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D'ALEMBERT AND THE *VIS VIVA* CONTROVERSY

THE usual date cited for the conclusion of the controversy over the measure of force is 1743, the year of publication of Jean d'Alembert's *Traité de Dynamique.*[1] The controversy, however, lingered on for many years after this date. A recent study by L. L. Laudan has documented its existence through the remainder of the eighteenth century.[2] This and other articles have questioned the priority of d'Alembert's solution of the controversy.[3] However, none of these analyses has pointed out that there are significant differences between d'Alembert's discussions of the controversy in the first edition of the *Traité de Dynamique* of 1743 and in the revised second edition of 1758. The crucial argument that *vis viva* is the measure of a force acting through a distance while momentum is the measure of a force acting through a time was not given until 1758. As Pierre Costabel has shown, this argument had already been presented by Roger Boscovich in 1745.[4] The 1743 edition of the *Traité* goes only as far as distinguishing dead from living forces and characterizing the controversy as a dispute over words. As Thomas Hankins has pointed out, 'sGravesande called it a verbal debate as early as 1729.[5] The intention of this paper is to discuss the differences between the two editions of d'Alembert's *Traité* as regards the controversy over living force.

The first edition of 1743 accepts two valid measures of force: (*a*) the measure mdv for the case of equilibrium (*i.e.*, dead force), which d'Alembert equates misleadingly with quantity of motion, and (*b*) the measure mv^2 (living force) for the case of retarded motion where the 'number of obstacles overcome' is as the square of the velocity. Here force is defined as 'a term used to express an effect':

> Nevertheless as we have only the precise and distinct idea of the word force, in restricting this term to express an effect, I believe that the matter should be left to each to decide for himself as he wishes. The entire question cannot consist in more than a very futile metaphysical discussion or in a dispute of words unworthy of still occupying philosophers.[6]

To the second edition is added a section in which a third meaning is given

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to the measure of force. Here the valid measures of force are described as being (1) dead force, (2) the space traversed up to the total extinction of motion (mv^2) and (3) the space traversed uniformly in a given time (mv). Let us discuss the details of each of the two editions.

In his preface to the 1743 edition of the *Traité de Dynamique*, d'Alembert stated that he would consider the motion of a body only as the traversal of a certain space for which it uses a certain time. He rejected a discussion of the causes of motion and the inherent forces of moving bodies as being obscure, metaphysical and useless to mechanics. It was for this reason, he said, that he refused to enter into an examination of the question of living forces. Mentioning in passing the part played by Leibniz, Bernoulli, Maclaurin and a lady 'famous for her spirit' (Madame du Châtelet), d'Alembert proposed to expose succinctly the principles necessary to resolve the question.[7]

It is not the space uniformly traversed by a body, nor the time needed to traverse it, nor the simple consideration of the abstract mass and velocity, by which force should be estimated. Force should be estimated solely by the obstacles which a body encounters and by the resistance it offers to these obstacles. The greater the obstacles it can overcome or resist, the greater is its force, provided that by 'force' one does not mean something residing in the body.

One can oppose to the motion of a body three kinds of obstacle.[8] First, obstacles that can completely annihilate its motion; second, obstacles that have exactly the resistance necessary to halt its motion, annihilating it for an instant, as in the case of equilibrium; and third, obstacles that annihilate its motion little by little, as in the case of retarded motion. Since the insurmountable obstacles annihilate all motion they cannot serve to make the force known. 'One must look for the measure of the force either in the case of (a) equilibrium or (b) in that of retarded motion.' Concerning these two possibilities for a measure:

Everyone agrees that there is equilibrium between two bodies, when the products of their masses by their virtual velocities, that is the velocities by which they *tend* to move, are equal. Thus in equilibrium the product of the mass by its velocity, or, what is the same thing, the quantity of motion, can represent the force.[9] Everyone agrees also that in retarded motion, the number of obstacles overcome is as the square of the velocity. For example, a body which compresses one spring with a certain velocity can with a double velocity compress, all together or successively, not two but four springs similar to the first, nine with a triple velocity, *etc.*[10]

In this second case, continues d'Alembert, the force of a body is as the product of the mass by the square of the velocity. Should not then 'force' mean only the effect produced in surmounting an obstacle or resisting it? Force should be 'measured by the absolute quantity of the obstacles or by the sum of their resistances'. Hence we have the precise and distinct idea of 'force' as a term to express an effect.

In the above quotation d'Alembert incorrectly identifies the product of the mass and the virtual velocity with quantity of motion. In the case of equilibrium the measure of force is the product of the mass of the body and its virtual velocity, mdv.[11] This measure of force was what Leibniz had called dead force, or *vis mortua*, although he did not use the term virtual velocity or the expression mdv. For the case of moving bodies the measure of force was given by mv or quantity of motion, later called momentum. In the early years of the controversy, according to Leibniz the erroneous identification of mdv and mv was made by Cartesians such as Father Honoratius Fabri, Father Ignatius Pardies, Father Malebranche, Marcus Marci and Claude Deschales.[12] The error was also made by Abbé Catalan[13] and later by the British scientist J. T. Desaguliers.[14] D'Alembert does not make the distinction clear in the above paragraph, but in the 1758 edition an added section distinguishes between them. From this misleading use of the term quantity of motion may have arisen the idea that d'Alembert resolved the controversy in 1743.[15]

In summary, the 1743 edition of the *Traité* distinguishes two meanings of force. *Vis viva* is defined by the effect it can produce and is proportional to the square of the velocity. Secondly, force is defined for the case of equilibrium as the product of the mass and the virtual velocity, mdv.

D'Alembert's discussion concluded with the much quoted statement that 'the question cannot consist in more than a completely futile metaphysical question, or a dispute over words unworthy of still occupying philosophers'.[16] At this point in the 1758 edition of the *Traité de Dynamique*[17] are inserted what the foreword to that edition describes as 'several reflections on the question of living forces', 'added to the preliminary discourse'. In this insert three, rather than two, meanings of force are described.

The three cases are: (1) [dead force], where a body has a tendency to move itself with a certain velocity, but the tendency is arrested by some obstacle; (2) [quantity of motion], in which the body actually moves uniformly with this certain velocity; and (3) [living force], where the body moves with a velocity which is consumed and annihilated little by little by some cause. The effect produced in each case is different, because in each the action of the same cause is differently applied. The body in itself, however, possesses nothing more in one case than the other. 'In the first case, the effect is reduced to a simple tendency which is not properly a measure since no motion is produced; in the second *the effect is the space traversed uniformly in the given time* and this effect is proportional to the velocity; in the third case, *the effect is the space traversed up to the total extinction of motion*, and this effect is as the square of the velocity.'[18]

The two parties, d'Alembert added in 1758, are entirely in accord over the fundamental principles of equilibrium and motion, and their solutions are in perfect agreement. Thus the question is a 'dispute over words' and is 'entirely futile for mechanics'.

Thus, although the 1743 edition of d'Alembert's *Traité* had been cited by many authors as resolving the dispute, it provided little more clarification than contrasting dead with living forces and calling the argument a 'dispute over words'. 'sGravesande in 1729 had also called it a dispute over words but neither he nor d'Alembert (in 1743) really defined in what way this was true.

Although the 1758 edition of the *Traité* did point out that momentum could be considered as a force acting during a given time, and *vis viva* as a force acting over the space traversed, d'Alembert likewise was anticipated in this insight by Roger Boscovich. Pierre Costabel has shown that Boscovich's *De Viribus Vivis* (Rome, 1745) suggested a separate graphical representation for each of the two measures of force.[19] The *De Viribus Vivis* is a fifty-page work of difficult Latin dealing with two separate subjects involving living force. According to Costabel it shows that Boscovich possessed a very thorough understanding of the history of the quarrel before his own intervention, from Leibniz and Bernoulli to Voltaire, de Mairan and du Châtelet. Boscovich does not cite d'Alembert's *Traité de Dynamique*, but this had been published only two years earlier.[20] He did not meet d'Alembert until a visit to Paris in 1759. Nor does he mention Euler, whose *Mechanica* of 1736 contained ideas suggestive of a general treatment of mechanics. The reflections of Euler on the nature of forces did not take form until 1749–50. For comparative purposes certain aspects of Costabel's discussion will be summarized here with additional interpretations.

Employing both the ancient scholastic categories and the new mathematical methods of his time, Boscovich discussed the graphical representation of a pressure applied through a time and a force applied over a distance. *Vis activa*, for Boscovich, was the 'instantaneous action' by which a pressure (*pression*) passes into action and engenders a new velocity. He said that it corresponded to Leibniz's dead force (*vis mortua*).[21] This instantaneous pressure passes to a velocity, not by multiplication of effects in the course of an instant, but only by continuous application. In the same way a line produces a surface not by its own multiplication but by its continual motion along a path. Thus a pressure is related to the velocity produced as a straight line to the surface engendered. The pressure, an active presence (*puissance*), passes into action not by multiplication of effects but by generating a two-dimensional image adequately rendered only by geometry.

Without taking a position on the definition of force, Boscovich measured the velocity acquired as a ratio composed of the pressure and its duration. A geometrical image is generated by the line representing the pressure with time as the second dimension of the diagram. The pressure is thus a function of time.[22] Interpreting this in modern terminology, the momentum mv would be represented as the integral of these instantaneous pressures (or impulses) over a time, or $\int m\mathrm{d}v = \int p\mathrm{d}t$.[23]

Boscovich suggested that, if the time coordinate is replaced by the space traversed and the pressure coordinate by the force which at any instant produces the velocity proportional to it, a second aspect of the phenomenon is represented. Boscovich, however, explained neither this substitution nor the introduction of the concept of force. The new term 'force' must be interpreted as an entity proportional to the velocity engendered at any instant. If the pressure coordinate is changed to the force and the time coordinate to the space then the new geometrical image producing the velocity would be represented in modern notation as $\int F\mathrm{d}s$. We would then interpret *vis viva* as $\int mv\mathrm{d}v = \int F\mathrm{d}s$ (where $\mathrm{d}s = v\mathrm{d}t$).[24] Boscovich does not bring the mass into this analysis.

Although not explicitly stated, Boscovich's analysis contains the necessary elements for distinguishing force, mv, as a time-dependent function and mv^2 as a space-dependent function. It brings together aspects of force analysed previously by Bernoulli, Louville and others (see notes 23 and 24). On the question of elastic and inelastic collisions briefly discussed in his paper, Boscovich used the principle of action and reaction and its equivalent, the conservation of quantity of motion taken in an algebraic sense. In verifying the conservation of living force in the sense of Leibniz he said that living force, being formed as it is by the square of the velocity, destroys the sign of that velocity, whereas the quantity of motion conserves all its characteristic elements. Boscovich concludes in paragraph 39 of

De Viribus Vivis that the question of living force is a question of language and completely useless.[25] In spite of this analysis of 'force', however, Boscovich believed that momentum was the true measure of force, *vis viva* being valid only as a method of calculation. In his *De Viribus Vivis* as well as in his later *Philosophiae Naturalis Theoria* (1758), he argued that there were no living forces in nature.[26]

Thus Boscovich, while providing an insight which theoretically helped to resolve the *vis viva* controversy, did not claim equal status for the two principles in treating physical problems. Since d'Alembert also preferred momentum to *vis viva*,[27] it would be of historical interest to inquire when equal status was given to both principles by practising scientists in the solution of mechanical problems, particularly in cases of elastic impact. A sampling of textbooks through the eighteenth and nineteenth centuries shows that most authors who treated the problem of collision employed the two principles, conservation of momentum and conservation of relative velocities.[28]

A small number of scientists in the eighteenth and early nineteenth centuries accepted the use of both momentum and *vis viva* for problems of elastic collisions. It is probable that the mutual acceptance of both principles had to await the fuller understanding and development of energy relations which took place in the 1840s, but this should be investigated in detail. The simultaneous use of both momentum and kinetic energy was advocated in textbooks at least by the 1860s.[29]

It seems, therefore, that d'Alembert had very little to do with the termination of the *vis viva* controversy either theoretically, practically or historically. He was not the first to call it a 'dispute over words'. He was not the first to contrast momentum as a force acting through a time interval with *vis viva* as a force acting over the space traversed. He did not advocate the simultaneous use of momentum and *vis viva* to solve impact problems. He did not give a complete discussion of the uses of *vis viva* in solving problems relating to compressed springs and falling bodies. Nor did he deal with some of the important philosophical and theological issues regarding conservation of *vis viva* which were basic to the arguments of some participants. Finally, in the year 1743 d'Alembert did not present the argument which had heretofore been cited as resolving the controversy. The date 1743 is therefore of no significance as a terminus for the *vis viva* controversy.

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NOTES

[1] Jean d'Alembert, *Traité de Dynamique*, first edition (Paris, 1743), preface. References to the date 1743 are made in the following works: Ernst Mach, *Science of Mechanics*, sixth edition (La Salle, Ill., 1960), 365; William Whewell, *History of the Inductive Sciences*, third edition, 2 vols. (New York, 1872), vol. I, 361; J. Bernard Stallo, *The Concepts and Theories of Modern Physics* (New York, 1884), 72; J. E. Montucla, *Histoire des Mathématiques*, 3 vols. (Paris, 1799–1802), vol. III, 641.

[2] L. L. Laudan, 'The *Vis Viva* Controversy, a Post-Mortem', *Isis*, 59 (1968), 131.

[3] Thomas Hankins, 'Eighteenth-Century Attempts to Resolve the *Vis Viva* Controversy', *Isis*, 56 (1965), 281; Pierre Costabel, 'Le *De Viribus Vivis* de R. Boscovich ou de la Vertu des Querelles de Mot', *Archives Internationales d'Histoire des Sciences*, 14 (1961), 3.

[4] Costabel, *loc. cit.*

[5] Hankins, *loc. cit.*

[6] D'Alembert, *op. cit*, xxi.

[7] *Ibid.*, xvi, xvii.

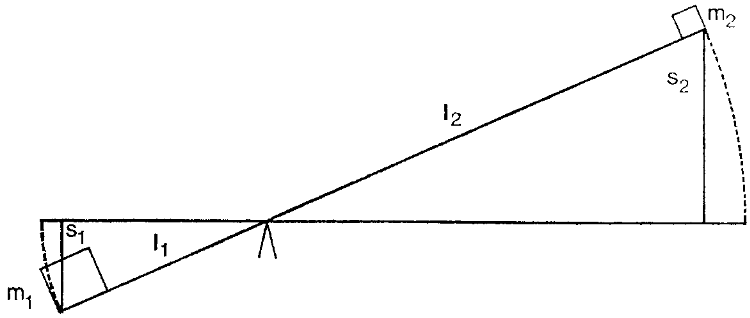
[8] *Ibid.*, xix, xx. 'Ceci bien entendu, il est clair qu'on peut opposer au Mouvement d'un Corps trois sortes d'obstacles; ou des obstacles invincibles qui anéantissent tout-à-fait son Mouvement, quel qu'il puisse être; ou des obstacles qui n'ayent précisément que la résistance nécessaire pour anéantir le Mouvement du Corps, et qui l'anéantissent dans un instant, c'est le cas de l'équilibre; ou enfin des obstacles qui anéantissent le Mouvement peu à peu, c'est le cas du Mouvement retardé. Comme les obstacles insurmontables anéantissent également toutes sortes de Mouvements, ils ne peuvent servir à faire connoître la force: ce n'est donc que dans l'équilibre, ou dans le Mouvement retardé qu'on doit en chercher la mesure. Or tout le monde convient qu'il a équilibre entre deux Corps, quand les produits de leurs masses par leurs vitesses virtuelles, c'est-à-dire par les vitesses avec lesquelles ils tendent à se mouvoir, sont égaux de part et d'autre. Donc dans l'équilibre le produit de la masse par la vitesse, ou, ce qui est la même chose, la quantité de Mouvement, peut représenter la force. Tout le monde convient aussi que dans le Mouvement retardé, le nombre des obstacles vaincus est comme le quarré de la vitesse; ensorte qu'un Corps qui a fermé un ressort, par exemple, avec une certaine vitesse, pourra avec une vitesse double fermer, ou tout à la fois, ou successivement, non pas deux, mais quatre ressorts semblables au premier, neuf avec une vitesse triple, et ainsi du reste. D'où les partisans des forces vives concluent que la force des Corps qui se meuvent actuellement, est en général comme le produit de la masse par le quarré de la vitesse.'

[9] Italics mine. The case of equilibrium was not equivalent to the quantity of motion. This was pointed out by Leibniz in 1686: 'We need not wonder that in the common machines the lever, windlass, pulley, wedge, screw and the like there exists an equilibrium since the mass of one body is compensated for by the velocity of the other. . . . For in this special case the quantity of the effect or the height risen or fallen will be the same on both sides no matter to which side of the balance the motion is applied. It is therefore merely accidental here that the force can be estimated from the quantity of motion.' (Gottfried Wilhelm Leibniz, 'Brevis Demonstratio Erroris Memorabilis Cartesii', *Acta Eruditorum* (1686), 161.) In a supplement to the above, written in 1695, Leibniz says 'Even if some of these [*i.e.*, the laws of the inclined plane and acceleration of falling bodies] seem reconcilable with that hypothesis which estimates power by the product of mass by velocity, this is only accidentally, since the two hypotheses coincide in the case of dead forces [*potentia mortuus*] in which only the beginning or end of conatuses is actualized. But in living forces, or those acting with an actually completed impetus, there arises a difference just as the example shows which I have given above in the published paper. For living power is to dead power or impetus (actual velocity) is to conatus, as a line is to a point or as a plane is to a line.' (Leibniz, *Philosophical Papers and Letters*, trans. and ed. Leroy E. Loemker (Chicago, 1956), vol. I, 460.

[10] D'Alembert, *op. cit.*, xix–xx. The idea of moving balls compressing elastic springs (*ressorts*) and thus surmounting an obstacle was used by Jean Bernoulli in 1727 in a paper supporting *vis viva* submitted for a contest on the communication of motion sponsored by the Académie des Sciences (Paris). Subsequently these springs were used in the arguments of many other participants in the controversy, among them Jean Jacques de Mairan (1728), Camus (1728), Louville (1729), Abbé Deidier (1741) Madame du Châtelet (1740). For a detailed discussion of this problem see

Carolyn Iltis, *The Controversy Over Living Force: Leibniz to d'Alembert* (Dissertation, The University of Wisconsin, 1967).

[11] In the case of the lever, for example, $F_1l_1 = F_2l_2$ or $F_1s_1 = F_2s_2$. But $F = mg$ and $ds = dv.dt$. Thus $m_1gdv_1.dt_1 = m_2gdv_2.dt_2$. For the case of the lever in equilibrium the times are equal ($dt_1 = dt_2$) and hence $m_1dv_1 = m_2dv_2$, dead force. But the dv's are virtual velocities and not the actual velocities in the momentum expression mv for moving bodies. (Example mine.)



The term 'virtual velocity' was first used by John Bernoulli in 1717 in a letter to Varignon Bernoulli, in his 'Discours sur les Loix de la Communication du Mouvement', *Recueil des Pièces qui a Remporté les Prix de l'Académie Royale des Sciences*, **2** (1727), stated that 'the fundamental principle of statics lies in the equilibrium of "powers", the moments being composed of absolute forces and their virtual velocities'. He argued that by extending this principle to the forces of bodies which have actual velocities, philosophers had gone too far. Here Bernoulli gave a definition of virtual velocities: 'I call virtual velocities [*vitesses virtuelles*] those acquired by two or more forces taken in equilibrium when a small movement is imprinted upon them . . . The virtual velocity is the element of velocity already acquired that each body gains or loses in an infinitely small time along its direction' (p. 19). For a history of the virtual velocity concept, see Erwin Hiebert, *Historical Roots of the Principle of Conservation of Energy* (Madison, Wisc., 1962).

[12] Leibniz, 'Brevis Demonstratio', *op. cit.*, note 9, 162.

[13] Abbé Catalan, 'Courte Remarque', *Nouvelles de la République des Lettres*, **8** (1686), 1002

[14] J. T. Desaguliers, 'Animadversions upon some Experiments relating to the Force of Moving Bodies', *Phil. Trans.*, **32** (1724), 271.

[15] Although d'Alembert in 1758 distinguished mv and mdv he did not delete the mis-identification in the above quotation from the 1758 edition.

[16] D'Alembert, *op. cit*, xxi.

[17] Jean d'Alembert, *Traité de Dynamique* (1758 edition, Paris, 1921), xxx. This second edition was expanded and revised by d'Alembert. The three definitions of force discussed below were added to this edition.

[18] *Ibid.*, xxx. My italics.

[19] Pierre Costabel, *op. cit.*, note 3, 3

[20] Here Costabel recognizes that d'Alembert's own contribution to the controversy did not really occur until the 1758 edition of the *Traité*. However, he does not specify that here d'Alembert added the section to the preface concerning the difference between a force acting through a time and a force acting over a distance. He indicates rather that this was due to d'Alembert's addition of a section generalizing the principle of living force to the main body of the *Traité* See Costabel, *op. cit.*, 4.

[21] Costabel, *op. cit.*, 6. In his *Specimen Dynamicum* Leibniz said that in dead force 'motion does not yet exist . . . but only a solicitation to motion'. It is a pressure or a tension. Loemker, *op. cit*, note 9, vol II, 717.

[22] Costabel, *op. cit.*, 6, 7.

[23] In 1729 Jacque de Louville somewhat confusedly presented a definition with reference to compressed springs of what we would interpret as the impulse of a Newtonian force. He defined

the force of each impulsion, f_i, communicated in an instant as the 'instantaneous force'. 'Actual force' is the 'product of the force of each impulsion by the number [*i.e.*, the sum] of impulsions the moving body receives in equal times [or $\int_{t_2}^{t_1} f_i \, dt$]'. To Louville this meant that mv and not mv^2 was the measure of force. Since the impulse, f_i, is equivalent to the pressure, p (and to the Newtonian force), Boscovich's analysis is similar to Louville's. Boscovich, however, also recognized the usefulness of mv^2 in calculations whereas Louville did not. (Jacque Eugène de Louville, 'Sur la théorie des Mouvements variés', *Histoire de l'Académie Royale des Sciences* (1729), 154.)

[24] In 1727 Jean Bernoulli presented an analysis of *vis viva* in terms of balls moved by releasing compressed springs. The velocity increment is represented by the pressure of the spring, p, or dead force, and the increment of time. Thus $dv = pdt$. Since $v = dx/dt$, $dt = dx/v$. Therefore $dv = pdx/v$ or $vdv = pdx$. The integral is $v^2/2 = \int pdx$. Bernoulli then adds the concept of mass showing that living force, mv^2, is as the square of the velocity. Since pressure and Newtonian force are equivalent, this can be interpreted as $mv^2/2 = \int Fdx$. This is essentially Boscovich's argument (Jean Bernoulli, 'Discours sur les Loix de la Communication du Mouvement', in the *Recueil des Pièces qui a Remporté les Prix de l'Académie Royale des Sciences*, **2** (1727), separate pagination.)

[25] *Ibid.*, 9.

[26] Roger Boscovich, *A Theory of Natural Philosophy*, trans. J M. Child, from the second edition of 1763 (London, 1922), section 293: '. . . it will be sufficiently evident, both from what has already been proved as well as from what is to follow, that there is nowhere any sign of such living forces nor is this necessary. For all the phenomena of Nature depend upon motions and equilibrium, and thus from dead forces and the velocities induced by the action of such forces For this reason, in the dissertation *De Viribus Vivis*, which was what led me to this theory thirteen years ago, I asserted that there are no living forces in Nature, and that many things which were usually brought forward to prove their existence, I explained clearly enough by velocities derived solely from forces that were not living forces.' For a discussion of Boscovich's views on living force and on momentum, see Hankins, *op. cit.*, note 3, 291.

[27] See Hankins, *op cit.*, 284.

[28] The standard approach utilizing momentum and relative velocities followed the lines of Wallis's and Wren's solution to the problem in 1668. See John Wallis, 'A Summary Account of the General Laws of Motion', *Phil. Trans.*, 3 (1669), 864; John Wallis, *Mechanica sive de Motu*, 2 vols. (London, 1669–71), vol. 1, 660; Christopher Wren, 'Lex Naturae de Collisione Corporum', *Phil. Trans* , 3 (1669), 867. Examples of textbooks using conservation of momentum and conservation of relative velocities, but not *vis viva*, are as follows: Richard Helsham, *A Course of Lectures in Natural Philosophy* (London, 1743), 58, 68; Anonymous, *La Physique Expérimentale et Raisonnée* (Paris, 1756), 16; Abbé Para du Phanjas, *Théorie des Etres sensibles ou Cours complet de Physique* (Paris, 1772), 304; Denison Olmsted, *A Compendium of Natural Philosophy* (New Haven, Conn., 1833), 25; James Renwick, *First Principles of Natural Philosophy* (New York, 1842), 68.

[29] For example see James Wylde, *The Circle of the Sciences* (London, 1862–69), vol. 1, 745 This second approach, using both momentum and kinetic energy in the solution of elastic impact problems, had its origins in Christian Huygens's paper of 1668 'Extract d'une Lettre de M. Huygens', *Journal de Scavans* (1669), 19, and *Phil. Trans* , **4** (1669), 925. A general solution to the problem of impact was presented by Leibniz in 1692. Leibniz set down three equations equivalent to conservation of relative velocities, conservation of momentum, and conservation of *vis viva*. Any two of these used simultaneously, he said, would be sufficient for elastic impacts. For inelastic impacts conservation of *vis viva* does not hold 'but this loss of the total force, or this failure of the third equation does not detract from the inviolable truth of the law of the conservation of the same force in the world. For that which is absorbed by the minute parts is not absolutely lost for the total force of the concurrent bodies.' (Gottfried Wilhelm Leibniz, 'Essay de Dynamique sur les Loix du Mouvement', *Mathematische Schriften*, ed. C. I. Gerhardt (Halle, 1860), series 2, vol. 2, 215.) The manuscript containing these equations was not published until 1860 when Leibniz's works were collected and published by Gerhardt. The three equations, however, did appear in the work of Leibniz's follower Jean Bernoulli in an essay submitted to the Académie des Sciences in 1727. (Jean Bernoulli, 'Discours sur les Loix de la Communication du Mouvement', *Recueil des Pièces qui a Remporté les Prix de l'Académie Royale des Sciences* (Paris, 1727), **2,** 1; see p. 29.) L. L. Laudan, *op. cit.*, note 2, 137, has pointed out that Desaguliers in 1744 was convinced that 'the phaenomena of the Congress of Bodies may be equally solv'd according to

the Principles of the Defenders of the new [mv^2], as well as those of the old [mv] Opinion'. (Desaguliers, *Course of Experimental Philosophy* (London, 1735–44), vol. 2, 63.) Thomas Young in a chapter 'On Collision' from his book, *A Course of Lectures on Natural Philosophy and the Mechanical Arts* (London, 1807, vol. 1, 78), admits the utility of both the momentum and energy principles, but he does not suggest their simultaneous application as the most general method of attacking elastic collision problems. In 1824, William Whewell mentioned the validity of both mv and mv^2 in his discussion of impact. (William Whewell, *Elementary Treatise on Mechanics* (Cambridge, 1824), 258.)