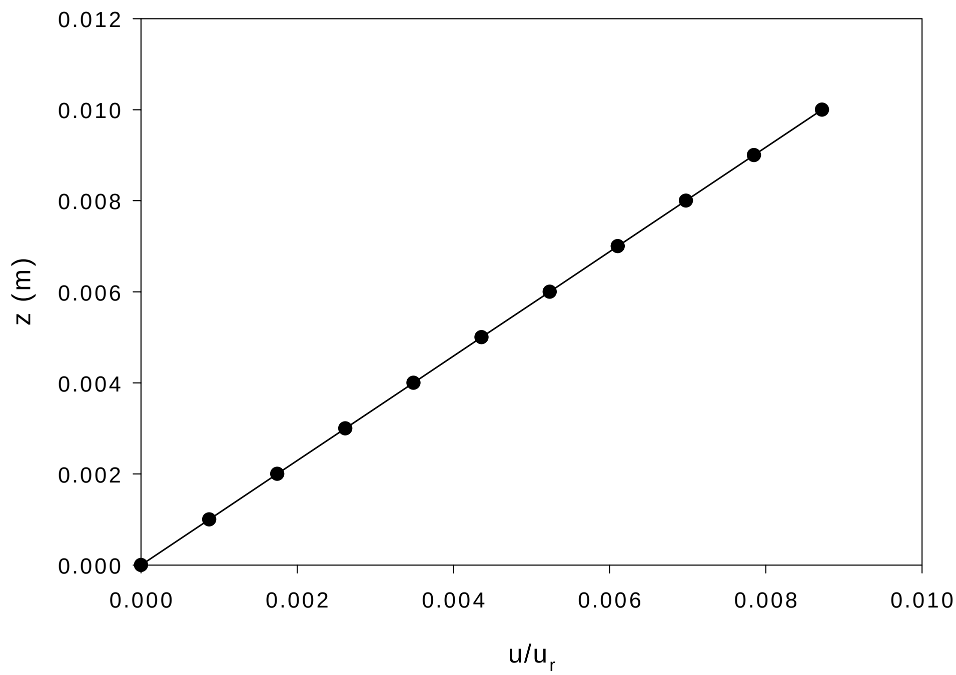

Figure 2 Wind profile in a laminar boundary layer

The classic view of evolution of flow over a leaf starts with a uniform and laminar stream of air upwind from a plate or leaf. As the air encounters the leaf there is drag at the surface and shear starts. A wind velocity profile and a boundary layer evolves. Initially the flow remains laminar throughout the boundary layer. But after a distance into the edge, flow becomes tripped and turbulence is generated. A logarithmic wind profile developes in the turbulent zone. But there is always a laminar boundary layer in close contact with the leaf, as z goes to zero. In the turbulent zone, there is a turbulent and laminar boundary layer.

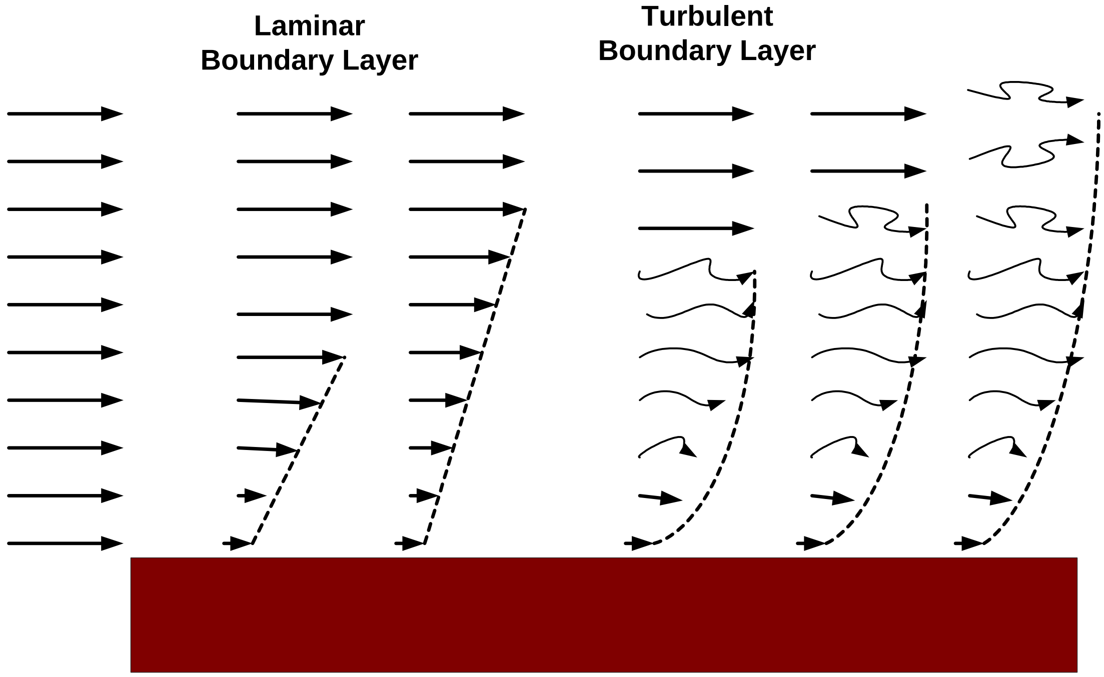

Figure 3 Evolution of laminar and turbulent boundary layers. Adapted from Monteith and Unsworth (Monteith and Unsworth 1990)

In the laminar zone, the diffusivity profile is constant with height, while in the turbulent zone, the turbulent diffusivity increases with height.

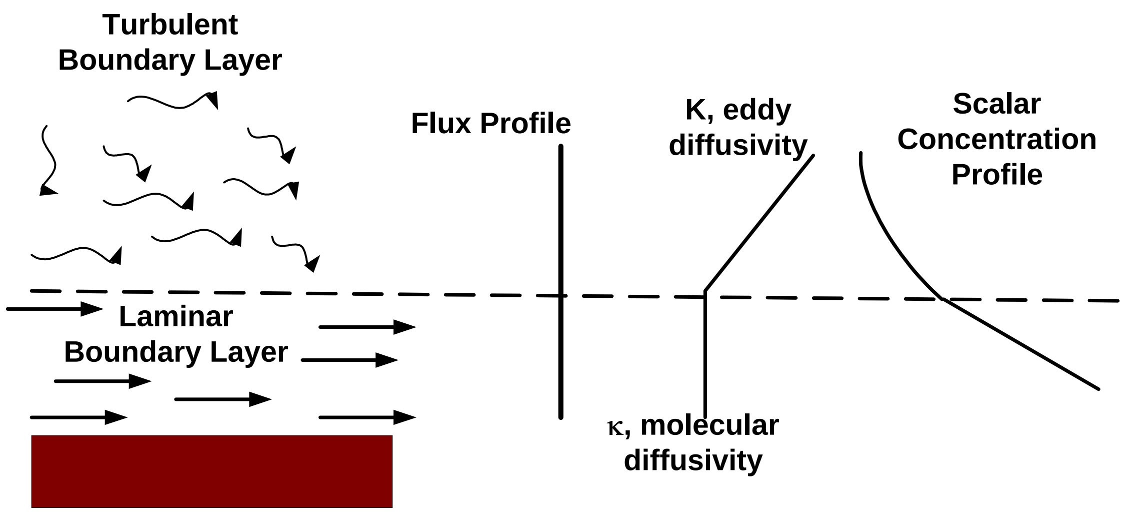

Figure 4 Conceptual diagram of flux, diffusivity and scalar profiles through adjacent laminar and turbulent boundary layers. Adapted from Oke(Oke 1987)

L24.4 Leaf Boundary Layer Resistance for Momentum Transfer

With regards to momentum transfer, the skin friction force exerted by the air onto the surface can be computed as:

$$\tau = \frac{\rho u}{r_m}$$

r_m is the resistance to momentum transfer (s m^{-1}). The resistance, or inverse conductance, is defined in relation to the vertical distance momentum or a scalar must be transferred. In laminar flow it is expressed as:

$$r_m = g^{-1} = \int_{z_1}^{z_2} \frac{dz}{\nu}$$

From engineering studies, a common representation for r_m is:

$$r_m = 1.5(\frac{l}{u\nu})^{0.5} = 1.5u^{-1}\, Re^{0.5}$$

Form drag is an additional force imposed on air flow. It is a force that is transferred across the wind velocity streamlines and is greatest when the object is perpendicular to the flow. It is a function of the size, shape and orientation of the object immersed in the fluid.

The drag force can be estimated by considering momentum transfer to a stagnation point that is perpendicular to the wind flow, which would be the case for maximal form drag. The initial momentum of the fluid would be ρu. The mean wind speed across the transition would be u/2, so the rate of momentum transfer would be:

$$\tau = 0.5\rho u^2$$

In normal conditions, fluid slips around the surface. Hence, the drag force is parameterized as:

$$\tau = \rho C_d u^2$$

C_d is the drag coefficient. Its value ranges between 0.4 and 1.2 for cylindrical and spherical objects. It is a function of leaf size, shape and orientation. As a word of caution, some drag coefficients consider the factor 0.5 noted above and others do not. It is critical to consult the fundamental equation from which reported drag coefficients are derived.

In practice, momentum transfer to leaves consists of a combination of skin friction and form drag. One relation for a drag coefficient is:

$$C_d = C_f + n\sqrt{u}$$

The drag coefficient of an isolated leaf is not equal to that of real leaves that are arranged in clumps (Landsberg and Thom, 1971, Thom, 1971). A concept of shelter factors is used to translate leaf drag information to the canopy scale. Shelter factors increase with friction velocity. For soybeans, their magnitude can range between 1.4 to 2.2. Woody trees can posses shelter factors upwards towards ten. Sheltering in conifers differs from broadleaves (Grant, 1984). First there is sheltering among needles in shoots, then there is sheltering among shoots.

L24.5 Leaf Boundary Layer Resistances/Conductances for Heat and Mass Transfer

For heat and mass transfer the flux density for some scalar, c, (mole m^{-2} s^{-1}) can be expressed using the resistance-analog algorithm:

$$F_x = \frac{\rho_{x,a} - \rho_{x,s}}{r_x} = g_x(\rho_{x,a} - \rho_{x,s})$$

From this relation, a resistance or conductance can be defined as:

$$r_x = g^{-1} = \int_{z_1}^{z_2} \frac{dz}{D_x}$$

In this form the resistance has units of s m^{-1}. Another way of expressing a resistance is as the ratio over a length scale, *l*, and the molecular diffusivity for the transfer of a specific molecule in a specific gas mixture, D_c:

$$r_c = \frac{1}{g_c} = \frac{l}{D_c}$$

If the flux density is measured in terms of mixing ratios rather than molar densities then one uses a slightly different definition for resistance/conductance:

$$r_c = \frac{1}{g_c} = \frac{V \cdot l}{D_c}$$

where *V* is the molar volume. In this situation, resistances and conductances are expressed in flux density units (mol m^{-2} s^{-1}). The resistances for heat and mass transfer differ from momentum transfer as they are not affected by form drag.

One can obtain information of the molecular diffusivity from physicochemical handbooks. The challenge with regards to evaluate resistances/conductances comes in trying to assess the length scale in terms of the gradient and its curvature (*dc/dn*):

$$l_c = \frac{c_{x,a} - c_{x,s}}{\partial c / \partial n}$$

Engineers use dimensionless numbers for mass transfer scale information and to compare results from various systems. For mass transfer one can express the boundary layer conductance for mass transfer as:

$$g_c = \frac{D_c Sh}{d}$$

where Sh is the Sherwood number and *d* is a characteristic length scale of the leaf, or object under investigation. The integrated length scale depends on the shape and size of the object and the direction of the wind. Technically, it can be defined as a weighted integral (Monteith and Unsworth 1990)):

$$d = \left(\frac{\int_0^l d(y)dy}{\int_0^l \sqrt{d(y)}dy} \right)^2$$

For an animal, d is related to its volume: $d=V^{1/3}$
For a rectangle or cylinder pointed along the wind: d=length
For a circular disk: d =0.81 w
For a sphere and perpendicular cylinder: d=diameter

For heat transfer, the boundary layer conductance has a similar form and is expressed as;

$$g_h = \frac{D_h Nu}{d}$$

Nu is the Nusselt number.

A summary of Dimensionless numbers used in the next two lectures is presented in Table 1. Details on how to assess *Sh* and *Nu* are given in the next section.

Table 1. A summary of dimensionless numbers used to compute leaf boundary layer conductances and a description of their physical meaning, see (Schuepp 1993).

Reynolds number	Re	$\mathrm{Re} = \frac{ul}{\upsilon}$	Inertial to visous forces
Schmidt	Sc	$Sc = \frac{\upsilon}{D_c} = \frac{\mu}{\rho D_c}$	Kinematic viscosity to molecular diffusivity
Prandtl	Pr	$\mathrm{Pr} = \frac{\upsilon}{D_t}$	Kinematic viscosity to thermal diffusivity
Sherwood	Sh	$Sh = \frac{g_c l}{D_c}$	Dimensionless mass transfer conductance (conductance divided by the ratio of the molecular diffusivity and a length scale, l)
Grasshof	Gr	$Gr = \frac{l^3 \rho^2 g \beta \Delta T}{\mu^2}$	Buoyant force times an inertial force to the square of the viscous force
Nusselt	Nu	$Nu = \frac{g_h l}{D_t}$	Dimensionless heat transfer conductacne

Summary of Concepts

- A laminar sublayer always exists close to the surface of leaves, even when experiencing turbulent flow
- The Reynolds' number quantifies whether a leaf is experiencing turbulent or laminar flow and increases with characteristic leaf size.
- The conductance for mass transfer is proportional to the molecular diffusivity and the Sherwood number and is inversely proportional to the characteristic leaf size.
- A constant flux layer exists for heat and mass transfer through the laminar and turbulent boundary layers. The products of molecular (or turbulent) diffusivities and concentration gradients interact to preserve this constancy.
- $r_m = g^{-1} = \int_{z_1}^{z_2} \frac{dz}{\nu}$
- $r_x = g^{-1} = \int_{z_1}^{z_2} \frac{dz}{D_x}$

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