

## Lecture 28, Leaf Energy Balance, Part I

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### Topics to be Covered

- A. Introduction
  - a. History of leaf energy balance studies
  - b. Role of leaf energy balance on Evolution and leaf shape
  - c. Resistance/conductance networks
  
- B. Leaf Energy Balance: Steady-State Linear Theory
  - 1. Dry leaf
    - a.  $T_l - T_a$ , leaf-air temperature differences
  - 2. Wet and transpiring leaf
    - a.  $T_l - T_a$ , leaf-air temperature differences
    - b. Evaporation,  $f(\text{net radiation})$
    - c. Evaporation,  $f(\text{isothermal radiation})$
  - 3. Night, dew and frost

### L27. 1 Introduction and History

If one goes to the San Francisco Botanical Garden one can view a wide array of leaf sizes. Large leaves exist that have diameters approaching a meter. Others are tiny and fragile, less than a millimeter. How do all of these plants co-exist? We can address this question and other related ones by examining the energy balance of leaves and by determining the factors that affect its temperature.

The temperature of leaves is often different than the temperature of the air. Under some circumstances leaves are warmer than air temperature. Under other conditions they may be cooler. Many biophysical processes respond non-linearly to temperature. Hence we need to evaluate the temperature where the processes are occurring, e.g. the leaf. Leaf temperature affects saturation vapor pressure at leaf surface, it is correlated with isotope fractionation, it affects molecular diffusivity, respiration, photosynthesis, hydrocarbon emission rates, enzyme kinetics. So if we want to evaluate these important processes correctly we must evaluate the temperature where the action is, at the leaf surface.

It is evolutionarily imperative for a leaf to manage its temperature to prevent a runaway energy exchange that could cause lethal and catastrophic temperatures. A way by which this has occurred is through the evolution of stomata, active pores that regulate the loss of water and diffusional uptake of CO<sub>2</sub>. Other factors that can contribute to the status of the leaf energy balance include its stomatal density and capacity, transpiration, leaf size, orientation and shape, its spatial position in the canopy, leaf optics, pubescence and roughness. If one examines the plants with huge leaves in the botanical garden, one will notice that they tend to be associated with tropical plants growing in the humid understory of the forest and the plants I saw were growing in pools of water.

The subject of leaf temperature was quite controversial in the early 20<sup>th</sup> Century. Early historical bias and conventional wisdom concluded that leaf temperature was warmer than air (Curtis, 1926). But this concept came from studies in the humid east and glass houses. By 1938 Wallace and Clum had reported that leaves could be 7 C below air temperature (Wallace, Clum, 1938). However, Curtis refuted these data and argued they were based on poor experimentation and faulty logic (Curtis, 1938). He was especially critical of results from wilted leaves with vasoline that prevented transpiration and still resulted in 'transpirational cooling'. To bring closure to this problem and explain why do large range of values in leaf-air temperatures exist, one must look at theory and wait for proper measurements of leaf energy exchange. Klaus Rasche (Raschke, 1960) and David Gates (Gates, 1965; Gates, 1966) are credited with formulating one of the first models that evaluate the energy balance of a leaf. Raschke's work was stimulated while stationed in India. There he became curious about correlations between leaf size, climate zone and position in the canopy. He was particularly interested in the relation between leaf size, temperature and transpiration (eg the role of leaf boundary layers and temperature). It was uncertain whether smaller leaves cooled easier, so they were able to conserve water or they possessed a thinner boundary layer, so they were able to transpire more efficiently. Gates was a physicist interested in biophysics of plants. He made early measurements of net radiation and spectral reflectances of leaves and extended these measurements to study leaf energy balance. A thorough evaluation of his work is available in his book *Biophysical Ecology* (Gates, 1980).

By the 1970s ecologists were using energy exchange measurements to understand the form and function of leaves (Parkhurst, Loucks, 1972; Taylor, 1975). In a classic study on optimal leaf form S.E. Taylor (Taylor, 1975) cites example of *Quercus douglassii*, the species I work with, with having larger leaves on sheltered northeast slopes than on sunnier southwest slopes. We also find they have very erect leaves, to minimize the amount of incident sunlight.

To better understand how leaves behave and evolved, one needs to consider how the environment has changed over the evolutionary history of a leaf. Several hundred million years ago, CO<sub>2</sub> levels were about 2000 ppm. In this environment, temperatures were elevated due to a greenhouse effect, stomatal conductances were lower and solar radiation was less than today. Leaf energy exchange had to be plastic to survive that initial environment, as well as today's. McElwain et al (McElwain et al., 1999) argue that leaf temperatures could have exceeded air temperature by 10C in the elevated

environment of the Triassic-Jurassic period, when CO<sub>2</sub> concentrations could have been 2000 ppm plus and air temperature 3 to 4 oC warmer on a global basis. If true this effect could have forced natural selection towards smaller leaves, that would not reach lethal temperatures of 45 oC. It could be a reason why there was a 95% turnover in megafloora in Europe. The reduction in taxa with respect to leaf size at the Triassic-Jurassic boundary was 99, 80, and 60 to 10% for leaves 5, 2 to 3 and 0.5 to 1 cm wide, respectively. Now this is a truly interesting and exciting Biometeorological application!!

In our modern era, scientists are able to simulate complex temperature patterns across leaves of varying shapes as was produced recently by Roth-Nebelsick (Roth-Nebelsick, 2001).

636 A. Roth-Nebelsick

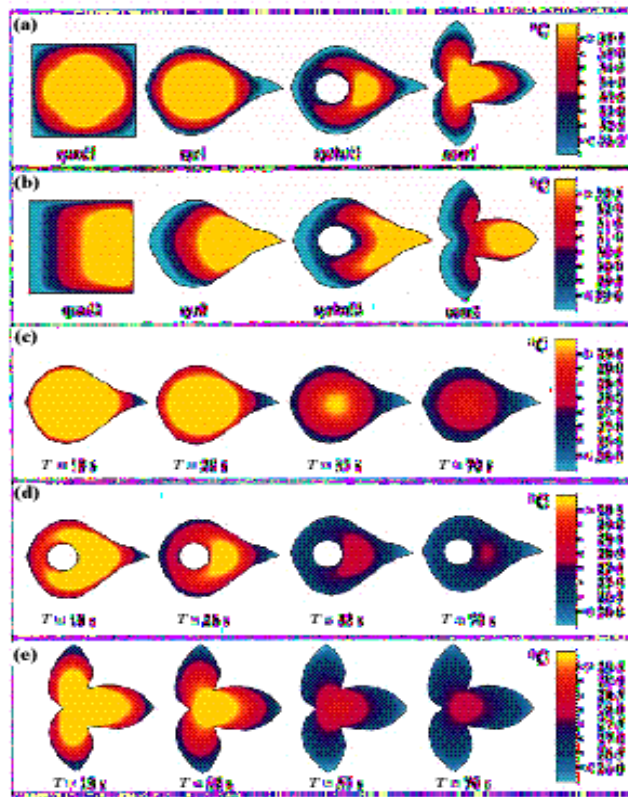


Figure 5. Contour plots of temperature distribution of various model structures. Steady-state heat flux: (a) surface temperature pattern of quad1, syr1, syrho1 and acer1; (b) surface temperature pattern of quad2, syr2, syrho2 and acer2. Transient heat flux: (c) temporal development of surface temperature pattern of model structure syr5; (d) temporal development of surface temperature pattern of model structure syrho5; (e) temporal development of surface temperature pattern of model structure acer5.

**Figure 1 adapted from Roth-Nebelsick 2001. Impact of leaf size on the transient development of heat transfer across a spectrum of leaf shapes.**

Geochemists are able to deduce information about temperature by measuring the amount of stable isotopes in their leaf, water and ice samples. Recently, a group of researchers inferred that leaf temperatures seem to be stable and that there is subtropical to boreal convergence in tree leaf temperatures, approaching 21 C (Helliker, Richter, 2008). Whether or not this finding holds up to scrutiny can be conducted with leaf energy balance models and knowing something about the physics and biology that controls leaf temperature.

The energy budget of a leaf is one of the most fundamental subjects of biometeorology. Knowledge of leaf temperature is critical to compute evaporation, heat flux, photosynthesis and respiration and pollutant transfer. Status of leaf temperature affects the presence or absence of insects and the incubation of pathogens. As we will see towards the end of this lecture, we can also use leaf energy balance arguments to explain certain aspects of evolution! To achieve this feat, leaf temperature should be maintained at non-lethal level (< 45 °C). This can be done by altering leaf angle to the sun (heliotropism, leaf angle), leaf position (clumping, shoots), the energy balance (reflectance), surface aerodynamics (aerodynamic resistance via leaf width and shape, leaf, needle), physiology and function (amphi- or hypostomatous stomatal arrangement,  $C_3, C_4, V_{cmax}$ ).

If a leaf is **amphistomatous**, stomata are arranged on both sides of the leaf. Then  $r_a=r_b$ . This leads to:

$$R=r/2$$

If a leaf is **hypostomatous** stomata are on only one side (usually the lower side). In this case we can assume that  $r_b$  essentially goes to infinity. In this case  $R=r_a, G=g_a$

We will examine several cases of leaf energy balances. Our goal is to examine leaf temperature. But one can use a similar approach to evaluate leaf evaporation and sensible heat exchange.

## 28.2 Dry, Non-Transpiring Leaves, Computing Leaf Energy Budget and Leaf-Air temperature differences

Under steady steady-state conditions the net radiation balance ( $R_n, W m^{-2}$ ) is the sum of incoming and outgoing fluxes of short ( $R$ ) and longwave energy ( $L$ ). For a dry leaf, this energy is consumed as sensible heat ( $H$ ):

$$R_n = R \downarrow - R \uparrow + L \downarrow - L \uparrow = H$$

where sensible heat flux density ( $H$ ) is expressed in terms of an aerodynamic conductance,  $g_a$  or  $g_h$ , air density and the specific heat at constant pressure.

$$H = \rho_a C_p (T_l - T_a) g_h$$

Assuming we can measure  $R_n$  we can derive a very simple relation for the leaf-air temperature difference:

$$(T_l - T_a) = \frac{R_n}{g_h \rho_a C_p}$$

In actuality,  $R_n$  will vary with leaf temperature, so this equation is too simple (Paw U, Gao, 1988; Schuepp, 1993). Instead one prefers to derive leaf temperature as a function of the net input of short radiation and incoming longwave radiation. We also have to consider the reflection and emission of long wave energy at the leaf surface:

$$L\uparrow = \varepsilon\sigma T_l^4 + (1 - \varepsilon)L\downarrow$$

Substituting the equation for longwave losses into  $R_n$  yields:

$$R_n = R\downarrow - R\uparrow + L\downarrow - [\varepsilon\sigma T_l^4 + (1 - \varepsilon)L\downarrow]$$

Combining terms and simplifying produces

$$R_n = R\downarrow - R\uparrow + \varepsilon L\downarrow - \varepsilon\sigma T_l^4$$

For convenience we now define  $Q$  as the sum of the available short and longwave radiation flux densities, as this will be a quantity we can measure and is external to the system under study:

$$Q = R\downarrow - R\uparrow + \varepsilon L\downarrow$$

Further simplification is accomplished by defining the outgoing radiation flux density in terms of leaf reflectance,  $\rho$

$$R\uparrow = \rho R\downarrow$$

$$Q = (1 - \rho)R\downarrow + \varepsilon L\downarrow$$

At this stage the leaf energy balance is re-expressed as:

$$Q = (1 - \rho)R\downarrow + \varepsilon L\downarrow = \varepsilon\sigma T_l^4 + \rho C_p g_h (T_l - T_a)$$

We do not know leaf temperature, but we know air temperature. We can use Taylor's Expansion theory to evaluate the long wave emission as a function of air and leaf temperature and solve for the leaf-air temperature difference

$$f(x) \sim f(x_0) + (x - x_0) \frac{df}{dx}$$

$$\varepsilon\sigma T_l^4 = \varepsilon\sigma T_a^4 + 4\varepsilon\sigma T_a^3 (T_l - T_a)$$

Substitution now produces:

$$Q - \varepsilon\sigma T_a^4 - 4\varepsilon\sigma T_a^3(T_l - T_a) = \rho_a C_p (T_l - T_a) g_h$$

Re-arranging terms and solve for the leaf-air temperature difference yields:

$$T_l - T_a = \frac{Q - \varepsilon\sigma T_a^4}{\rho_a C_p g_h + 4\varepsilon\sigma T_a^3}$$

### 28.3 Leaf Energy Budget, Computing Leaf-Air Temperature Differences and Evaporation Rates of Transpiring Leaves

#### a. Leaf-Air Temperature Differences

To compute the evaporation rate of leaves we start with two fundamental equations. One is the leaf energy balance is defined by how net radiation is consumed by sensible and latent energy ( $\lambda E$ ) exchange:

$$R_n = H + \lambda E$$

The other is the equation for the flows (flux densities) of solar and terrestrial energy, that comprises the net radiation balance.

$$Q = (1 - \rho)R \downarrow + \varepsilon L \downarrow = \varepsilon\sigma T_l^4 + H + \lambda E$$

We make the fundamental assumption that the leaf is exposed to steady steady-state conditions.

The resistance network for water vapor transfer consists of a route through the leaf boundary layer and the stomata. These two resistances are in series, so their conductances are in parallel:

$$r_a + r_s = \frac{1}{g_a} + \frac{1}{g_s} = \frac{g_a + g_s}{g_a g_s} = \frac{1}{g_w} :$$

$$\begin{aligned} \lambda E &= \frac{(m_v / m_a) \lambda \rho_a g_w (e_s(T_l) - e_a)}{P} = \\ &= \frac{(m_v / m_a) \lambda \rho_a (e_s(T_l) - e_a)}{P} \frac{g_s g_a}{g_a + g_s} = \\ &= \frac{(m_v / m_a) \lambda \rho_a (e_s(T_l) - e_a)}{P(r_s + r_a)} \end{aligned}$$

and  $m_v / m_a$  is 0.622.

To solve this set of equations we apply a relation that linearizes the Saturation Vapor Pressure relation and the Stefan Boltzmann Equation for the emission of long wave radiation. By apply Taylor's Expansion Theory we estimate the saturation vapor pressure at leaf temperature as:

$$e_s(T_l) = e_s(T_a) + e_s'(T_l - T_a) + \frac{e_s''}{2}(T_l - T_a)^2$$

The slope of the saturation vapor pressure curve is denoted:

$$e_s(T)' = \frac{de_s(T)}{dT} = s$$

and the second derivative is denoted:

$$e_s(T)'' = \frac{d^2e_s(T)}{dT^2}$$

The squared term in the Taylor series expansion is negligible leaving:

$$e_s(T_l) = e_s(T_a) + e_s'(T_l - T_a)$$

At this stage our goal is twofold, to derive analytical equations that solve for latent heat exchange,  $\lambda E$ , and leaf temperature  $T_l$ . The beauty of the equation for latent heat exchange is that the unknown,  $T_l$ , is eliminated. Value of deriving analytical equations for leaf evaporation is that the relation can be used in sensitivity tests to understand how evaporation responds to changes in forcing variables and model parameters, such as the leaf and aerodynamic conductances.

Let's start with the simplest case for evaporation from a wet leaf where  $g_w$  is the leaf water vapor conductance, combining the stomatal and boundary layer effects (next we will derive an equation for the expanded version of  $g_w$ ).

$$Q - \varepsilon\sigma T_a^4 - 4\varepsilon\sigma T_a^3(T_l - T_a) = \rho_a C_p (T_l - T_a) g_a + \frac{0.622\lambda\rho_a g_w (e_s(T_a) - e_a)}{P} + \frac{0.622\lambda\rho_a g_w e_s'(T_a)}{P} (T_l - T_a)$$

Applying the equation for the vapor pressure deficit makes a simplification,  $D = e_s(T_a) - e_a$ .

Next we re-arrange the equations and solve for  $T_l - T_a$ . This manipulation yields:

$$Q - \varepsilon\sigma T_a^4 - 4\varepsilon\sigma T_a^3(T_l - T_a) = \rho_a C_p (T_l - T_a) g_a + \frac{0.622\lambda\rho_a g_w D}{P} + \frac{0.622\lambda\rho_a g_w e_s'(T_a)}{P} (T_l - T_a)$$

and

$$\rho_a C_p (T_l - T_a) g_a + 4 \varepsilon \sigma T_a^3 (T_l - T_a) + \frac{0.622 \lambda \rho_a g_w e_s(T_a)'}{P} (T_l - T_a) = Q - \varepsilon \sigma T_a^4 - \frac{0.622 \lambda \rho_a g_w D}{P}$$

$$T_l - T_a = \frac{Q - \varepsilon \sigma T_a^4 - \lambda g_w 0.622 D / P}{\rho_a C_p g_h + 4 \varepsilon \sigma T_a^3 + \lambda 0.622 e_s(T_a)' \rho_a g_w / P}$$

So far we have been concerned with the **top-side** of a leaf. We must also derive equations for cases where the stomata are on both or one side of a leaf and the energy exchange is occurring on both sides.

1: **Amphistomatous** leaves have stomata on both sides. Their net radiation balance is:

$$R_n = 2H + 2\lambda E = (R \downarrow - R \uparrow + L \downarrow - L \uparrow)_{top} + (R \uparrow - R \downarrow + L \uparrow - L \downarrow)_{bottom}$$

2: **Hypostomatous** leaves have stomata on one side. Their net radiation balance is:

$$R_n = 2H + \lambda E = (R \downarrow - R \uparrow + L \downarrow - L \uparrow)_{top} + (R \uparrow - R \downarrow + L \uparrow - L \downarrow)_{bottom}$$

Evaluation of leaf-air temperature differences for hypostomatous leaves follows:

$$Q_n = (R \downarrow - R \uparrow + L \downarrow)_{top} + (R \uparrow - R \downarrow + L \uparrow)_{bottom}$$

$$Q_n - 2\varepsilon \sigma T_a^4 + 8\varepsilon \sigma T_a^3 (T_l - T_a) = 2\rho_a C_p (T_l - T_a) g_a + \frac{0.622 \lambda \rho_a g_w (e_s(T_a) - e_a)}{P} + \frac{0.622 \lambda \rho_a g_w (de_s / dT)}{P} (T_l - T_a)$$

$$Q_n - 2\varepsilon \sigma T_a^4 - \frac{0.622 \lambda \rho_a g_w (e_s(T_a) - e_a)}{P} = (T_l - T_a) \left[ 2\rho_a C_p g_a - 8\varepsilon \sigma T_a^3 + \frac{0.622 \lambda \rho_a g_w (de_s / dT)}{P} \right]$$

$$(T_l - T_a) = \frac{Q_n - 2\varepsilon \sigma T_a^4 - \frac{0.622 \lambda \rho_a g_w (e_s(T_a) - e_a)}{P}}{\left[ 2\rho_a C_p g_a - 8\varepsilon \sigma T_a^3 + \frac{0.622 \lambda \rho_a g_w (de_s / dT)}{P} \right]}$$

### *b. Evaporation Based on the Net Radiation Balance*

So far we have been focusing on leaf-air temperature differences. We now want to use this information to compute evaporation rates (Jarvis, McNaughton, 1986; Paw U, Gao, 1988). We can do this in terms of the net radiation budget. But this version needs to know already the surface temperature or must have an independent measure of  $R_n$ .

For this analysis we start with the leaf energy budget, which defines that the net radiative flux density is partitioned into sensible and latent heat flux density:

$$R_n = H + \lambda E$$

Solving for latent heat flux density yields:

$$\lambda E = R_n - H$$

Next we invoke the conductance/resistance relations for sensible and latent heat exchange. The conductance equation for sensible heat exchange is:

$$H = \rho_a C_p (T_l - T_a) g_h$$

The relation for latent heat exchange is:

$$\begin{aligned} \lambda E &= \frac{(m_v / m_a) \lambda \rho_a g_w (e_s(T_l) - e_a)}{P} = \\ &= \frac{(m_v / m_a) \lambda \rho_a (e_s(T_l) - e_a)}{P} \frac{g_s g_a}{g_a + g_s} = : \\ &= \frac{(m_v / m_a) \lambda \rho_a (e_s(T_l) - e_a)}{P(r_s + r_a)} \end{aligned}$$

We simplify the equation for  $\lambda E$  by applying the linearized form of the equation for  $e_s(T_l) - e_a$ :

$$\begin{aligned} \lambda E &= \frac{\rho_a (m_v / m_a) \lambda g_w (D + e_s'(T_l - T_a))}{P} \\ e_s(T_l) - e_a &= D + e_s'(T_l - T_a) = \frac{\lambda E \cdot P}{\rho_a \lambda (m_v / m_a) g_w} \end{aligned}$$

As an aside, it should be noted that many derivations include the psychrometric constant, defined in terms of air pressure, the specific heat of dry air at constant pressure and the latent heat of evaporation.

$$\gamma = \frac{PC_p}{0.622\lambda}$$

The next key step is to eliminate the leaf-air temperature difference. This is accomplished by solving for it in terms of latent heat exchange:

$$T_l - T_a = \Delta T = \frac{\lambda E \gamma}{s \rho_a C_p g_w} - \frac{D}{s}$$

$$\lambda E = R_n - \left( \frac{\lambda E \gamma}{s \rho_a C_p g_w} - \frac{D}{s} \right) \rho_a C_p g_h$$

$$\lambda E = R_n - \left( \frac{\lambda E \gamma g_a}{s g_w} - \frac{D \rho_a C_p g_h}{s} \right)$$

Next we re-arrange the equation so we can solve for  $\lambda E$ :

$$\lambda E + \frac{\lambda E \gamma g_a}{s g_w} = R_n + \frac{D \rho_a C_p g_a}{s}$$

$$\lambda E \left( 1 + \frac{\gamma g_h}{s g_w} \right) = R_n + \frac{D \rho_a C_p g_h}{s}$$

A final simplification is made by multiplying the equation by  $s$  to remove it from the denominator:

$$\lambda E \left( s + \frac{\gamma g_a}{g_w} \right) = s R_n + D \rho_a C_p g_h$$

And Voila', we now have an equation for the evaporation from the single side of a leaf, knowing its net radiation balance, without needing to know its temperature"

$$\lambda E = \frac{s R_n + D \rho_a C_p g_h}{\left( s + \gamma \frac{g_h}{g_w} \right)}$$

Breaking  $g_w$  into  $g_s$  and  $g_{av}$ , while retaining  $R_n$  yields:

$$\lambda E = \frac{s R_n + D \rho_a C_p g_h}{\left( s + \gamma \left( 1 + \frac{g_h}{g_s} \right) \right)}$$

### c. Evaporation base on Isothermal Radiation Analysis

Typically do not know  $R_n$ , or  $T_l$ . Furthermore, for many analyses we are concerned about the effects of elevated or suppressed leaf temperature on longwave emission, and net radiation, it is preferred instead to identify the isothermal net radiation budget and how it

feedbacks on evaporation. For example if leaf temperature, becomes elevated with respect to air temperature it can increase D, but it also **reduces** the longwave emission and the amount of net radiation that is **available** for evaporation.

To proceed along these lines we define the Isothermal net radiation (Jarvis, McNaughton, 1986). It is the radiation flux density that occurs when the leaf and air temperature are equal.

$$R_{ni} = Q - \sigma \epsilon T_a^4$$

$$\begin{aligned} R_n &= Q - \sigma \epsilon T_l^4 = \\ &Q - (\sigma \epsilon T_a^4 + 4\sigma \epsilon T_a^3 (T_l - T_a)) = \\ &R_{ni} - 4\sigma \epsilon T_a^3 (T_l - T_a) \end{aligned}$$

$$R_n = H + \lambda E$$

Solving for latent heat flux density yields:

$$\lambda E = R_n - H$$

Substituting the isothermal radiation balance produces:

$$\lambda E = R_{ni} - 4\sigma \epsilon T_a^3 (T_l - T_a) - H$$

Which allows us to produce an equation for latent heat exchange that is a function of the leaf-air temperature difference:

$$\lambda E = R_{iso} - 4\epsilon \sigma T_a^3 (T_l - T_a) - \rho C_p g_h (T_l - T_a)$$

$$\Delta T = (T_l - T_a)$$

$$\lambda E = R_{iso} - \Delta T (4\epsilon \sigma T_a^3 - \rho_a C_p g_h)$$

$$\Delta T = \frac{\lambda E \cdot P}{\rho_a \lambda (m_v / m_a) s g_w} - \frac{D}{s}$$

or

$$\Delta T = \frac{\lambda E \gamma}{s \rho_a C_p g_w} - \frac{D}{s}$$

Substituting the leaf-air temperature difference equation into the relation for  $\lambda E$  produces:

$$\lambda E = R_{iso} - \left( \frac{\lambda E \gamma}{s \rho_a C_p g_w} - \frac{D}{s} \right) (4 \sigma \varepsilon T_a^3 - \rho_a C_p g_h)$$

$$\lambda E = R_{iso} - \left( \frac{\lambda E \gamma}{s \rho_a C_p g_w} \right) (4 \sigma \varepsilon T_a^3 - \rho_a C_p g_h) + \frac{D}{s} (4 \sigma \varepsilon T_a^3 - \rho_a C_p g_h)$$

$$\lambda E \left( 1 + \frac{\gamma 4 \varepsilon \sigma T_a^3}{s \rho_a C_p g_w} + \frac{\gamma g_h}{s g_w} \right) = R_{iso} + \frac{D 4 \varepsilon \sigma T_a^3}{s} + \frac{\rho_a C_p D g_h}{s}$$

$$\lambda E \left( s + \frac{\gamma 4 \varepsilon \sigma T_a^3}{\rho_a C_p g_w} + \frac{\gamma g_h}{g_w} \right) = s R_{iso} + D 4 \varepsilon \sigma T_a^3 + \rho_a C_p D g_h$$

An algebraic simplification can be made using the radiative conductance:

$$g_r = \frac{4 \varepsilon \sigma T_a^3}{\rho C_p}$$

This term can be thought of as the resistance defining the conditions where sensible heat and emissive energy flux densities are in balance:

$$H = L \uparrow = 4 \varepsilon \sigma T_a^3 (T_l - T_a) = \rho C_p (T_l - T_a) g_h$$

Substitution of  $g_r$  produces:

$$\lambda E \left( s + \frac{\gamma g_r}{g_w} + \frac{\gamma g_h}{g_w} \right) = s R_{iso} + D \rho_a C_p (g_r + g_h)$$

$$\lambda E = \frac{s R_{iso} + D \rho_a C_p (g_r + g_h)}{\left( s + \frac{\gamma g_r}{g_w} + \frac{\gamma g_h}{g_w} \right)}$$

Next, we can refine the derivation by breaking the water vapor conductance into its components.

$$\frac{1}{g_w} = \frac{1}{g_s} + \frac{1}{g_{av}} = \frac{g_s + g_{av}}{g_s g_{av}}$$

Assuming, that the conductances for heat and water vapor are identical,  $g_h = g_{av}$ , leads to:

$$\lambda E = \frac{sR_{iso} + D\rho_a C_p (g_r + g_h)}{(s + \gamma \frac{g_s + g_{av}}{g_s g_{av}})(g_r + g_h)}$$

or:

$$\lambda E = \frac{sR_{iso} + D\rho_a C_p (g_r + g_h)}{(s + \gamma(1 + \frac{g_r}{g_{av}})(1 + \frac{g_{av}}{g_s}))}$$

## 27.4 Nocturnal Energy Balance

During the day the vapor pressure at the leaf surface is greater than the air, so the flux of water is upward, from the leaf to the atmosphere. At night, the surface of a leaf radiates to the atmosphere, so its temperature can be cooler than the air above (Leuning, 1988; Leuning, 1989). The free air, being warmer can retain more moisture than the air near the surface of the leaf. If the moisture comes from the underlying soil, then we have distillation. Hence the gradient of humidity is reversed, moisture is directed downward and can condense on the surface of the leaf as dew. Condensation or frost formation leads to a heat gain by the leaf surface, rather than a loss (as with evaporation)

At night leaf temperature is a balance between radiative energy losses to a cold sky and a gain of sensible and latent heat from the surrounding air (Leuning, 1988). Leaves with condensation can be 1 to 2 degrees C warmer than dry leaves. Free convection is unimportant at night as forced convection is the main mechanism for energy transfer.

At night, energy balance is computed assuming that stomata are closed, so there is no transpiration. The energy balance is driven by long wave energy emanating from the sky above and the ground underneath.

$$R \downarrow_{ad} + R \downarrow_{ab} = \epsilon \sigma ((T_{sky}^4 - T_l^4) + (T_{ground}^4 - T_l^4))$$

If we think about incoming and reflected long wave we have to consider

$$R \downarrow_{ad} + R \downarrow = \epsilon_{sky} \sigma T_{sky}^4 - (1 - \epsilon_{leaf}) \epsilon_{sky} \sigma T_{sky}^4$$

If there is condensation:

$$H + \lambda E = 2\rho C_p g_h (T_a - T_l) + \frac{2M_w \lambda_{s,f} g_w}{M_a P} (e_a - e_s(T_l))$$

The surface conductance goes to infinity as the surface is wet with condensation.

$$\lambda E = \frac{sR_{iso} + D\rho_a C_p (g_r + g_h)}{(s + \gamma(1 + \frac{g_r}{g_{av}}))(1 + \frac{g_{av}}{\infty})}$$

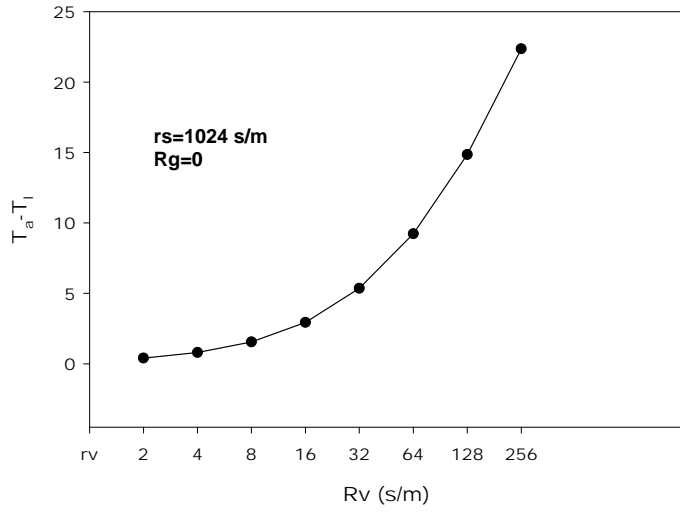
$$\lambda E = \frac{sR_{iso} + D\rho_a C_p (g_r + g_h)}{(s + \gamma(1 + \frac{g_r}{g_{av}}))}$$

Dew will fall when  $\lambda E < 0$  or  $-sR_{iso} > D\rho_a C_p (g_r + g_h)$

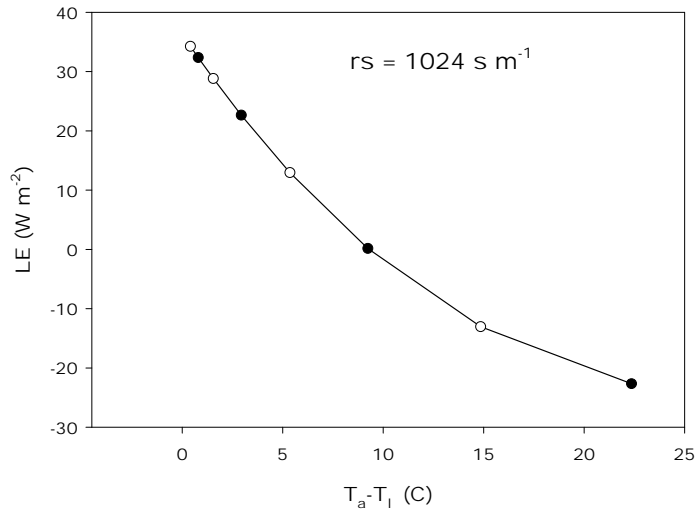
Here we must use the corrected latent heat coefficient, one for the condensation, fusion or distillation of water vapor.

The maximum rate of dew fall will be less than  $0.1 \text{ mm h}^{-1}$ , as for the case when  $R_{iso}$  is  $-100 \text{ W m}^{-2}$ . More often dew fall is on the order of 0.1 to 0.4 mm per night. Actual dewfall does not reach levels computed by theory. There is latent heat released when there is condensation. This heat offsets the radiative heat loss to the sky. Calm winds at night also limit the exchange of water vapor. Here, dew fall can only occur as the whole atmospheric surface layer becomes saturated. The lower atmosphere at a saturated state cannot meet the computed dewfall requirement. This fact leads to the role of distillation, the contribution of moisture from the soil.

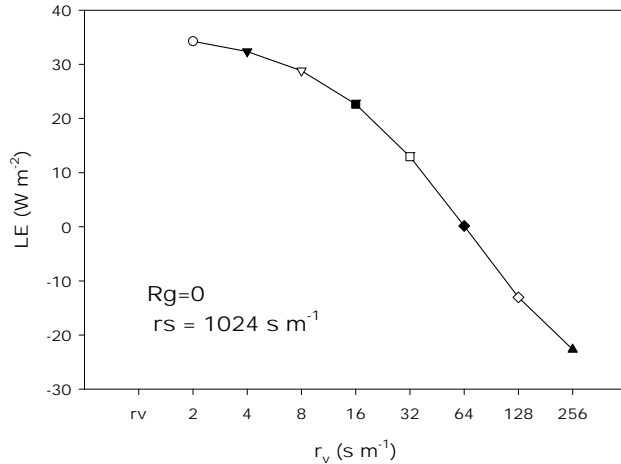
Leuning performed experiments of leaf temperature using paired energy balance of transpiring and non transpiring (coated with vasolene) can be used to assess transpiration rates from information on leaf temperature differences. This approach removes need for independent assessment of conductances. Leaf boundary layers conductances can also be assessed from pairs of heated and unheated plates



**Figure 2 Leaf-air temperatures at night with closed stomata**



**Figure 3 Evaporation/dew formation at night with closed stomata**



**Figure 4** Evaporation/dew formation at night with turbulent mixing.

### Summary

- Net radiation balance of a wet leaf is partitioned into sensible and latent heat exchange
- The leaf energy balance equation is derived using four principles and operations:
  - The leaf energy balance
  - Ohm's Law Resistance analogy for transpiration and sensible heat exchange
  - Linearization of the saturation vapor pressure relationship
  - Numerical elimination of leaf-air temperature differences
- The Isothermal energy balance derivation accounts for differences between leaf and air temperature, the loss of longwave energy and its impact on available energy
- Leaves can be hypostomatous or amphistomatous, a fact that affects the derivation of the leaf energy balance equation..

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