*CAROLYN ILTIS*

# MADAME DU CHÂTELET'S METAPHYSICS AND MECHANICS*

## Introduction

GABRIELLE Émelie du Châtelet's historical identity has all too often centered on her role as the witty temperamental mistress of Voltaire.[1] Accounts of their life delight in relating the gossip of their relationship: Voltaire's anger when she remained closeted in her locked study with Clairaut, claiming difficulties with a mathematical proof; or his hysterical weeping causing his fall down a flight of stairs upon hearing of her death during childbirth, to be picked up at the foot by the young soldier Saint-Lambert who had made her pregnant.

In an age when marriage was a convenience and affairs—one at a time—the rule, when women unaided by aristocracy or money had little prospect of intellectual achievement, she recognized her worth and asked to be evaluated on her own grounds as a scholar:

> Judge me for my own merits or my lack of them but do not look upon me as an appendage to this great general or that renowned scholar. This star that shines at the court of France or that famed author. I am in my own right a whole person, responsible to myself alone, for all that I am, all that I say, all that I do.[2]

*Natural Sciences Interdisciplinary Programme, University of San Francisco. Thanks are expressed to Dr. Dierdre LaPorte for sending me portions of her doctoral dissertation, 'Theories of Fire and Heat in the First Half of the Eighteenth Century', Harvard, 1970, relating to du Châtelet and Voltaire; and to Keith Symon for reading and evaluating du Châtelet's contributions to mechanics as discussed in my dissertation, 'The *Vis Viva* Controversy: Leibniz to d'Alembert', University of Wisconsin, 1967. I am indebted to Gerd Buchdahl for comments on du Châtelet's ideas in relation to the philosophy of Leibniz and Kant. I wish to thank the Frederick E. Brasch Collection on Sir Isaac Newton and the History of Scientific Thought, Special Collections, Stanford University Libraries, Stanford, California, for permission to reprint the accompanying plate from Mme Du Châtelet's *Institutions de Physique*, Paris, 1740.

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[1] For accounts of the life of Gabrielle Émelie du Châtelet (1706–1749) and her relationship with Voltaire see Mme de Graffigny, *La vie privée de Voltaire et de Mme du Châtelet*, Paris, 1820; Frank Hamel, *An Eighteenth Century Marquise: A Study of Émile du Châtelet and Her Time*, Paris, 1910; André Maurel, *La Marquise du Châtelet, Amie de Voltaire*, Paris, 1930; Jean-Baptiste Capefigue, *La Marquise du Châtelet et les amies des philosophes du XVIII^e Siecle*, Paris, 1868; and Nancy Mitford, *Voltaire in Love*, London: Hamilton, 1957.

[2] Letter to Frederick of Prussia, quoted in Samuel Edwards [pseud. Noel Bertram Gerson], *The Divine Mistress*, New York: David McKay, 1970, p. 1. This recent popular account of her life fails to make a serious evaluation of her scientific achievements.

Recent scholarship has begun to produce an evaluation of her contributions to science and philosophy which treat her own unique accomplishments. Her influence on Voltaire's thought, her mathematical achievements, her translation of Newton's *Principia*, her dissemination of the Leibnizian philosophy in France, her dissertation on the nature of fire have all been researched and discussed.[3]

Here I shall present an analysis of her natural philosophy and mechanics as it appeared in the anonymously published *Institutions de Physique* of 1740 (although some title pages say London, 1741).[4] The *Institutions* was meant as a textbook for her son's use. In part an attempt to popularize Leibniz's views, it was successful in creating immediate interest and excitement. Du Châtelet was probably introduced to Leibnizian ideas in 1736–1737 through manuscripts sent to Voltaire by Frederick of Prussia containing translations of Christian Wolff's Leibnizian metaphysics and from Voltaire's copy of Mairan's 'Dissertation sur l'estimation et la mesure des forces matrices du corps' (1728) against the Leibnizian concept of *vis viva*, $mv^2$.[5] Through Voltaire she was also familiar with the Leibniz–Clarke correspondence. In 1738 she read Jean Bernoulli's 'Discours sur les lois de la communication du movement', and after an enquiry and a response from Maupertuis which favoured Bernoulli over Mairan was converted to the Leibnizian position, at least in dynamics.[6] In a letter of April 30, 1738 to Maupertuis, she relates that she has read what Leibniz has written in the *Acta Eruditorum* on *vis viva*.[7]

[3] For evaluations of du Châtelet's intellectual achievements see Ira O. Wade, *Voltaire and Madame du Châtelet: An Essay on the Intellectual Activity at Cirey*, Princeton: Princeton Univ. Press, 1941; W. H. Barber, 'Mme du Châtelet and Leibnizianism: The Genesis of the Institutions de Physique', in *The Age of Enlightenment: Studies Presented to Theordore Besterman*, Edinburgh–London, 1967, pp. 200–222; W. H. Barber, *Leibniz in France from Arnauld to Voltaire: A Study in French Reactions to Leibnizianism, 1670–1760*, Oxford, 1955, pp. 135–140, 182–186. On her work on chemistry and the nature of fire see Dierdre LaPorte, 'Theories of Fire and Heat in the First Half of the Eighteenth Century', doctoral dissertation, Harvard Univ. 1970, pp. 296–343; and Robert L. Walters, 'Chemistry at Cirey', *Stud. Voltaire 18th Cent.* 58 (1967): 1807–1827. On her French edition of Newton's *Principia* see I. Bernard Cohen, 'The French Translation of Isaac Newton's *Philosophiae naturalis principia mathematica* (1756, 1759, 1966), *Archs. Int. Hist. Sci.* 21 (1968): 261–290. On her mathematics see Julian L. Coolidge, 'Six Female Mathematicians', *Scripta Mathematica* 17 (1951): 20–31; and Florian Cajori, 'Madame du Châtelet on Fluxions', *Mathematical Gazette* 13 (1926): 252.

[4] Gabrielle Émelie du Châtelet, *Institutions de Physique*, Paris, 1740.

[5] Jean Jacques de Mairan, *Mem. Acad. Sci.*, Paris, 1728, pp. 1–49. On her Leibnizian background see Barber, 'Mme du Châtelet and Leibnizianism', pp. 205–206, and René Taton, article on 'Gabrielle Émelie du Châtelet', in *Dictionary of Scientific Biography*, Charles Coulston Gillispie, (ed.) III, 215–217, New York: Charles Scribner's Sons.

[6] Theodore Besterman (ed.), *Les Lettres de la Marquise du Châtelet*, Geneva: Institut et Musée Voltair, Les Delices, 1958, vol. I, lettres 1–231, 1733–1739; vol II, lettres 232–486, 1740–1749. See Lettre 118 à Maupertuis, 2 février 1738, vol. I, p. 213. Jean Bernoulli, 'Discours sur les lois de la communication du mouvement', *Recueil des pièces qui a remporté les prix de l'Academie Royale des Sciences*, Paris, 1727, II, 1–108 (separate pagination).

[7] Besterman, *Lettres*, lettre 122, vol. I, p. 220. Probably Leibniz's 'Brevis demonstratio', *Acta Erud.*, March, 1686, or 'Specimen dynamicum', *Acta Erud.* April, 1695.

Then in March 1739 Samuel Koenig was brought to Cirey by Maupertuis as a tutor for her and Voltaire in mathematics. By way of Koenig she became converted to Leibniz's philosophical views through their expression in the work of Christian Wolff. In the *Institutions* she states her indebtedness to Wolff's *Ontologia* of 1729, the author of which she had earlier called '*un grand bavard en metaphysique*'.[8] As a result of Koenig's teachings she revised the philosophical chapters of the *Institutions de Physique* which had been written in secret and approved for publication by September 1738. Although Newtonian in its basic mechanical principles, the resulting work followed Leibniz on the subject of dynamics, while the natural philosophy of the early chapters presented an integration of elements from the thought of Leibniz, Descartes, and Newton.

It is this integrative character of Madame du Châtelet's thought which sets her *Institutions* apart from other attempts to disseminate Newtonian mechanics. The metaphysical foundations of scientific theory, the place of physics in the context of a broader philosophy of nature, and the problem of the freedom of the will stimulated her intellectual imagination. Her attempts to integrate aspects of Cartesianism and Newtonianism with Leibnizian ideas reflected a need among natural philosophers of the 1740s to reconcile the conflicts among these systems. But these same integrative tendencies left her uncritical of logical inconsistencies in her account of the metaphysics underlying her system. In addition, her Leibnizian convictions in dynamics prevented her from achieving a full recognition of the validity of both measures of 'force', $mv$ and $mv^2$. The following is a discussion of her synthesis and its strengths and weaknesses.

## I

## Madame du Châtelet's Philosophy of Nature

A metaphysical question had first stimulated du Châtelet's interest in Leibniz's *vis viva* principle. In her correspondence with Maupertuis she exhibited great concern for the implications of the conservation of living force for free will. Stating early in 1738 that she had read much

[8] Besterman, *Lettres*, lettre 146 à Maupertuis, 29 September 1738, vol. I, p. 246. Barber, *Leibniz in France* [3], pp. 135–140, and 'Mme du Châtelet and Leibnizianism' [3], pp. 208–209. On her debt to Leibniz and Wolff see the *Institutions*, 'avant-propos', pp. 12, 13. (Christian Wolff, *Philosophia Prima Sive Ontologia* [1st edn, Frankfurt, 1729], critical edition J. Ecole (ed.) in *Gesammeltte Werke*, Abt. II, Bd. 3, Hidesheim: Georg Olms, 1962). In a footnote she declared her indebtedness principally to the following chapters of the *Ontologia*: 'De Principio Contradictionis, de Principio Rationis Sufficientis, de Possibili et Impossibili (1962 edn, pp. 15–87), de Necessario et Contingente (pp. 223–260); de Extensione, Continuitate, Spatio, Tempore etc. (pp. 425–492). On the relations of Wolff's philosophy to that of Leibniz see Jean Ecole, 'Cosmologie wolffienne et dynamique leibnizienne', *Les Études philosophiques* 19 (1964) pp. 3–10.

on the subject of *forces vives*, she asked whether the freedom of living beings to create motion must not be a violation of conservation. 'I believe myself free', she wrote, 'and I do not know whether the same quantity of force in the universe does not destroy freedom'.[9] In the commencement of motion, she reasoned, is it not true that a force is produced which hitherto did not exist? If we do not have the power to produce motion, then there is no free will. But if there is free will then it is absolutely necessary that the will can initiate motion.[10]

The issue of the free will of individuals in initiating motion was related to the larger question of God's will in creating motion in the universe and in subsequently causing particular motions. One of the central issues of the *vis viva* controversy had been the question of God's role in the natural order. Was God's nature to be characterized by his power and volition or by his wisdom and foresight?[11] The distinction between the omnipotence of God's will and omniscience of his logic formed the framework within which Madame du Châtelet formulated the natural philosophy of her *Institutions de Physique*.

A basic question discussed by Leibniz and Clarke in their famous letters on the nature of God, space, matter, and force was the kind of world God could create.[12] For Newton and Clarke the world could have been otherwise, for it depended on the free exercise of God's will and its continued sustenance through God's providence.[13] For Leibniz it was the best of all possible worlds since God operated rationally within the laws of logic to create the actual existing world.[14]

Rejecting the voluntarist theology of Newton and Clarke, Madame du Châtelet held that the logical possibility for the existence of the natural world was to be explained by Leibniz's principle of non-contradiction, and its actual existence by the principle of sufficient reason which accounts for the existence of some things and not others.[15] A logically possible world is a noncontradictory world, whose

[9]Besterman, *Lettres*, lettre 122 à Maupertuis, 30 avr. 1738, vol. I, p. 220.

[10]*Ibid.*, lettre 124 à Maupertuis, 9 mai 1738, vol. I, p. 226.

[11]See Carolyn Iltis, 'The Leibnizian–Newtonian Debates: Natural Philosophy and Social Psychology', *Br. J. Hist. Sci.* 6 (December 1973): 343–377; David Kubrin, 'Newton and the Cyclical Cosmos: Providence and the Mechanical Philosophy', *J. Hist. Ideas* 28 (July–September 1967): 325–346.

[12]Samuel Clarke, *A Collection of Papers which Passed Between the Late Learned Mr. Leibniz and Dr. Clarke*, London, 1717.

[13]On voluntarism and 17th century natural philosophies see J. E. McGuire, 'Boyle's Conception of Nature', *J. Hist. Ideas* 33 (1972) pp. 523–542. On God's will and direct action in Newton's natural philosophy see J. E. McGuire, 'Force, Active Principles, and Newton's Invisible Realm', *Ambix* 15 (1968) pp. 154–168, esp. pp. 161–164. See also P. M. Heimann, 'Nature Is a Perpetual Worker', *Ambix* 20 (1973) pp. 1–25.

[14]Gottfried Wilhelm Leibniz, *Philosophical Papers and Letters*, Leroy E. Loemker (ed.), Chicago, 1956, vol. II, pp. 1099, 1100.

[15]Du Châtelet, *Institutions* (4), pp. 21–22 (section 7, 8); p. 55 (section 34). On Leibniz's

beings could exhibit nothing, within themselves, mutually destructive or incompatible. However, such a world may not actually exist; the actual world is created by God's volition (*volonté*). 'Thus the divine understanding is the eternal region of truths and the source of possibilities; the same as his volition is the source of actuality and existence'.[16] God's understanding as the origin of all that is possible is thus prior to divine volition, the source of existence and actuality. 'If the possibility of things depended on his volition, then it would be necessary to say that God had been without understanding while his volition was occupied in creating the possibles'.[17]

In asserting that God's 'understanding contains all that is possible [while] all that is not found is impossible',[18] she failed to give adequate consideration to the area lying between logical possibility and actual existence which had been stressed by Leibniz in his mature years. According to Leibniz that which does not actually exist is not logically impossible but it may not be part of the 'best of all possible worlds', i.e., the most perfect. She did not argue, as had Leibniz, that the external possibilities must also be those which are the most perfect if they are to become actual.[19]

Criticizing the arbitrariness of natural explanation based on no other reason than God's volition, she rejected Malebranche's occasionalist doctrine of God's immediacy in the action of moving bodies.[20] In the occasionalist system, she said, natural law as expressed through secondary causation has no efficacy, because bodies are never the *causes* of activity but only the receptors.[21] God's immediate concourse would be an immanent manifestation of Divine Causation external to nature. Such continual miracles are not consistent with the laws of motion and the essence of bodies.

Madame du Châtelet's philosophy of nature in the *Institutions de Physique* was basically a Leibnizian exposition, yet it showed the influence of Cartesian substance philosophy—the position that whatever exists is a substance or a modification of a substance.[22] For Descartes

principle of noncontradiction see Margaret D. Wilson, 'Leibniz and Locke on First Truths', *J. Hist. Ideas* 28 (1967) pp. 347–366.

[16] Du Châtelet, *Institutions*, p. 68 (section 48); see also p. 134 (section 121).

[17] *Ibid.*, p. 68 (section 48).

[18] *Ibid.*, p. 67 (section 49).

[19] Gerd Buchdahl, *Metaphysics and the Philosophy of Science*, Blackwell, Oxford, 1969, pp. 399, 404, 454.

[20] Du Châtelet, p. 69 (section 49); p. 154 (section 138). On Malebranche see E. J. Aiton, *The Vortex Theory of Planetary Motions*, Elsevier, New York, 1972, p. 71; Thomas L. Hankins, 'The Influence of Malebranche on the Science of Mechanics During the Eighteenth Century', *J. Hist. Ideas* 28 (1967) pp. 193–210.

[21] Du Châtelet, p. 154 (section 138).

[22] On Cartesianism and substance philosophy see Richard A. Watson, *The Downfall of Cartesianism, 1673–1712*, The Hague: Martinus Nijhoff, 1966.

the two substances, matter and mind, differed in essence. The essence of matter was extension, the essence of mind was thinking.[23] Material things could be modified by the attributes size, shape, and motion; minds by sensations and ideas.

Du Châtelet had schooled herself in the Cartesian distinctions between essences and their attributes and modes (or invariable and variable properties). When she came to treat these matters in her *Institutions*, her account revealed the influence of the Leibnizian view that attributes and properties are contained in the concept of each individual substance and not amenable to external influences.[24] She argued that the attributes of existing bodies are logically deduced from and harmoniously related to their essence in that they are possible, noncontradictory, and not mutually destructive, although they are not always found in actual existing beings.[25] When the essentials are posed, the attributes follow by the law of sufficient reason.[26] In any being, essence and attributes are constant; essences are invariable and attributes incommunicable. Modes, however, are variable properties; their existence is contingent. From the essence it can be understood why a mode is possible, but not why it becomes actual. Modes do not depend logically on attributes because attributes do not contain the reason for the existence of antecedent modes or external existing beings.[27]

In general, then, God's understanding was the source of what was possible, the essences themselves being founded on the principle of noncontradiction. If, on the other hand, essences depended on God's will, they would be arbitrary and would be possible or impossible simply because God willed it so.[28] Hence, to achieve a rational understanding of nature, it was necessary first to demonstrate the intrinsic possibility of a body from the principle of noncontradiction, secondly to establish its external possibility from the principle of sufficient reason, and then to deduce the attributes and the modes of which it was susceptible.[29] Placing the body within the order of nature

[23] René Descartes, *The Meditations*, La Salle: Open Court, 1952, pp. 52, 53. On Descartes' physics and its reception in 18th century thought see M. Mouy, *Le developpement de la physique cartesienne*, Vrin, 1934; Jean Ehrard, *L'Idée de nature en France dans la première moitieé du XVIII^e siècle*, Paris, 1964, 2 vols.; Daniel Mornet, *Les sciences de la nature en France au XVIII^e siècle*, Paris: Armand Colin, 1911. On his philosophy see Gerd Buchdahl [19], chap. 3, pp. 79–180.

[24] Leibniz, 'Correspondence with Arnauld', in *Discourse on Metaphysics* etc., La Salle: Open Court, 1957, pp. 103–136.

[25] Du Châtelet, pp. 59–60 (section 39, 41).

[26] *Ibid.*, p. 61 (section 42); p. 66 (section 47).

[27] *Ibid.*, p. 59 (section 40); p. 65 (section 47); p. 62 (section 44).

[28] *Ibid.*, p. 67 (section 48).

[29] *Ibid.*, p. 69 (section 50).

it was finally necessary to show how it depended on its neighbours and which causes gave actuality to the many possible modes.[30] Relationships between bodies could thus be understood through internal relations within the order of nature not as externally imposed upon unrelated particulars.

In his analysis of nature, Leibniz had distinguished between primitive and derivative force ($mv^2$). The monads, or primitive active force, were true substances; derivative force was a *phenomenon bene fundatum*, not fully real, yet derived from primitive force and subject to the order and laws of nature. Derivative force, found in the impact of bodies, was attached to and was 'like' the primitive force or true substance—it was 'substantialized'.[31] Leibniz had also assigned extension to the world of well-founded phenomena, arguing that extension and motion are only attributes of phenomenal bodies, whereas primitive force is a true substance.[32] He held that space was a relation: the order of simultaneous states in the unfolding lives of monads, and that extension was the magnitude of space. 'It is wrong', said Leibniz, 'to confuse extension with what is extended as is commonly done, and so to consider it a substance'.[33]

Madame du Châtelet maintained the Leibnizian distinction between primitive force and derivative force which 'derives from the former but is only a phenomenon', grounding primitive force in the 'simple unextended beings' Leibniz had called monads.[34] These simple beings consisted of a continual tendancy to act, an indestructible tendency which constantly produced change when there was no sufficient reason to prevent the completion of the action.[35] Sensible changes were the result of a continued perpetual succession of internal states in the simple beings. That the internal action of each simple being differed from that of all others followed from Leibniz's principle of the identity of indiscernibles.[36]

She argued that extension could be grounded in nonextended beings and that extended beings existed because there were simple unextended beings. However, it was impossible to represent these simple un-

[30] *Ibid.*, p. 70 (section 50).

[31] Buchdahl [19], pp. 410, 417, 420, 423. On Leibniz's dynamics see also Carolyn Iltis, 'Leibniz and the *Vis Viva* Controversy', *Isis* 62 (1970) pp. 21–35. Margaret D. Wilson, 'Leibniz's Dynamics and Contingency in Nature', and J. E. McGuire, '*Labyrinthus continui*: Leibniz on Substance, Activity and Matter', in *Matter, Motion and Time Space and Matter*, Ohio State University Press, 1975.

[32] Leibniz, *Philosophical Papers*, Loemker (ed.) 14, I, 417; II, 641, 843, 845, 978.

[33] *Ibid.*, II, 1084.

[34] Du Châtelet, p. 172 (section 158–159).

[35] *Ibid.*, pp. 137–138 (section 126).

[36] *Ibid.*, p. 139 (section 128); p. 155 (section 139).

extended beings in the imagination or to detect them with the senses.[37] Mechanical phenomena discernible by the senses derive from higher metaphysical principles; the metaphysical union of elements produces the mechanical union of bodies which we see.[38]

> The repugnance which one has at conceiving how simple non-extended beings can, by their own assemblage, compose extended beings is not a reason to reject them. This revolt of the imagination against simple beings comes simply from habit in which we represent our ideas as sensible images. . . . Perhaps someday there will be found a calculus of true metaphysics in which by the sole substitution of characters one can come to truths, as done in algebra. M. Leibniz believed he had found such a calculus but died before communicating his ideas on it. . . .[39]

In attempting to relate extension to the non-extended she held that phenomenal extension is a composite of substances; both extension and force have their origin in the same simple beings.[40] The extension of matter arises from the aggregate of simple beings; force and resistance arise from the harmonious conspiring of active and passive principles within the aggregated elements.[41] Just as colours and the sensible phenomena of gross bodies result from confusion and the imperfection of our sense organs, the confusion decreases by degrees when the real origin of phenomena is found.[42] The gradation leads to simple beings or monads, the real substances which are the origin of what we see. Thus phenomena are born from the confusion of the several realities.[43] Abstraction allows us to focus our attention either on the whole confused aggregate or on the real elements of the aggregate.[44]

Although du Châtelet attempted to present a Leibnizian analysis of the relationship between the phenomenal and substantial levels of nature and between the parts of an aggregate, she did not really try to clarify the logic of these relations. This left her account incomplete and unconvincing. For Leibniz the elements of a composite whole were connected by ideal or external relations as opposed to the necessary intrinsic connections between substances. Thus physical connections such as those between a pile of stones did not provide a 'true unity'. The link between substances and *phenomena bene fundata* was for Leibniz ultimately explained in analogical language and while far more

[37] *Ibid.*, p. 136 (section 124); p. 132 (section 120); p. 135 (section 123); p. 170 (section 156).
[38] *Ibid.*, p. 148 (section 133).
[39] *Ibid.*, pp. 150–151 (section 135).
[40] *Ibid.*, p. 166 (section 152).
[41] *Ibid.*, p. 157 (section 142); p. 169 (section 155).
[42] *Ibid.*, pp. 166–168 (section 153).
[43] *Ibid.*, p. 169 (section 154).
[44] *Ibid.*, p. 171 (section 157).

subtle than du Châtelet's account represented one of the weaknesses of his system.[45]

In her discussion of the phenomenal world Madame du Châtelet argued that matter was to be described in terms of extension, force, and inertia. Extension combined with the passive force of inertia and the active moving force of *vis viva* (derivative force) is what we 'call' matter. However, the terms of her analysis were often inconsistent. In some places she stated that the nature of body 'consists' of these three principles, 'subsisting together' and mutually independent.[46] In other places she wrote that force is 'different from matter', but 'inseparably attached to it'.[47] Nevertheless, the main point of her philosophy was that force and matter must be placed on the same ontological level. Force is to be found in all matter; one is unknown without the other. Bodies cannot be described solely in terms of simple extension as Descartes and Malebranche had believed.[48] It was therefore necessary to join the 'power to act' to extension.[49] In insisting on the activity of matter she contributed to a new synthesis emerging from the older Cartesian philosophy long held in France.

Another source of major confusion in her philosophy was her use of the term substance in describing the elements of her ontology. In some places she referred to extension, force, and inertia as *phenomes substantiés*, a term she attributed to Wolff, asserting that the three were only substantial phenomena, 'which appear to us as substances but which are not'.[50] But in other places she directly labelled them substances:

> At first it appears strange that bodies should be composed of two substances, extension and active force and to admit of a species of action of an immaterial substance such as active force on matter, (*Sur la matière*), but as on the one hand, the phenomena show the substantiality of active force the same as that of matter and on the other there are insurmountable difficulties which oppose if one should conclude that neither matter nor active force are true substances, it is necessary to mount higher and look for their source in something prior from which one can show why active force and matter should be substances.[51]

This left it unclear as to whether extension and active force were

[45] See Buchdahl [19], chapt. 7, esp. pp. 393, 414, 417, 420, 422.

[46] Du Châtelet, p. 159 (section 143, 144); p. 160 (section 145).

[47] *Ibid.*, pp. 164, 165 (section 149).

[48] *Ibid.*, p. 152 (section 137); p. 155 (section 139), 'La force est donc aussi nécessaire a l'essence du corps que l'étendre'; pp. 164–165 (section 149, 150, 151).

[49] *Ibid.*, p. 159 (section 143); p. 165 (section 150); p. 165 (section 151).

[50] Du Châtelet, p. 170 (section 156). See Christian Wolff, *Cosmologia generalis*, Francofurti et Lipssiae, 1737, p. 119 (section 138): 'Per extensionem, vim inertiae, et vim activam omnes corporum mutationes explicari possunt'; p. 121 (section 141): 'Materia est extensum vi inertiae praeditum'; p. 144 (section 178): 'Materia igitur et vis activa substantiae non sunt quoniam tamen instar substantiarum concipi debent substantiae apparent'.

[51] Du Châtelet, p. 165 (section 151).

substantial phenomena or true substances. To further compound the problem, du Châtelet, like Descartes, sometimes referred to extension as a principal property of substance and sometimes as substance itself.[52] Force, inertia, and extension were likewise referred to sometimes as principles and other times as essences or essential properties of matter. Her failure to use terms carefully thus left considerable confusion as to the exact status of the components of her ontology.

In evaluating Madame du Châtelet's philosophy of nature we may say that she did not claim originality for the philosophical ideas she expressed. She regarded herself as the disseminator and translator of the work of others. Philosophically her ideas were often confused and inconsistent. She did not grasp the subtleties of the full Leibnizian doctrine but this was due in part to the limited availability of texts, the failure of Leibniz himself to provide a systematic exposition of his own philosophy, and its dissemination through the secondhand accounts of Wolff and Koenig. In addition, substance philosophy, which formed her framework, was dying out as a viable mode of analysis. Yet the *Institutions de Physique* represented one of many attempts in the mid-eighteenth century to integrate the central ideas of the Cartesian, Leibnizian, and Newtonian systems of nature. In recognizing the activity of matter and in placing matter and force on the same ontological level, she contributed to an emerging view of nature which ultimately became important in the development of the general law of energy conservation.

## II

### Du Châtelet's Dynamics

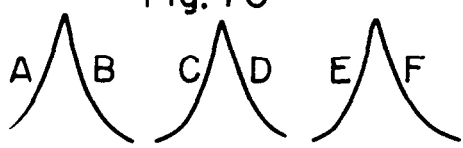
The *Institutions de Physique* also presented a Leibnizian interpretation of the dynamics of bodies in motion, or *vis viva*. It discussed the standard examples used by the participants in the *vis viva* controversy and defended the Leibnizian position. Du Châtelet began her analysis of dynamics with Leibniz's distinction between dead (or Newtonian) and living force. Living force can arise from dead force when a body is continually subject to a series of infinitely small forces or pressures (*pressions*). If a body yields to these dead forces, it conserves them and acquires a force which is the sum of all these accumulated pressures.[53]

She gave two examples of the relationship of dead to living force: elasticity and gravity. Following the elastic spring argument introduced

[52] See Buchdahl [19] pp. 89–91.
[53] Du Châtelet, p. 420 (section 567).

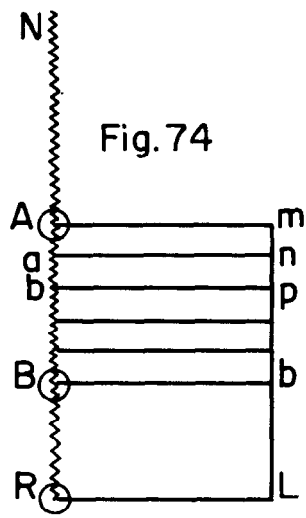
by Jean Bernoulli (1727) she pictured a set of three similar sections of elastic springs (*ressorts*) 'equally strong and equally tense'[54] (see du Châtelet's Figure 73). If a body receives the force held in one of these elastic springs, a second body receiving the force held in two similar elastic springs equal to the first will acquire two times more force. A body receiving the force of three equal and similar springs will acquire three times the force.

Fig. 73




An analogous example was the force of gravity.[55] Gravity presses uniformly on heavy bodies at each instant and at all points of their fall. Gravity can be considered as an infinite elastic spring *NR* pressing equally on body *A* in the space *AB* and acting at all points between *A* and *B*, (see du Châtelet's Figure 74). The forces that the bodies have received at *A* and *R* are as the lines *AB* and *AR* since the living forces are as the number of equal elastic springs communicating by expansion their forces to the bodies in motion. In a double space there are two times as many elastic coils as in a single space, and the number of coils are in the ratio of the spaces *AB* to *AR*. Thus the living forces of the

Fig. 74




[54]Bernoulli, 'Discours sur les lois de la communication du mouvement' [6]. On Bernoulli's springs and their role in the *vis viva* controversy see Carolyn Iltis, 'The Decline of Cartesianism in Mechanics: The Leibnizian–Cartesian Debates', *Isis* 64 (1973) pp. 356–373.

[55]Du Châtelet, pp. 420–422 (section 567).

body descending by gravity are as the spaces $AB$ to $AR$. But these spaces are as the squares of the velocities, and thus the living forces of the bodies at $B$ and $R$ are as the squares of their velocities.

Space and not time was the basis for the measure of 'force':

> Time should enter into the consideration of force no more than into the measure of riches of a man, which are the same whether dispensed in a day, a year, or a hundred years.[56]

For force to be real and not merely a metaphysical notion, therefore, a resistance was necessary by which its effects could be seen. If a body encountered other bodies which it set in motion, or if it bent elastic springs or compressed or transported other masses, then the presence of the force was known and could be estimated by the quantity of the effects it produced.

Madame du Châtelet then proceeded to reduce the arguments of Jean Jacques Mairan (1728), supporter of the Cartesian measure of force, $m|v|$, to nonsense.[57] The outcry which followed rekindled the famous *vis viva* controversy, inspiring Mairan, Voltaire, Abbé Deidier, and d'Alembert to re-examine the issue.[58]

Mairan had followed the Cartesian definition of force, $m|v|$ (i.e. the quantity of matter multiplied by the uniform velocity of the body). In his 1728 paper he attempted to reduce accelerated and retarded motion to cases of uniform motion. In this way, force in the Newtonian sense, $\Delta mv$, could be eliminated from the analysis of nature, a philosophical position which followed from Malebranche's rejection of the concept of force.[59] Mairan had defined $m|v|$ in terms of the 'elastic bands *not* lifted, the objects *not* flattened, and in general the objects *not* overcome which would be under uniform motion'. 'Force', $m|v|$, was therefore proportional to the simple velocity i.e. Cartesian force.

He had argued that the momentum of a moving body could be retarded by degrees by elastic bands placed at equal intervals in its path. Each one of these bands would offer a resistance equal to that of a body of mass 1, moving with velocity 1, so that the moving body lost momentum at each encounter. Mairan calculated the $mv$ lost by the

[56] *Ibid.*, pp. 423–424 (section 568–569).

[57] Jean Jacques de Mairan, 'Dissertation sur l'estimation et la mesure des forces motrices des corps', *Mem. Acad. Sci.*, Paris, 1728, pp. 1–49. For an analysis of Mairan's arguments see Carolyn Iltis, 'The Decline of Cartesianism in Mechanics', [54], pp. 370–373.

[58] De Mairan, Lettre à Madame *** sur la question des fores vives en reponse aux objections, Paris, 1741, pp. 1–37. François Voltaire, 'Doutes sur la mesure des force motrices et sur leur nature, presentés à l'academie des sciences de Paris, en 1741', *Oeuvres completes*, Paris, 1819–1825, vol. 28, pp. 420–420. A summary of this essay appears in *Histoire de l'Academie Royale des Sciences* (1741), Hist., pp. 149–153. Abbé Deidier, *Nouvelle refutation de l'hypotheses des force vives*, Paris, 1741, 145 pp. Jean d'Alembert, *Traité de dynamique*, 1st edn, Paris, 1748, preface.

[59] See Hankins, 'The Influence of Malebranche' [20], pp. 205–207.

body in successive instants by the number of bent strips. He also calculated the number of bands the body would have passed if it were moving uniformly during the same time. He then measured the total *mv* by the difference or total number of bands *not* bent.[60]

Madam du Châtelet pointed out that Mairan was analyzing nature not as it was, but as it was not. He appealed to events which did not occur rather than those which did. In refuting his supposition that force is measured by the spaces not transversed which would be under uniform motion, du Châtelet argued that two contradictory ideas were being used simultaneously. If a body exhausted a part of its force in compressing three elastic springs in the first second of its retarded motion, and only had enough force remaining to compress one more in the next second, then it would have to take back some of its force if it could compress two springs in the second second of uniform motion. It is contradictory to suppose that a force can remain constant and yet at the same time that it can produce a portion of the effects which consume it. A body cannot, at one and the same time, be considered as moving under uniform motion and retarded motion.[61]

In general, she said, the effects produced by uniform motion and retarded motion are different and cannot be compared. The effect of the first is only the space traversed, without obstacles encountered within it; that of the second consists in the displacement of these obstacles. In all those cases which are *possible*, the force of bodies should be evaluated by the obstacles which it is *possible* to overcome. As she put it in a later reply to Mairan, it is not permissible to substitute for real parts actually overcome or consumed, imaginary parts that cannot be surmounted, without supposing contradictions.[62]

Equally forcefully, Madame du Châtelet also exposed the 'error' in an argument contrived by James Jurin, who supported Descartes' quantity of motion (*mv*) as the measure of 'force'.[63] Jurin had supposed a plane moving in a straight line with a velocity of 1. On this plane is a body of mass 1 acquiring its velocity from the moving plane and consequently having a 'force' of 1. Now suppose that a spring capable of giving the body a velocity of 1 is fastened to the plane and in being released pushes the body in the same direction as the plane. In so doing it communicates one degree of velocity and consequently one

[60]Mairan, 'Dissertation sur l'estimation et la mesure des forces motrices des corps' [57] (section 40–41).

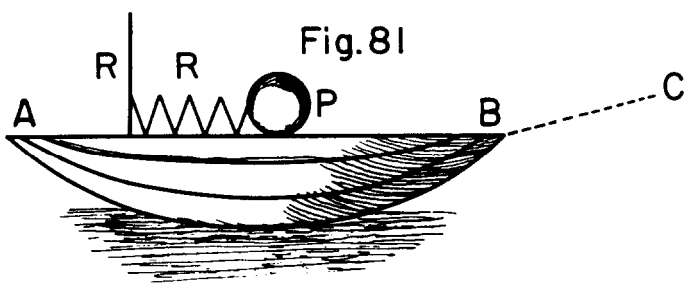
[61]Du Châtelet, p. 432 (section 574).

[62]Gabrielle Émelie du Châtelet, 'Reponse de Madame la Marquise du Châtelet à la lettre que M. de Mairan, secretaire perpetuel de l'académie royale des sciences, lui à écrite le 18. Février, 1741, sur la question des forces vives', Brussels, 1741, 37 pp., bound with *Dissertation sur la nature et propagation du feu*, Paris, 1744, pp. 18, 19, 21.

[63]Du Châtelet, *Institutions*, p. 441, (section 584).

degree of force to the body. Now, asks Jurin, what will be the total force of the body? The total force adds to 2, but the total velocity is also 2. Thus the force of a body is proportional to the mass multiplied by the simple velocity.

The error which du Châtelet correctly and astutely finds in the above reasoning is this: Suppose for greater ease that in place of the plane of Jurin a boat $AB$ moves on a river in the direction $BC$, with velocity 1, (see du Châtelet's Figure 81). Body $P$ is transported on the boat, acquiring thereby the same velocity as the boat. The elastic spring touching the ball is supported at the other end by an immobile support. When released it pushes toward both directions, $A$ and $B$, and communicates to the body $P$ not a velocity of 1 but this velocity minus a second quantity which depends on the proportion between the mass of the boat, $AB$, and the mass of body $P$. The quantity of living force residing in the coiled spring will, after its release, be found in the body and the boat taken together. Thus Jurin's case is founded on the false supposition that the elastic $R$ will communicate to body $P$ transported on a movable plane the same force that it communicated to it when the spring was supported by an immovable obstacle at rest.[64] In other words, this is a case in which one must consider the energy exchange ($\Delta E = 1/2\ mv^2$) between the moving object and the moving plane or ship. In failing to recognize this, Jurin had made the same logical error as had Colin Maclaurin in an earlier argument (1724) comparing bodies released with equal velocities on a moving ship and on the shore.[65] Both had made calculations which supported the measure of 'force', $mv$, and both had neglected the recoil energy supplied to the moving ship. Madame du Châtelet was alone in recognizing the inconsistency in this type of argument.




[64] *Ibid.*

[65] Colin Maclaurin, 'Demonstration des loix du choc des corps', *Recueil des pièces qui a remporté les prix de l'academie royale des sciences* (1724), vol. 1, pp. 1–24 (separate pagination). See pp. 7, 8. Maclaurin's argument was as follows: Suppose that two persons, one on a ship which advances with uniform motion with a velocity of 2 and the other at rest on the shore, thrown two equal bodies $A$ and $B$ with equal efforts in the direction of motion of the

Du Châtelet's *Institutions de Physique* was sufficiently provocative and controversial that it caused Mairan, secretary of the *Académie des Sciences* in Paris, to respond to her criticisms in a 'Lettre à Madame *** sur la question des forces vives en reponse aux objections', in 1741.[66] She in turn submitted a reply later in the same year which was published with her 'Dissertation sur la nature et propagation du feu'.[67]

The publication of the *Institutions* also brought to a head deep philosophical differences which existed between du Châtelet and Voltaire. Voltaire, who joined the controversy in 1741, was the author of the popular French presentation of the principles of Newtonianism and was in every way opposed to the Leibnizian way of thinking.[68] His sceptical, practical, and empirical approach to science led him to impatience with any explanation of the world which went beyond the strictly material. The philosophy of Leibniz and that of his follower Christian Wolff, from whom Voltaire learned Leibnizian metaphysics, left him with little respect for Leibniz's views.

A French translation of one of Wolff's books appearing in 1736 was Voltaire's first introduction to Leibniz. His and Madame du Châtelet's association with Samuel Koenig in 1739 taught him more of Wolffian metaphysics and confirmed him in his opposition. His loyalty to Madame du Châtelet, however, caused him to restrain his attacks on Leibnizianism. Nevertheless, he regretted her conversion and made fun of her enthusiasm for Leibniz. In spite of this he seems to have appreciated the merits of the *Institutions de Physique.*[69] An article by Voltaire, presented to the *Académie des Sciences* in 1741 and entitled 'Doubts on the Measure of Motive Force and Their Nature', took issue with Madame du Châtelet and with Leibnizian dynamics:[70]

ship. Suppose that the body *B* which was at rest gains a velocity of 8. Body *A* advances on the ship with a velocity of 10, the sum of the boat's velocity and its own. According to Leibniz the *force* of body *A* before it was thrown forward was 4, the square of the boat's velocity. Its increase of force after being thrown is 8 or 64, making its total force 64 + 4 = 68. But since its total velocity after being thrown is 8 + 2 = 10, its force ought to be 100. This is contradictory and therefore forces cannot be proportional to the squares of the velocities. I am indebted to Professor Keith Symon for pointing out to me the inadequacy of Maclaurin's argument and the correctness of Madam du Châtelet's analysis of Jurin's example. For a discussion of Maclaurin's role in the controversy see Wilson L. Scott, *The Conflict Between Atomism and Conservation Theory, 1644 to 1860*, MacDonald: London; Elsevier: New York 1970, pp. 24–30.

[66]See note [58].

[67]See note [62].

[68]Voltaire, *Lettres philosophiques*, 1734, and *Elements de la philosophie de Newton*, Amsterdam, 1738. On Voltaire see Ira O. Wade, *Studies on Voltaire*, Princeton: Princeton Univ. Press, 1947. See also note [3].

[69]Barber, *Leibniz in France* [3], pp. 174–183, 191. Du Châtelet apparently concealed the writing of the *Institutions* from Voltaire just as she had done with the dissertation on fire. See Barber, 'Mme du Châtelet and Leibnizianism' [3], p. 212.

[70]See note [58].

> Force is not . . . an internal principle [*un principe interne*] a substance which animates bodies and is distinguished from bodies as some philosophers have maintained [i.e. Leibniz]. Force is nothing but the action of bodies in motion and does not exist primitively in simple beings called monads which these philosophers say are without extension and yet constitute extended matter . . . They can no more produce moving force than zeros can form a number. If force is only a property it is subject to variation as are all modes of matter. And if it is in the same ratio as the quantity of motion, is it not obvious that its quantity alters if the motion augments or diminishes?[71]

In attempting to prove this point, Voltaire gave an interesting incorrect example which followed from Descartes' concept of the quantity of motion, $m|v|$. The quantity of motion is always *increased*, he said, when a small *elastic* body collides with a larger one at rest. For example, an elastic body $A$ of mass 20, in motion with velocity 11 ($mv = 220$), hits $B$ at rest whose mass is 200 ($mv = 0$). $A$ rebounds with a quantity of motion of 180 ($mv = 180$) and $B$ goes forward with $mv = 400$ (i.e., $220 + 0 = -180 + 400$). But Voltaire reasoned that $A$, which originally had a 'force' of 200, had produced a total 'force' $m|v|$ of 580. 'On the other hand, as everyone agrees, a great deal of motion is *lost* in the collision of *inelastic* bodies. Thus force in particular parts of matter increases and decreases'.[72]

In Leibniz's philosophy 'force' ($mv^2$) was conserved in elastic collisions. Voltaire is arguing that 'force' ($m|v|$) is not conserved either in this elastic case or in inelastic collisions; rather, it varies as do other properties of matter. 'Force', therefore, cannot be a primary or invariant property of matter. Voltaire's disagreement with Madam du Châtelet rested in part on a difference in the mathematical definition of force and in part on metaphysical differences within the context of substance philosophy. For du Châtelet force itself was a substance, while for Voltaire it was only a property of the universal substance, matter.

In his first published work, *'Thoughts on the True Estimation of Living Forces'* (1747), Immanuel Kant commented on Madame du Châtelet's role in the long controversy.[73] He gave his own account of her objections to Mairan and Jurin, arguing that there was justification

[71] Voltaire, *Oeuvres* [58], pp. 428–429. See Leibniz, 'Correspondence with Arnauld' [24], p. 221: 'For I think rather that everything is full of animated bodies, and in my opinion there are incomparably more souls than M. Cordemoy has atoms. His atoms are finite in number while I hold that the number of souls, or at least of forms is wholly infinite, and that matter being divisible without end, no portion can be obtained so small that there are not in it animated bodies, or at least such as are endowed with primative entelechy, and (if you will permit to use the word life so generally), with vital principle, that is to say, with corporeal substances, of all of which it may be said in general that they are alive'.

[72] Voltaire, *Oeuvres*, pp. 428–429.

[73] Immanuel Kant, 'Gedanken von der wahren Schätzung der lebendigen Kräfte', in *Immanuel Kant's Werke*, Bruno Berlin Cassirer: 1922, vol I, pp. 1–187.

for both the Cartesian and Leibnizian positions.[74] For his own purposes he classified forces into two types: internal, or metaphysical, force which causes a body to move unless hindered but which, since it is not subject to the law of conservation of force, is *for that reason* not capable of mathematical treatment despite the fact that one can provide an 'estimate' for its magnitude, namely, $mv^2$, in all of which it bears an uneasy resemblance to a mixture of Leibniz's 'primitive active' and 'derivative active' (*vis viva*) forces. The other type of force was externally produced, phenomenal, and subject to conservation, hence 'mathematical', and expresses as $mv$, the Cartesian measure of 'force'.[75]

Moreover, in a section entitled 'Vindication of the Thoughts of Mairan against Mme du Châtelet', Kant chastized the Marquise for failing to show more respect to such a great man as Mairan. But, he concluded, since she stood far above those of her own sex, and most of the opposite sex as well, it was understandable that she did not avail herself of the flattery and praise especially reserved for the fairer sex![76]

Although both Kant, in his own manner, and Voltaire, in his, paid tribute to Madame du Châtelet's outstanding abilities, the limits of their own conceptual systems prevented either man from giving adequate recognition to her mechanical analysis of the *vis viva* controversy.

## Conclusion

In an earlier paper on 'The Leibnizian–Newtonian Debates', I argued that adherents to the Newtonian and Leibnizian measures of force in the 1720s were influenced by intellectual commitments to the Newtonian and Leibnizian world views and by social commitments to the groups and institutions surrounding the two scientists.[77] These commitments rendered it difficult or impossible to entertain the possibility that the opposing interpretations of the experiments and mechanical examples might have any validity. I suggested that such commitments would be weakened in the succeeding generation of scientists which had begun to integrate elements of the two philosophies of nature by the 1740s.

[74]*Ibid.*, p. 55 (section 44); p. 67 (section 57); p. 127 (section 111); pp. 133–136.

[75]*Ibid.*, pp. 27–28 (section 16, 17); pp. 39–40 (section 28, 29). See Buchdahl [19], p. 553, and Max Jammer, *Concepts of Force*, Cambridge, Mass.: Harvard Univ. Press, 1957, p. 179.

[76]Kant, [73] p. 136. Although Kant argued philosophically for the equality of the sexes, especially on the issue of property rights, on an experimental level he fell back on the 'fact' of the 'natural superiority of the husband's faculties compared with those of the wife' (Immanuel Kant, *The Philosophy of Law*, W. Hastie (trans.), Edinburgh, 1887, vol. I, i (section 24–26), excerpts in Julia O'Faolain and Lauro Martines (eds.) *Not in God's Image*, New York: Harper & Row 1973, pp. 284–286).

[77]Iltis, 'The Leibnizian–Newtonian Debates' [11].

Madame du Châtelet's *Institutions de Physique* represented such an integration. In her metaphysics she adopted and unified arguments from the Cartesian view that extension was the defining characteristic of matter and from the Leibnizian philosophy in which force was viewed as the primary substance. In her mechanics she presented the Leibnizian position in dynamics along with a Newtonian exposition of basic mechanics. Although her analysis of mechanical problems was of very high quality, she did not reach the conclusion that both the Newtonian and Leibnizian measures of force were valid in the solution of problems.

However, other natural philosophers, namely, d'Alembert, Desaguliers, Boscovich, and Reid, writing during the 1740s, did arrive at integrations of various elements of the three world views such that it became possible for them to admit of the validity of both measures of force. Even so, it cannot be claimed that through these integrations they 'resolved' the *vis viva* controversy. Thus, d'Alembert in 1743 accepted as valid measures of force $mdv$, or dead force, for cases of equilibrium, and $mv^2$, or living force, while calling the controversy a 'dispute over words'.[78] Not until 1758 did he add to this the measure $mv$ defined in terms of the space traversed uniformly in a given time. But his *Traité de dynamique* was basically a text in Cartesian kinematics. Forces were rejected as 'obscure and metaphysical'; mechanics was properly the study of motion or 'observed effects' only.[79]

In 1744, John Theophilus Desaguliers finally published the second volume of his *Course of Experimental Philosophy*, explaining the nine-year delay between the two volumes as due to his inability to resolve the 'question about the Force of Bodies in Motion'.[80] At the conclusion of his discussion of the opposing arguments, he stated: 'I am now convinc'd that all the Phaenomena of the Congres of Bodies may be equally solv'd according to the Principles of the Defenders of the new [$mv^2$] as well as those of the old [$mv$] opinion'.[81] In spite of this admission, however, Desaguliers was a committed Newtonian who accepted and operated within a conceptual framework which made

[78] Carolyn Iltis, 'D'Alembert and the *Vis Viva* Controversy', *Stud. Hist. Phil. Sci.* 1 (1970) pp. 135–144; see pp. 135–138. Jean d'Alembert, *Traité de dynamique*, 2nd edn, Paris, 1758 (1743).

[79] Thomas Hankins, *Jean d'Alembert: Science and the Enlightenment*, Clarendon Press: Oxford 1970, p. 153. See also Hankins, 'Eighteenth Century Attempts To Resolve the *vis viva* Controversy', *Isis* 56 (Fall 1965) pp. 281–297; see pp. 284–285.

[80] L. L. Laudan, 'The *Vis Viva* Controversy, a Post Mortem', *Isis* 59 (Summer, 1968): 131–143; see p. 137. John Theophilus Desaguliers, *A Course of Experimental Philosophy*, London, 1734, II, v.

[81] Desaguliers, *Course of Experimental Philosophy*, II, 63.

force external to matter rather than an internal principle of activity.[82]

Boscovich, in his *De Viribus Vivis* of 1745, had analyzed $mv$ as a time dependent function and $mv^2$ as a space dependent function using geometric diagrams.[83] But he did not claim equal status for the two measures of 'force'; momentum was the true measure while *vis viva* was valid only as a calculating device. He could not accept Leibniz's philosophy of the internal activity of matter.[84]

Thomas Reid's *'Essay on Quantity'* of 1748 likewise stated acceptance of both $mv$ and $mv^2$ as valid principles in mechanics.[85] An early adherent to momentum, over a period of several years he gradually came to the realization that the various problems and experiments discussed in the *vis viva* controversy could be explained equally as well by using the principle $mv^2$.

In spite of the fact that during the 1740s Desaguliers, Boscovich, Reid, and d'Alembert independently came to the conclusion that both $mv^2$ and $mv$ were valid mechanical principles, this did not imply their equal acceptance philosophically and mechanically. Only a few scientists utilized both principles in the solution of mechanical problems until after the enunciation of the general law of energy conservation one hundred years later in the 1840s.[86]

While Madame du Châtelet did not adopt the validity of both measures of force in her mechanics, her insistence on the equal status of matter and 'force' contributed to a new view of nature. During the eighteenth century, the Newtonian–Cartesian dualism of passive matter and external mechanical forces was replaced by an ontology of active substances within extended matter.[87] The view that the essence of matter consisted not only of extension but also of attractive and repulsive forces was important in the emergence of the general law of the conservation of energy in the 1840s.[88] Thus, a philosophical

[82]Iltis, 'Leibnizian–Newtonian Debates' [11], p. 367.

[83]Iltis, 'D'Alembert and the *Vis Viva* Controversy' [78], pp. 138–140. Pierre Costabel, 'Le *De Viribus Vivis* de R. Boscovich ou de la vertue des querelles de mot', *Arch. Int. Hist. Sci.* 14 (1961) pp. 54–57. Roger Boscovich, *De Viribus Vivis*, Rome, 1945.

[84]Hankins, 'Eighteenth Century Attempts' [79], p. 295.

[85]Laudan, 'The *Vis Viva* Controversy' [80], pp. 138–143. Thomas Reid, 'An Essay on Quantity', *Phil. Trans.* 45 (1748) pp. 505–520. Unabridged version in Laudan, pp. 140–143.

[86]Iltis, 'D'Alembert and the *Vis Viva* Controversy', see notes [28, 29].

[87]P. M. Heimann and J. E. McGuire, 'Newtonian Forces and Lockean Powers: Concepts of Matter in Eighteenth-Century Thought', *Historical Studies in the Physical Sciences*, Russell McCormmach (ed.), 3 (1971) pp. 233–306; see pp. 236–237.

[88]Charles Coulston Gillispie, *The Edge of Objectivity*, Princeton: Princeton Univ. Press, 1960, p. 385. Thomas S. Kuhn, 'Energy Conservation as Simultaneous Discovery', in *Critical Problems in the History of Science*, Marshall Clagett (ed.), Madison: Univ. of Wisconsin Press, 1959, pp. 321–356; see pp. 336–339. On Helmholtz, see Yehuda Elkana, 'Helmholtz' Kraft: An Illustration of Concepts in Flux',in *Historical Studies in the Physical Sciences*, II, 263–298; esp. p. 264; P. M. Heimann, 'Helmholz and Kant: The Metaphysical Foundations of *Uber die Erhaltung der Kraft'*, *Stud. Hist. Phil. Sci.* 5 (1974) pp. 205–238.

reorientation as well as a mechanical 'solution' were both necessary before it could be said that the *vis viva* controversy had been 'resolved'.

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