CHAPTER I

Descartes: The Heterogeneity of Motion and Direction

> Descartes rightly distinguished between velocity and direction and also saw that in the collision of bodies that state results which least changes the prior conditions. But he did not rightly estimate this minimum change, since he changes either the direction alone or the velocity alone, while the whole change must be determined by the joint effect of both together. He failed to see how this was possible, however because two such hetergeneous things did not seem to him to be capable of simultaneous treatment.
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> Leibniz, "Specimen Dynamicum" (1695)[1]

Expressed in his Principia Philosophiae (1644) was Descartes' belief that God, the general cause of all motion in the universe, preserves the same quantity of motion and rest put into the world at the time of creation. The measurement of this quantity of motion, Descartes implies, is given by the product of the quantity of matter and its velocity, i.e. *mv*. The term momentum to designate this quantity was not introduced into the scientific literature until late in the seventeenth century. In the Principles Descartes states that "we must reckon the quantity of motion in two pieces of matter as equal if one moves twice

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[1] Gottfried Wilhelm Leibniz, "Specimen Dynamicum," (1695) *Philosophical Papers and Letters*, trans. and ed. Leroy E. Loemker, 2 vols., Chicago, 1956, 2, 718, and *Leibnizens Mathematische Schriften*, ed. C. I. Gerhardt, Halle, 1860, /2/, 2, 234-246.

as fast as the other, and this in turn is twice as big as the first."[2] The conservation of this quantity of motion, Descartes argues, is derived from God's perfection for he is in himself unchangeable and all his operations are performed in a perfectly constant and unchangeable manner. There exists an absolute quantity of motion which for the universe remains constant. If the motion in a particular part of matter is decreased, then that in another part is increased by a like amount. Motion, like matter, once created cannot be destroyed.

It is evident from Descartes' application of this principle in his rules governing the collision of bodies (principles 45-52), that the quantity, *mv*, conserves only the absolute magnitude of the quantity of motion of a body and not its direction, that is, velocity is always treated as a positive quantity, $|v|$, rather than as a vector quantity, $\vec{v}$, whose direction is either positive or negative. This, of course, is one aspect in which Descartes' quantity of motion differs from our modern concept of momentum and which makes his law incorrect from our modern point of view. Descartes however was definite in his opinion that motion and direction both could not be comprehended by the same law. He states "there is a difference

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[2] René Descartes, "Principia Philosophiae," *Oeuvres*, ed. Charle Adam and Paul Tannery, Paris, 1905, *8*, 61, principle 36. Translation by Anscombe and Geach in Descartes, *Philosophical Writings*, Edinburgh, London, 1954, 215.

between motion as such and its determinate direction; it is thus possible for the direction to change while the motion remains unaltered".[3]

Again in Le Monde (written by 1632, but published posthumously in 1664) he writes:

> I add nothing in respect to the direction which each part /of a flame/ takes; for if you consider that the power of moving itself, and that which determines the direction which the movement shall take, are two entirely different things, one of which might exist without the other (as I have explained in the second discourse of the Dioptrics) you will readily see that each moves in the way that is made least difficult to it....[4]

I do not think these are statements which were hastily formulated by Descartes or statements which can be brushed off by a simple announcement that he was wrong. This distinction between motion and direction was one around which all his laws of motion and rules for collision were structured and was basic to his understanding of the physical world. How then is this erroneous idea incorporated into his physical principles?

In his Principia Philosophiae, Descartes formulated three laws which govern motion and direction[5] and by

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[3]Descartes, Oeuvres, 8, 65, principle 41. Contrast Aristotle, who held that a body's natural motion is determined by the direction it must naturally take, i.e. the natural place toward which it tends. A motion and its direction thus cannot be treated separately. Eg. see Aristotle, Physics, BK.4,211a3.

[4]Descartes, Oeuvres,6,9. Above translation Henry A.P. Torrey in The Philosophy of Descartes in Extracts from his Writings, New York, 1892, 212. For the statement in the "Dioptrics" see Oeuvres, 6,9 and this dissertation pp. 19-21 .

[5]Descartes, Oeuvres, 8, 62-65, principles 37, 39, 40.

means of which he intends us to understand the motions in particular parts of matter. Of these three secondary laws of nature, also based on God's immutability, the first is a statement about motion, the second is a statement about direction.[6] and the third is a statement of how the two operate jointly in particular bodies.

The first law is, "Every reality so far as it is simple and undivided, always remains in the same condition so far as it can and never changes except through external causes."[7]

Concerning this law Descartes writes,

> If a piece of matter is at rest, one thinks it will never begin to move, unless impelled by some cause. Now there is equally no reason to believe that if a body is moving its motion will ever stop, spontaneously that is, and apart from any obstacle. So our conclusion must be: A moving body so far as it can goes on moving.[8]

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[6] Gerd Buchdahl has also called attention to this difference between Descartes' first two laws of motion: "...he breaks up the concept of motion into what we now call its 'scalar' and 'vector' components, i.e. velocity is divided into speed and direction, pronouncements concerning the first and second giving him his 'first law' and 'second natural law' respectively. (Prin. ii, 37-39)".
Gerd Buchdahl, "The Relevance of Descartes' Philosophy for Modern Philosophy of Science," *British Journal for the History of Science*, *1* (1963) 236.

[7] Descartes, *Oeuvres*, *8*, 62, principle 37.

[8] *Ibid*. 62. That this motion is continued uniformly is made explicit in Principle 43: "What is moving has some power of persisting in its motion- in a motion constant as regards velocity and direction." In *Le Monde* the first law is stated, "The first is, that each individual particle of matter remains always in one and the same state so long as contact with others does not compel it to change. ...if it be at rest in any place, it will never leave it, unless

This is the well-known first portion of Descartes' statement of the law of inertia. But it is not the complete statement. It lacks the concept that a moving body continues in a straight line unless acted on by another body. This portion concerning the body's direction is contained in Descartes' second law (principle 39) which is; "Any given piece of matter considered by itself tends to go on moving, not in any circular path, but only in straight lines."[9]

In his discussion of this law, Descartes gives us a clue as to why motion and direction should be considered separately. He says: (see diagram p. 16)"For example, a stone A is moving in a sling EA in a circle ABF. At the moment when it is at the point A it has motion in a definite direction, viz, in a straight line toward C where the straight line AC is a tangent to the circle. It cannot be imagined that the stone has any definite curvilinear

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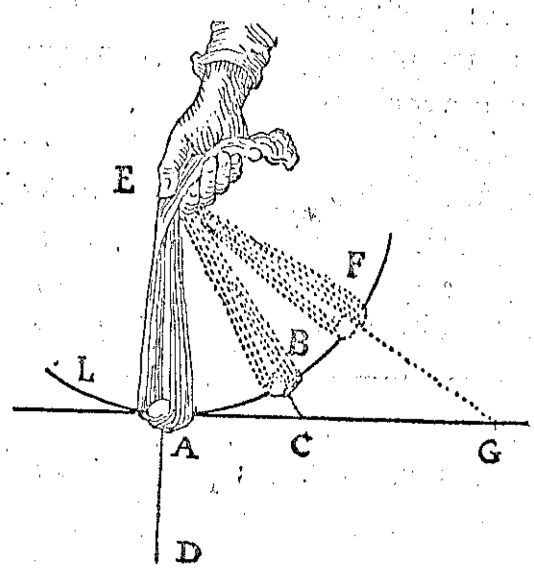
others drive it therefrom; and if it has once begun to move, it will continue always to move with uniform energy (avec une egale force) until others stop or retard it." Oeuvres, 11, 38.

[9] Descartes, Oeuvres, 8 , 64, principle 39.

In Le Monde this law is given (as his third law) as follows: I will add for a third, that when a body moves, although its movement is most frequently in a curved line and can never be otherwise than circular in some degree, as has been said above, nevertheless each one of its particles in particular tends always to continue its own motion in a straight line. And so their action--that is to say, their inclination to move--is different from their movement". Oeuvres, 11, 43-44.

conſervat, nullâ habitâ ratione ejus qui fortè fuit paulò antè. Ac quamvis nullus motus fiat in inſtanti, manifeſtum tamen eſt omne id quod movetur, in ſingulis inſtantibus quæ poſſunt deſignari dum movetur, determinatum eſſe ad motum ſuum continuandum verſus aliquam partem, ſecundùm lineam rectam, non autem unquam ſecundùm ullam lineam curvam. Ut, exempli cauſâ, lapis A, in fundâ E A per circulum A B F rotatus, eo inſtanti, quo eſt in puncto A, determinatus quidem eſt ad motum verſus aliquam partem, nempe ſecundùm lineam rectam verſus C, ita ſcilicet ut linea recta A C ſit tangens circuli. Non autem fingi



poteſt illum determinatum eſſe ad ullum motum curvum : etſi enim priùs venerit ex L ad A per lineam curvam, nihil tamen iſtius curvitatis intelligi poteſt in eo remanere, dum eſt in puncto A. Hocque etiam experientiâ confirmatur, quia ſi | tunc è fundâ egrediatur, non perget moveri verſus B, ſed verſus C. Ex quo ſequitur, omne corpus quod circulariter movetur, perpetuò tendere ut recedat à centro circuli quem deſcribit. Ut ipſo manûs ſenſu experimur in lapide, dum illum fundâ circumagimus. Et quia conſideratione iſtâ in ſequentibus ſæpe utemur, dilimotion; it is true that it arrived at A from L along a curved path, but none of this curvature can be conceived as inherent in its motion at the point A.[10] Thus he implies that *if* motion and direction could be treated simultaneously, a circular path might be an inherent part of a motion, and a stone if released could continue on a curve rather than fly off on a straight line. Now in the Aristotelian tradition of motion, the direction of a moving body was an integral part of its natural form of motion. All natural bodies had within themselves a source of their own motion, in one or a combination of three natural directions: upwards, downwards, or in a circle. Descartes revolutionized this concept of motion in two ways: Motion is not a power in the mover, but a state of the moved;[11] and direction is

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[10] Descartes, *Oeuvres*, *8*, 64, principle 39. Italics mine.

[11] Descartes, *Oeuvres*, *8* , 53-54, principle 25.

Norman Kemp Smith in his *Studies in the Cartesian Philosophy*, London, 1902, 70, 71, 75-59, has indicated an inconsistency in the position Descartes takes in his physics that motion is a mode (a geometrical interpretation) and the position assumed in his metaphysics that motion is as important a reality as matter and mind (a dynamical interpretation).

Descartes' principle 25 states: "And I say that it is the *transportation* and not either the force of the action which transports, in order to show that the motion, is always in the mobile thing, not in that which moves [it] (movente); for these two do not seem to me to be accurately enough distinguished. Further I understand it is a mode of the mobile thing and not a substance, just as figure is a mode of the figured thing and repose of that which is at rest."

not inherent in a body's motion. Thus circularity is not a part of the motion of an object moving on a circular path. At any given instant when a constraining force is removed a body's tendency is to continue in a straight line. This is a denial of the Aristotelian theory that a body released from an "unnatural motion" will tend to move in its own "natural" direction--upwards, downwards, or in a circle.

This second law therefore corrects the erroneous belief in the natural tendency of a body to move in a circle held by most natural philosophers up to and including Galileo by showing that for the case of circular motion (excluding the forces themselves) direction is not an integral part of that motion. Descartes' law is true for the case of inertia, a highly abstract situation where an object moving in the absence of any forces can be treated without considerations of direction. Here then Descartes was correct in maintaining a distinction between motion and direction. However in the case of impact, motion and direction cannot be treated separately. Descartes did not recognize this and dealt with them as distinct variables rather than taking the velocity as a vector. The question arises whether his correct formulation for the case of inertia led him into the same but incorrect analysis for impact?

I think the answer must be no, for he seems to have decided there was a difference between motion and direction in quite another context before writing either Le Monde (written by 1632, but published posthumously in 1664) or the Principles of Philosophy (published 1644). This reference to the difference between motion and direction appears in his discussion of the law of refraction in La Dioptrique (published in 1637 as one of the illustrative essays accompanying his Discours de la methode.). But he was aware of the law of refraction and the distinction he was to maintain between motion and direction, at the time of the compilation of Le Monde about 1629. Here he writes that he had explained it in the second discourse of his Dioptrics.[12]

The second discourse of the Dioptrics is directed first to a discussion of reflection in preparation for the derivation of the law of reflection. The motion of a light ray striking a reflecting surface is better understood, argues Descartes, if compared to the motion of a ball, struck by a raquet, and hitting at an angle a surface CBE from which it rebounds. (see figure, p. 20).[13] It is necessary to remark, he says, that the power which continues the movement of the ball is different from that

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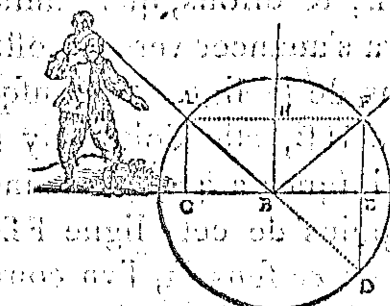
[12]Descartes, Oeuvres, 6, 9. See quotation from Le Monde, p. 2 of this thesis. See also J. F. Scott, The Scientific Work of Descartes, London, 1952, 36.

[13]Descartes, Oeuvres, 6, 93-98.

bien que le mouuement & generalement que toute autre sorte de quantité, estre diuisée entre toutes les parties desquelles on peut imaginer qu'elle est composée; & qu'on peut aysement imaginer que celle de la balle qui se meut d'A vers B est composée de deux autres, | dont l'vne la fait descendre de la ligne AF vers la ligne CE, & l'autre en mesme temps la fait aller de la gauche AC vers la droite FE, en sorte que ces deux, iointes ensemble, la conduisent iusques a B suiuant la ligne droite AB. Et en suite il est aysé a entendre, que la rencontre de la terre ne peut empescher que l'vne de ces deux determinations, & non point l'autre en aucune façon. Car elle doit bien empescher celle qui faisoit descendre la balle d'AF vers CE, a cause qu'elle occupe tout l'espace qui est au dessous de CE; mais pourquoy empescheroit elle l'autre, qui la faisoit auancer vers la main droite, vû qu'elle ne luy est aucunement opposée en ce sens là? Pour trouuer donc iustement vers quel costé cete balle doit retourner, descriuons vn cercle du centre B, qui passe par le point A, & disons qu'en autant de temps qu'elle aura mis a se mouuoir depuis A iusques a B, elle doit infalliblement retourner depuis B iusques a quelque point de la circonference de ce cercle, d'autant que tous les points qui sont aussy distans de cetuy cy B qu'en est A, se trouuent en cete circonference, & que nous supposons le mouue-



which determines it to move more toward one side than toward another. This is true because the ball received both its motion and its direction from the racquet and by considering the racquet it is easy to see the distinction. The force with which the racquet pushes the ball is the same no matter in what direction the ball might be hit. But it is the situation of the racquet which determines the ball, out of all the possible directions, toward the particular point B. Now because the motion and the determination of the ball are two different things, when the ball hits the earth, its direction can be changed but the force of its motion, stemming from a different cause, will not be altered.[14]

In deriving the law of refraction he again repeats his conviction that the movement of the ball differs entirely from its determination to move itself, so that the two quantities should be examined separately.

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[14]Descartes, Oeuvres, 6, 95. Italics added. Descartes then goes on to state that the motion of the ball and its determination each can be divided into all the parts of which they are composed, so that one can imagine one motion and a direction which make it descend from the line AF toward the line CE and a second which makes it go from the left line AC towards the right line FE. When the ball meets the earth the direction of the vertical component, but not its motion is altered, whereas the horizontal component is not changed in any way. Then the law of reflection is derived from the consideration that the time of descent from A to B is the same as the ascent from B up to some point on a circle about B with radius AB.

It would thus seem that at the basis of Descartes' derivation of the laws of reflection and refraction based on the false premise that light travels faster in the more dense medium $\left[\frac{\sin i}{\sin r} = \frac{v_{water}}{v_{air}}\right]$ / there lies another more subtle error which here has no effect on the true result obtained, $\left[\text{i.e. } \frac{\sin i}{\sin r} = \mu\right]$. However as will be seen, when used as a basis for the laws of impact that error has a definite and far-reaching effect. It consists in considering the motion of a body and its direction separately, so that in a collision either the body's motion or its direction can be altered without regard for the other, the result being that the former could be conserved while the latter remains unaffected.

Descartes' erroneous third law of nature, enunciated in the <u>Principia Philosophiae</u> together with the seven rules of impact derived from it, illustrates the effect of conserving quantity of motion without conserving total direction, (that is, without maintaining the same direction of the center of mass before and after collision).

> When a moving body collides with another, then if its own power of going on in a straight line is less than the resistance of the other body, it is reflected in another direction and retains the same amount of motion, with only a change in direction [part a], but if its power of going on is greater than the resistance, it carries the other body along with it and loses a quantity of motion equal to what it imparts to the other body [part b].[15]

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[15]Descartes, <u>Oeuvres</u>, <u>8</u>, 65, principle 50.

The two principles, one, the conservation of the motion of the attacking body and the other, the conservation of its forward direction, act separately to determine the effect. When the resistance of the obstacle is greater than the quantity of motion in the attacking body (part a), the effect is determined by the conservation of its mv, whereas the continuing direction of the attacking body is not retained. But when the resistance is less (part b), the effect is determined by conserving the same direction of the attacking body while the magnitude of its mv is not retained, but is decreased in such a way that the total quantity of motion is conserved between the two bodies.

Descartes' seven rules (principles 46-52) governing the action of colliding "hard" bodies can be explained on the basis of the third law.[16] All except the first are incorrect from an experimental viewpoint but all are consistent within the framework of the laws discussed above.[17]

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[16]The seven rules are given in the appendix to this chapter translated from the Latin text (Oeuvres 8, 68-70, Principles 46-52.) by Georgio de Santillana and Walter H. Pitts in a pamphlet for use at MIT.

[17]For a discussion of the background to these seven above, see R. Catsby Taliaferro, "The Concept of Matter in Descartes and Leibniz," Notre Dame Mathematical Lectures. Notre Dame, 1964, 9,17. "Strangely enough in the journal of Beeckman for Nov. 23 to Dec. 25, 1618 at a time when he and Descartes were meeting together for the discussion of problems in physics and mathematics there appear seven laws of percussion for perfectly soft bodies. /For Beeckman/ the atoms of his theory (for he accepts them) correspond to Descartes' perfectly hard bodies, except that for Beeckman they are perfectly soft in the sense that they do not rebound

In each of the seven cases, $m|v|$ is conserved; the final direction is determined in accordance with the third law above. In rules 1-3 the bodies B and C move toward each other, in rules 4-6, one is at rest, in rule 7 both move in the same direction.

These rules together with my analysis and mathematical expressions for them may be found in the appendix to this chapter. Descartes himself did not give mathematical equations for his concept and application of the quantity of motion. The bodies are considered to be perfectly hard, which for Descartes means they have no moving parts.[18]

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but move off together after impact. These laws are all correct from the point of view of classical mechanics (...), and well they might be, since Beeckman assumes the law of inertia, the conservation of momentum (...), and that the bodies are perfectly "inelastic."

[18]By hard (dura)body Descartes meant a body whose parts are at rest relative to each other since it takes force to move a particle at rest. Fluids on the other hand have their parts in motion and hence separate easily when an object is thrust into them. (Descartes, Oeuvres, 8, 70, principles 54, 55.) If the parts of bodies are at rest with respect to each other then there would be no elasticity and the bodies would be absolutely hard. Bodies of this sort are not found in nature. Were such theoretical bodies to collide, there would be no rebound, since there is no elasticity and they would tend to disintegrate. If absolute hardness is not meant, then the harder the bodies are taken to be, the more elastic they are. The elastic parts obey Hooke's law, $F/x=k$, the smaller distance of oscillation of the parts resulting in larger constants and greater elasticity. But if $x \to 0$ then no movement is involved and the parts are no longer little springs. In Descartes' rules for colliding bodies, the bodies B and C rebound as if elastic in rules 1, 2, 4, 6. However in rules 3, 5, and 7, the bodies stick together as if inelastic.

According to Wilson Scott, op. cit. 31-36 the paradox of Descartes' hard bodies rebounding is "partially resolved by recognizing that Cartesian hardness is a brittle

The difficulty with these seven rules for colliding bodies is, as has been said, that quantity of motion and direction are not handled simultaneously. Considerations drawn from inertia and from the derivation of the law of refraction as to how Descartes was led to this view have been discussed (see pp. 17,21.). There is one further possibility which may have governed Descartes' decision to treat them separately. This is given by Leibniz (1716):

> Descartes recognized that souls (<u>les ames</u>) cannot give force (<u>force</u>) to bodies because the same quantity of force is always conserved in matter. He believed however that the soul (<u>l'ame</u>) could change the direction of the body. But this was because the law of nature was still unknown in his day according to which matter conserves the same total direction. If he had noticed this he would have fallen on my system of pre-established harmony.[19]

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hardness. The particles are called hard because they have no elasticity. Yet being brittle they crack and may be pulverized....The controversy on hardness arose only when modern atomists like de Volder, Hartsoecker, Newton and Boerhaave began challenging the Cartesian defintion of the word <u>hard</u>. They insisted that a hard body should be not only inelastic but indivisible as well. ...the trio of <u>choc</u> members of the British Royal Society (Wallis, Wren, and Huygens) /did not/ question Descartes' definition of hardness with the possible exception of Wallis. They were indifferent to the puzzling implications of the problem, probably because they were not atomists themselves."

[19]Wilhelm Gottfried Leibniz, "The Monadology" (1716), <u>Philosophical Papers and Letters</u>, ed. and trans. Leroy E. Loemker, 2 vols., Chicago, 1956, <u>2</u>, 1058, sec. 80. In the essay, "On Vital Principles and Plastic Natures" (1705), Leibniz makes a similar statement as follows: "Descartes very well recognized that there is a law of nature by which the same quantity of force is conserved, though he made a mistake in applying this principle by confusing quantity of force with /quantity/ of motion; he therefore thought it unnecessary to give the soul the power of increasing or

It is reasonable that Descartes might have held such a view for he had difficulty in answering the questions and objections concerning the interaction of the mind and the body. Descartes' answer to the mind-body problem lay in his theory of the pineal gland which translated motions of the body by way of the animal spirits to the mind. Conversely the soul which has the characteristic of volition, in some way must be capable of acting on the body. But according to Norman Kemp Smith, Descartes never fully discussed this second problem of the soul's ability to act on the body:

> Secondly with regard to the action of the mind on body in volition, Descartes kept consistently to his dualism so far as to admit that the mind cannot originate motion in the brain. That would be a veritable creation of motion out of nothing by a mere fiat of the will. It would also be in direct conflict with Descartes' physical principle

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diminishing the force of the body but only that of changing its direction by changing the course of the animal spirits. And those Cartesians who have given vogue to the doctrine of occasional causes hold that since the soul can have no influence whatever upon the body, it is necessary for God to change the course and direction of the animal spirits in accordance with the wishes of the soul. But if this new law of nature which I have demonstrated had been known in Descartes' day according to which not only the same total force of bodies in interaction is conserved but also their total direction, he would undoubtedly have been led to my system of pre-established harmony, for he would have recognized that it is just as reasonable to say that the soul does not change the quantity of direction of the body as it is to deny to the soul the power of changing the quantity of its force, both being equally contrary to the order of things and the laws of nature, since both are equally inexplicable." Loemker, 2, 954, 955.

> that only motion can produce motion, and that the sum of motion in nature is constant and cannot be added to. Yet incomprehensible as is the action of mind on body, that does not prevent Descartes from most emphatically asserting that it takes place. 'That the mind which is incorporeal is capable of moving the body, neither general reasoning nor comparisons can teach us, yet none the less we cannot doubt it, since so certain and so evident experiences make it manifest to us everyday of our lives! (Lett., X,161 and Lett., IX, 132-4).[20]

Furthermore it is not clear that Descartes ever held the view attributed to him by Leibniz that the soul can change the direction of the body, while not altering its motion.[21] It is possible however, that Leibniz may have assumed it to be Descartes' position, if it was indeed held by his followers. Norman Kemp Smith writes:

> Some of Descartes' successors however, Clauberg, for instance (cf. Corporis et Animae Conjunctis, cap. XVI), did attempt by an analogy drawn from the material world to explain the action of mind on body. The driver of a wagon does not move the wagon, but only directs the motion of the horses that pull it. So too, the mind needs not to cause motion in the brain but only to direct the "animal spirits" that already exist in the brain and are continually circling about in it. This analogy however as has often been pointed out, is quite misleading since to be applicable at all to the relation between mind and body the driver of the wagon would have to guide the movements of the horses of his mere wish. That the mind should divert a motion of the brain in a new direction is not a whit less mysterious nor less at variance with Descartes' physical teaching that that it

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[20] Norman Kemp Smith, Studies in the Cartesian Philosophy, London, 1902, 82.

[21] I have not been able to find this idea in Descartes' major philosophical writings.

should originate an entirely new motion. Leibniz (Essais de Theodice, sec. 60, Gerhardt, VI, 135-6) and also many modern commentators assert that Descartes himself tried to escape the difficulty in this way. But though Descartes frequently speaks of the motion of the 'animal spirits' as being merely directed (not originated) by the movements of the pineal gland, he never so far as we are aware, suggests that those movements of the pineal gland, which are involved in voluntary action, can be explained in a similar manner as previously existing and merely guided by the mind.[22]

From this evidence it seems very unlikely that Descartes was led to the error of not conserving the same direction of the center of mass in the impact of bodies because of his philosophical position on the mind-body problem. However having once assumed motion and direction to have heterogeneous characteristics whether unthinkingly or from the reasons explained in the Dioptrics, it could have been a logical way for him to attempt an explanation of the action of the mind on the body.

Finally a brief comment must be made about the accusation sometimes made that Descartes was unconcerned about the experimental verification of his physical principles and that if he had performed experiments he immediately would have perceived the falsity of his rules of collision.[23]

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[22] Ibid., p. 82, footnote 2. Italics Kemp Smith's.

[23] For examples of this view see Ernst Mach, The Science of Mechanics, 6th ed., La Salle, 1960, 363: "Descartes however was infected with all the usual errors of the philosopher. He places absolute confidence in his own ideas. He never troubles to put them to experiential test. On the contrary a minimum of experience always suffices him for a

Perhaps it is more accurate to say that it was not so much Descartes' lack of empirical observations (for his writings contain many observations) as it was his conviction that the environment was too vastly inter-connected to be defined by simple mathematical expressions. Descartes looked at the conservation of motion from the point of view of the entire world system; to study an isolated system of colliding bodies was in the last analysis not possible because the necessary perfection could not be found in our complex environment:

> In fact no bodies in the universe can be thus separated from all others and in our environment we do not ordinarily get perfectly solid (dura) bodies; so it is much harder to calculate how the motions of bodies are changed by collision with others. For we have to take into account all the bodies that touch a body on every side; and the effect of these is very different according as they are solid or fluid.[24]

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maximum of inferences."

For a discussion of this view see Buchdahl (1963), op. cit.

See also Edward Jan Dijksterhuis, The Mechanization of the World Picture, trans. C. Dikshoorn, Oxford, 1961, 415: "/Descartes/ took a lively interest...in the empirical study of nature and in technology. He took infinite pains to have the instrument maker Ferrier grind hyperbolic lenses for him; he made independent physiological observations about the circulation of the blood, experimented with artificial rainbows, carried out experiments in order to determine the specific gravity of air, and observed meterological phenomena with the greatest accuracy. /Descartes, Les Meteores, Discourse VI, VII., Oeuvres VI, 298, 312./"

[24]Descartes, Oeuvres, 8 70, principle 53. See also Santillana and Pitts, op. cit., forward.

Within the complex inter-relationships of the world system it is understandable that assuming the constancy and simplicity of nature's (or God's) operations the total amount of motion could appear to be constant. Likewise within this complex system it would appear impossible that among the random fluctuating motions of bodies the same total direction could be conserved.

For in order to demonstrate conservation of momentum, an isolated system is needed. This would mean total direction is conserved because the center of mass of the system moves in a straight line with constant velocity before and after the collision. Thus instead of achieving his purpose of deriving simple mathematical expressions from abstract principles, Descartes finds he cannot separate bodies from their complicated relationships with other bodies to give verification to these simple rules.

Conclusion

The foregoing discussion has yielded the following points concerning Descartes' quantity of motion, $m|v|$. Descartes separates the ideas of motion and direction, and his universal law conserves the former but not the latter. That is, it conserves the quantity of motion as a scalar but not as a vector entity. The separation of motion and direction is an integral part of his sytem of physical laws, and seems to be a consequence of his own early scientific work in optics. But it also may be a reaction against the popular world view of his time which accepted the idea of the inherent circular motion of a body. It may not have been a consequence of his own answer to the mind-body problem. These deeply intertwined reasons which formed his thought vindicate him somewhat from the standard evaluation that his laws of colliding bodies are absurd because he did not take the trouble to check them experimentally.

Appendix I

Descartes' rules governing the action of colliding bodies, translated from the Latin text of the Principles of Philosophy:

> (1) If the two bodies B and C are equal and moving toward one another with equal speeds, but in opposite directions, they will rebound after the collision and each will move in the opposite direction from the original one, without change of speed.
>
> (2) If B is very slightly greater than C, but otherwise everything is as above, then only C will rebound, and both will move to the left with equal speed.
>
> (3) If B and C are equal in heaviness (mole), but B moves with slightly greater speed than C, not only do both move to the left afterwards, but B also imparts to C half the difference of their initial speeds. E. g., if B initially has six degrees of speed and C has only four, after collision they will both move to the left, each with five degrees of speed.
>
> (4) If body C is initially at rest and is a little greater than B in magnitude, then with whatever speed B may move toward C, it can never move it, but will itself be repelled in the opposite direction. For a resting body resists a fast moving body more than a slow one, and that in proportion to the excess of one fast moving in magnitude over the other; and on that account there is always more resistance in C than impulsion in B.

(5) If the resting body C is smaller than B, then, however slowly B moves toward C, it will carry C with it, after imparting so much of its motion to C that both continue together with equal speed. E.G., If B is twice as great as C, it transfers to C a third part of its speed, because that third part will move body C at the same speed as the remaining two-thirds will move B with its double heaviness. And thus after B has collided with C, it moves one-third more slowly than before; that is, it will take the same time to move two feet as it formerly took to move three feet. In the same manner, if B is three times as great as C it will impart a fourth part of its motion to it, and so on in other cases.

(6) If the body C at rest, is exactly equal in magnitude to B, which moves toward it, it is in part repelled by B and in part impels B forward; for example, if B moves against C with four degrees of speed, it will communicate one degree to C, but be reflected back in the opposite direction with the other three.

(7) Finally if B and C are moving in the same direction, but C is slower, with B following it at a greater speed, so as ultimately to collide with it, and if C is greater than B, but B exceeds C in speed more than C does B in magnitude, then in the collision B will transfer enough of its motion to C so that afterwards B and C will move in the same direction with the same speed. If, however, the excess of B over C in speed is less than that of C over B in magnitude, B will be reflected in the opposite direction at its original speed. These excesses may be computed as in the following example: if C is twice as great as B, and B moves less than twice as fast as C, B will not impel C at all, but be reflected in the opposite direction. If B does move more than twice as fast as C it will increase the motion of C. Thus, if C has only two degrees of speed and B has five, two degrees will be subtracted from B, which when added to C, produce only one degree more, since C has twice the magnitude of B: it is determined thus so that B and C will both have three degrees of velocity after their collision. Other cases can be determined in the same way. Nor do such cases require proof, because they are obvious in themselves.[25]

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[25] Descartes, Oeuvres, 8, 68-70, principles 46-52. Translation from the Latin is by Georgio de Santillana and Walter H. Pitts in a pamphlet for use at MIT, containing the complete text of Part II of the Principles of Philosophy.

Analysis and Mathematical Expressions for the Rules for Colliding Bodies[26]

In each of the seven cases, $m|v|$ is conserved; the final direction is determined in accordance with Descartes' third law (principle 50). In rules 1-3 the bodies B and C move toward each other; in rules 4-6 one body is at rest; in rule 7 both move in the same direction.

In rule 1, both bodies are equal in magnitude and speed. Hence both have the same "power of going on" and offer equal resistance. Both rebound retaining their quantities of motion.

$$m_B|v|_B + m_C|v|_C = m_B|v|_B + m_C|v|_C, \text{ where } m_B = m_C \text{ and } v_B = v_C.$$

In rule 2, the resistance, or bulk of B is greater but the speeds are equal, so part a of law 3 applies. (See this ch., p. 22). Body C changes its direction but retains its velocity and its mv.

$$M_B|v|_B + m_C|v|_C = M_B|v|_B + m_C|v|_C \quad \text{where } M_B > m_C, \ |v|_B = |v|_C$$

In rule 3, both bodies are equally heavy but B has a greater "power of going on" because it has a greater speed. Thus part b of Law 3 applies and both move together, with body B losing to C part of its quantity of motion.

$$m_B|V|_B + m_c|v|_c = 2m_B\left(\frac{|V|_B - |v|_c}{2}\right), \quad m_B = m_C, |V|_B > |v|_C .$$

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[26] Descartes does not give equations for the collision of bodies. The following analysis of the rules and the equations expressing them are my own.

In rule 4, body C is larger than B and is at rest. For Descartes, rest is the opposite of motion and a body requires power to stay at rest, just as it does to move. (Principles 43,44) Body C always has greater resistance, resisting a fast moving body more than a slow moving body. Thus part a of law 3 applies and B recoils.

$$m_C |v|_C + 0 = m_C |v|_C + 0,\ v_C = 0.$$

In rule 5, C at rest is smaller and has less resistance. Part b of law 3 applies and B carries C forward giving up a quantity of motion to C so that both continue with equal speed.

$$M_B |V|_B + 0 = \frac{(M_B\ |V|_B)}{M_B + m_C}(M_B + m_C),\ m_C < M_B.$$

Rule 6 hypothesizes equal magnitudes with C at rest. The result is a split between the effects of parts a and b because either is possible. B with speed 4 could push C along giving 2 units of speed to C (part b); or B could recoil without loss of speed. The actual outcome is a compromise, B giving one of the two units in question to C, and retaining the other, to recoil with velocity 3.

$$4m_B + 0 = (\overrightarrow{2 + 2})\ m_B$$

$$4m_B + 0 = \overleftarrow{4m_B} + 0 \qquad \text{equally probable}$$

$$4m_B + 0 = \overleftarrow{3m_B} + \overrightarrow{1m_B} \qquad \text{compromise where } m_B = m_c,\ |v|_c = 0$$

In Rule 7 where B and C move in the same direction such that B will overtake and collide with a larger body C, the results are predicted as in the above cases but the excess quantities are used. If the excess in magnitude is greater than the excess in speed, part a of law 3 applies and B recoils without loss of speed. But if the excess in speed is greater than the excess in resistance then part b applies, and motion is transferred such that both move together with equal speed.