

### CHAPTER III

#### Leibniz's Dynamics and the Initiation of the Controversy (1686-1716)

In March 1686, Leibniz published in the Acta Eruditorum, "A Brief Demonstration of a Notable Error of Descartes and Others Concerning a Natural Law, According to Which God is Said Always to Conserve the Same Quantity of Motion; A Law Which They Also Misuse in Mechanics".<sup>1</sup>

In this and subsequent papers Leibniz maintained that Descartes' "quantity of motion" ( $m|v|$ ) was conceptually in error. Knowing that this quantity is very close to the modern concept of momentum, excepting the sign of the velocity, we must ask how it was possible for Leibniz to have proved its falsity. If under certain conditions momentum is conserved, and is indeed a legitimate concept, how could Leibniz have demonstrated that it is not? In these two chapters

---

<sup>1</sup>Gottfried Wilhelm Leibniz, "Brevis demonstratio erroris memorabilis Cartesii et aliorum circa legem naturalem, secundum quam volunt a Deo eandem semper quantitatem motus conservari; qua in re mechanica abutuntur," Acta Eruditorum (1686) 161-163. Also in Leibnizens Mathematische Schriften, edited C. I. Gerhardt, Halle, 1860 7/2, 117-119. Hereafter cited as G. M. Translation by Leroy E. Loemker, Leibniz's Philosophical Papers and Letters, 2 vols., Chicago, 1956, 1, 455-458.

By "Descartes and Others," Leibniz indicated "Father Honoratius Fabri, Father Ignatius Pardies, Father Malebranche, Marcus Marci, and Claude Deschales." Loemker 1, 458.

then, the reasons for the initiation of the dispute will be explored. Several lines of argument will be followed as they developed historically. One of these is the confusion made by Descartes' followers over virtual velocities, a quantity applied in statics; they confused the mass of a body times its virtual velocity,  $mdv$ , with the quantity  $mv$ , a concept in dynamics. A second is Leibniz's identification of the measure of force and the conservation of force. A third is his application of the principle of the impossibility of perpetual motion to  $mv$  situations. Another is the desire of Leibniz to find a conserved dynamic quantity as a foundation for his philosophical system.

In his 1686 "Brief Demonstration", Leibniz stated that there was a difference between the concepts motive force /motricis potentiae/ and quantity of motion ( $mv$ ) /quantitas motus/ and that one cannot be estimated by the other. Leibniz, like many others, did not distinguish between mass and weight. He interchanged the Latin terms molem, corpus and libra, and the French terms masse, pesanteur and poids. Motive force should be designated mgs or ws (weight times height) since it is this which is equivalent (except for a factor of  $\frac{1}{2}$ ) to  $mv^2$ , which Leibniz called vis viva or living force. Leibniz however did not use different words for the m in motive force and the m

in mv and mv.<sup>2</sup> Leibniz's "motive force" is a rudimentary form of our concept of potential energy.

"It is reasonable," argued Leibniz,

that the sum of motive force/motricis potentiae/ should be conserved /conservari/ in nature and not be diminished--since we never see force lost by one body without being transferred to another--or augmented, a perpetual motion machine can never be successful because no machine, not even the world as a whole ~~because no machine, not even the world as a whole~~ can increase its force without a new impulse from without. This led Descartes who held motive force /vis motrix/ and quantity of motion /quantitatem motus/ to be equivalent, to assert that God conserves /conservari/ the same quantity of motion in the world.<sup>2</sup>

Leibniz's argument is based on two assumptions, both of which he claims are accepted by the Cartesians.

(1) "A body falling from a certain height /altitudine/ acquires the same force /vis/ necessary to lift it back to its original height if nothing external interferes." "Motive force" is thus taken to be the product of the body's weight and the height from which it fell. This statement is the idea of the impossibility of perpetual motion. If nothing

---

<sup>2</sup>Translation from Loemker 1, 455-456. Original latin from G.M./27 2, 117: Itaque cum ratione consentaneum sit, eandem motricis potentiae summam in natura conservari, et neque imminui, quoniam videmus nullam vim ab uno corpore amitti, quin in aliud transferatur neque augeri, quia vel ideo motus perpetuus mechanicus nusquam succedit, quod nulla machina ac proinde ne integer quidem mundus suam vim intenere potest sine novo externo impulsu; inde factum est, ut Cartesius qui vim motricem et quantitatem a Deo in mundo conservari.

external adds force to the process, mechanical perpetual motion cannot result.<sup>3</sup>

(2) The same force is necessary to raise body A (See Leibniz's fig. VI, p. 73) of one pound /libra/ to a height of 4 yards /ulnae/ as is necessary to raise body B of four pounds to a height of one yard. This is a work equivalency relation.<sup>4</sup> From these two assumptions Leibniz inferred that body A of one pound in falling a distance  $s = 4$ , will acquire the same force as body B of 4 pounds falling  $s = 1$ :

For in falling from C and reaching D, the body A will have there the force required to rise again to C by the first assumption; that is it will have the force needed to raise a body of

---

<sup>3</sup>This assumption had its beginnings in Jordanus' notion of gravitas secundum situm (gravity according to position). It is found in the writings of early seventeenth century authors as the experimental observation that no system of falling weights will produce perpetual motion in any of its parts. Galileo showed that no series of inclined planes will impart to a descending body a velocity sufficient to carry it to a vertical height greater than its initial height. See Hiebert, op. cit., 60, 61.

<sup>4</sup>The second assumption was stated by Descartes in a letter to Mersenne (1638) (A.T., 2, 228): "The proof of this depends solely on the principle which is the general foundation of all statics, that no more or less force /force/ is needed to lift a heavy body to a certain height /hauteur/ than to lift another less heavy to a height as much greater as it is less heavy or to lift one heavier to a height as much less. As for example, that the force which can lift a weight /poids/ of 100 pounds to the height of two feet, can also lift one of 200 pounds to the height of one foot, or one of 50 to the height of 4 feet and thus of others if it is so applied to them."

TAB. VI. ad a. 1686. pag. 159.



Fig. VII.

Fig. VI.

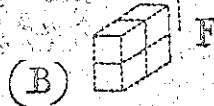


Fig. V.

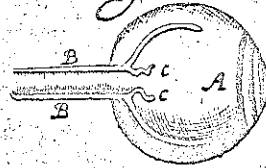


Fig. IV.

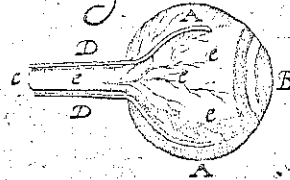


Fig. III.

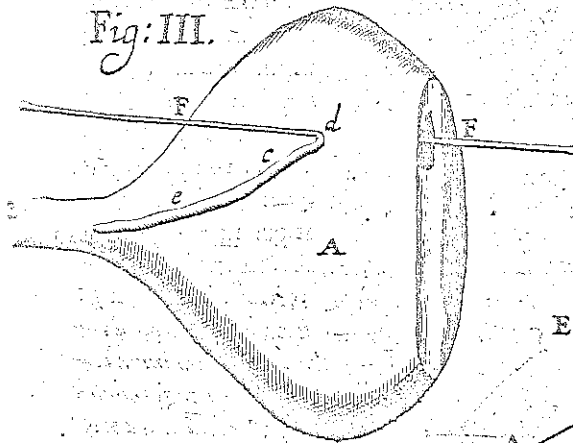
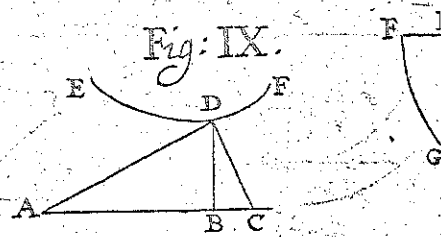


Fig. IX.



1 pound (namely itself) to the height of 4 yards. Similarly the body B after falling from E to F will there have the force required to rise again to E, by the first assumption; that is, it will have the force sufficient to raise a body of 4 pounds (itself namely) to a height of 1 yard. Therefore by the second assumption, the force of the body A when it arrives at D and that of the body B at F are equal.<sup>5</sup>

On the other hand the Cartesian quantities of motion are not equal. For as Galileo showed, body A in its fall will acquire twice the velocity of body B. (This is now written  $2gs = v^2 - v_0^2$ .) Body A, 1 pound falling from s = 4, will arrive at D with a velocity 2, hence its quantity of motion,  $mv = 2$ . Body B of four pounds falling from s = 1 arrives at F with velocity 1, its  $mv$  thereby being 4. Therefore the quantities of motion are unequal but the "motive forces" /vis motrix/, as proved above, are equal.<sup>6</sup>

Thus says Leibniz, the force of a body cannot be calculated by finding its quantity of motion, but rather is to be estimated from the quantity of the effect /quantitate effectus/ it can produce, i.e. from the height to

---

<sup>5</sup>Loemker, 1, 457.

<sup>6</sup>For the relationship  $mgs = mv^2$  implied here, Leibniz is indebted chiefly to Huygens who used it in his derivation of the law of the compound pendulum in his Horologium Oscillatorium (1673). Huygens also related the heights of fall of a body to the velocities acquired in Proposition VIII of his De Motu Corporum ex Percussione largely complete by 1656 but published posthumously in 1703. See this dissertation Ch. II, pp. 59-61.

which it can elevate a body of a given magnitude /mag-  
nitudinus <sup>7</sup>.

Several points are to be noted about this "Brief Demonstration", the first paper in a long series of discussions between Leibniz and his opponents on the subject of "living force".

First of all, Leibniz has not yet introduced the term, vis viva, i.e. "living force", or its mathematical equivalent,  $mv^2$ . He does not publically speak of living force until 1695 in the well-known "Specimen Dynamicum".<sup>8</sup> In these earlier papers the discussion involves the term motive force /vis motrix, vs, the equivalent  $mv^2$  being only implied by the use of the square root of the distance of fall in calculating the mv's of bodies A and B.<sup>9</sup>

Secondly he asserted that the Cartesians were led into error by confusing the force of motion which they est-

---

<sup>7</sup> Loemker, 1, 457.

<sup>8</sup> Leibniz first speaks of living force in his "Essay de dynamique" (1691) which was unpublished until discovered by Gerhardt in the papers at Hanover and included in the Mathematische Schriften, Halle, 1860, /27 2, 215. He also uses the term in an essay recently discovered by Pierre Costabel, and described in his Leibniz et la textes de 1692, Paris, 1960, 104.

<sup>9</sup> Loemker 1, 458. "It must be said therefore that forces are proportional jointly to bodies of the same specific gravity or solidity) and to the heights which produce their velocity or from which their velocities might be acquired).

imated by the quantity of motion, with the quantity used in statics in the case of the five simple machines which is also estimated by the mass times the /virtual/ velocity:<sup>10</sup>

Seeing that velocity and mass compensate for each other in the five common machines, a number of mathematicians have estimated the force of motion /vim motricem/ by the quantity of motion, or by the product of the body and its velocity /producto ex multiplicatione corporis in celeritatem suam/. Or to speak rather in geometrical terms the forces of two bodies (of the same kind) set in motion, and acting by their mass /mole/ as well as by their motion are said to be proportional jointly to their bodies /corporum/ or masses /molium/ and to their velocities /velocitatem/.<sup>11</sup>

---

<sup>10</sup> Neither Leibniz nor the Cartesians used the term virtual velocity. The virtual velocity,  $dv$ , of a body is the ratio of the virtual displacement,  $ds$ , to the time element,  $dt$ ,  $dv = ds/dt$ . Virtual displacement,  $ds$  is the distance through which a body in equilibrium or under constraint, would move if acted upon by a force which disturbs the equilibrium. Virtual velocity is the velocity the body would acquire in moving through the distance  $ds$ . On the use of the term virtual velocity, see Erwin Hiebert, *op. cit.*, 53: "Prior to the time of Varignon's Nouvelle mecanique of 1725, no name was attached to the principle we have been discussing /viz. virtual work/. John Bernoulli (1667-1748) of Basel supplied an expression in 1717 in an off-hand suggestion in a letter addressed to Varignon. In this letter Bernoulli introduced the term virtual velocity (vitesse virtuelle) .... He had used the term...to designate the velocity which is associated with any infinitesimal displacement which is compatible with the constraints imposed upon a system in the state of equilibrium where neither the constraints nor the displacements need be actualized." I am indebted to Professor Erwin Hiebert for his clarification of the way in which the Cartesians misused

<sup>11</sup> G. M. [27] 2, 117; Loemker 1, 455. and misunderstood the use of mv and mdv throughout the controversy.



This accusation Leibniz also makes in later papers. There is no evidence that Descartes himself made this error, although his followers certainly did. Quantity of motion, later known as momentum, is not the same as the quantity formed by the product of the mass and the virtual velocity as applied to static situations. For example in the case of the lever, the forces ( $F = ma$ ) on each lever arm are equal:  $m_1 dv_1 / dt_1 = m_2 dv_2 / dt_2$ . But because there is equilibrium the times are equal,  $dt_1 = dt_2$  and hence  $ds = dv$ . Thus  $m_1 dv_1 = m_2 dv_2$  or  $m_1 ds_1 = m_2 ds_2$ . But the  $dv$ 's are virtual velocities and not the actual velocities in the momentum expression,  $mv$ , for moving bodies. Furthermore  $mdv$  is an energy formula while  $mv$  is a momentum expression.<sup>12</sup>

---

<sup>12</sup>For a history of the principle of virtual velocities and virtual work, see Erwin Hiebert, *op. cit.*, 7-58. See especially p. 11: "The seed of the principle was contained in the *Mechanica* attributed to Aristotle: If movement has been imparted to a lever in equilibrium the velocities experienced by the weights at the ends of the lever arms, described as the arcs through which they are swept in equal times, will be in inverse proportion to the weights." p. 17: "Hero furnishes a clear and concise explanation of the action of the lever in terms of vertical displacements of weights at the end of lever arms...." p. 32: "Jordanus de Nemore in the early thirteenth century stated the law of the lever in a proposition which can be interpreted; 'The lesser weight on the longer lever arm will balance the greater weight on the shorter arm if the ratio of the weights is inversely proportional to the ratio of virtual displacements of the weights along the circular path described by the lever arms.'" p. 46: "In *Le Mechaniche* of Galileo (1649) it is stated: 'The moment (*momento*) of a body situated at the end of the lever arm of a balance is called its disposition (*propensione*) and is composed of its weight (*peso*) and distance (*lontananza*) from the fulcrum.'" p. 48: Finally Descartes "in his writings on statics found mostly

In this same "Brief Demonstration " Leibniz elaborated on the use of the mass times the virtual velocity in the five simple machines:

We need not wonder that in common machines, the lever, windlass, pulley, wedge, screw, and the like there exists an equilibrium, since the mass /magnitudo/ of one body is compensated for by the velocity of the other; the nature of the machine here makes the magnitudes of the bodies--assuming that they are of the same kind-- reciprocally proportional to their velocities, so that the same quantity of motion /quantitatem motus/ is produced on either side. For in this special case the quantity of the effect /quantitatem effectus/, or the height risen or fallen will be the same on both sides, no matter to which side of the balance the motion is applied. It is therefore merely accidental here that the force /vis/ can be estimated from the quantity of motion /motus quantitate/. There are other cases, such as the one given earlier, in which they do not coincide.<sup>13</sup>

Leibniz's point is an important one for as will be seen those Cartesians who replied to the "Brief Demonstration" continued to make this very error.

The third point to be noted about the "Brief Demonstration" concerns the measure of force versus the conservation of force. Were it not for the title and the introduction quoted above, one would have no quarrel with Leibniz's presentation. For he correctly shows that the force, defined by him as weight times height, (ws), is to be estimated by the height to which it can raise a body of a given magnitude.

---

in his correspondence, made frequent use of the principle of virtual displacements."

<sup>13</sup>G. M. /27 2, 119.

Quantity of motion mv is not the measure of a force so defined. However the title states that the Cartesians have made an error in asserting that quantity of motion is conserved. Similarly in the first paragraph, it is stated that "it is reasonable that the sum of motive force should be conserved in nature." Both of these statements imply, although Leibniz does not state this as a conclusion, that the "Brief Demonstration" has shown that quantity of motion, mv, is not conserved, whereas motive force, measured by ws is conserved. The only basis for these implications concerning conservation is that the Cartesian quantities of motion of bodies A and B were found to be unequal, while the motive forces, ws, of the two bodies were equal.

Now in order to discuss conservation, a mechanical connection between the two bodies is necessary, whereas to establish the mathematical measure of a force, it is not. If both mv and mv<sup>2</sup> are to be conserved, a two-body interaction would be necessary. To establish conservation of mv<sup>2</sup> but violate mv, a mechanical method of transferring the motive force from body A to body B such as a spring, would be necessary. But in Leibniz's example the bodies fall to the ground side by side and the forces of the two falling bodies are merely compared as to equality. The effect of the ground and the possibility of a mechanical connection or other method of transferring the force are ignored. Thus the implication of the title and introduction that the demon-

stration will yield information about conservation is not justified. The demonstration does successfully establish a mathematical measure of force since this requires only proportionality. Leibniz's implicit identification of measure and conservation is incorrect. He seems to have assumed conservation of motive force on the basis of the impossibility of perpetual motion. This however has nothing to do with conservation of quantity of motion. The confounding of measure and conservation, and the inattention to mechanical connections are a source of confusion in the ensuing controversy with the Cartesians.<sup>14</sup>

In another work of the year 1686, the Discourse on Metaphysics, Leibniz again refers to Descartes' error. In a presentation similar to that of the "Brief Demonstration", the same difficulty between the measure of a force and its conservation may be observed:

---

<sup>14</sup>Ernst Mach in his Science of Mechanics, also pointed out this confusion in Leibniz's thought, although not in great detail: "As to the rest, Leibniz's procedure is more in accordance with the methods of science than Descartes'. Two things however are confounded: the question of the measure /Kraftmaass/ of force and the question of the constancy of the sums mv and mv<sup>2</sup>. The two have in reality nothing to do with each other."

Ernst Mach, The Science of Mechanics 6th ed., LaSalle, 1960, 365.

Thanks are expressed to Professors Erwin Hiebert, Keith Symon and Joan Bromberg for helping to clarify my own thoughts on this subject.

Our new philosophers commonly make use of the famous rule that God always conserves the same quantity of motion in the world. This rule is indeed most plausible, and I have regarded it as beyond doubt. But more recently I have discovered wherein it is in error. This is that Descartes and many other able mathematicians believed that the quantity of motion, that is, the velocity multiplied by the magnitude of the moving body, coincides exactly with the moving force; or, to speak geometrically that the forces are proportional to the product of velocities and masses. Now it is reasonable that the same force should always be conserved in the universe. Also, when we attentively observe the phenomena, it is clear that perpetual mechanical motion cannot occur, because then the force of a machine, which is always diminished a little by friction and must therefore soon come to an end, would restore itself and consequently increase itself without any new impulsion from without....So these mathematicians have thought that what can be said of force can also be said of the quantity of motion.<sup>15</sup>

Leibniz then presents the same proof given in the "Brief Demonstration" and concludes:

So the quantity of motion of the body A at the point D is half the quantity of motion of the body B at the point E; yet their forces are equal. Thus there is a great difference between quantity of motion and force, which was to be proved. We may see from this that force must be estimated by the quantity of the effect which it can produce, for example by the height to which a heavy body of a certain size can be lifted; and this is quite different from the velocity which can be imparted to it.<sup>16</sup>

Here again Leibniz does not actually conclude from his proof that mv is not conserved, but only implies it by his

---

<sup>15</sup>Gottfried Wilhelm Leibniz, "Discours de Metaphysique", in Die Philosophischen Schriften von Gottfried Wilhelm Leibniz, edited C.I. Gerhardt, Berlin, 1875-1890, 4, 427-463; 442. Hereafter referred to as P.S. Loemker, 1, 482-483. Italics mine.

<sup>16</sup>Loemker, 1, 483. Gerhardt, P.S. 4, 442, 443.

initial statements which discuss conservation and which say that Descartes' rule conserving the same quantity of motion is in error.<sup>17</sup> Because he mentions this as if it were an integral part of his argument, we can only believe that he assumed that conservation of force bore a logical relation to the measure of force. He is however correct in his statement that motive force and quantity of motion are different and that the former is to be estimated by ws, granted his presuppositions.

It is in the Discourse on Metaphysics that Leibniz first elaborates on the content of the difference between force and quantity of motion, presenting an argument which is to become the spearhead of his attack on Cartesianism, and the basis of his own philosophy of monadology.

---

<sup>17</sup> Elsewhere Leibniz indicates that he considered his argument a proof that quantity of motion is not conserved. Cf. Leibniz, "Critical Thoughts on the General Part of the Principles of Descartes," (1692) Loemker, 2, 648. "On Article 36. The most famous proposition of the Cartesians is that the same quantity of motion is conserved in things. They give no demonstration of this however, for no one can fail to see the weakness of their argument derived from the constancy of God. For although the constancy of God may be supreme and he may change nothing except in accordance with the laws of the series already laid down, we must still ask what it is, after all, that he has decreed should be conserved in the series--whether the quantity of motion or something else, such as the quantity of force. I have proved that it is rather this latter which is conserved, that this is distinct from the quantity of motion and that it often happens that the quantity of motion changes while the quantity of force remains permanent. The arguments by which I have shown this and defended it against objections may be read elsewhere." Italics added.

Force is something different from size, from form, or from motion and the whole meaning of body is not exhausted in its extension together with its modification. ... Motion, if we regard only its exact and formal meaning, that is, change of place, is not something entirely real, and when several bodies change their places reciprocally, it is not possible to determine by considering the bodies alone to which among them movement or repose is to be attributed.... But the force, or the proximate cause of these changes is something more real, and there are sufficiently grounds for attributing it to one body rather than to another, and it is only through this latter investigation that we can determine to which one the movement must appertain.<sup>18</sup>

What is real in nature for Leibniz is primitive force, or striving, and this is to be developed by him, in the succeeding years, as the essence of the monad. Motion and extension, the essence of nature for Descartes, are to Leibniz merely relations and not realities at all. Thus at the root of his controversy with Descartes lies not a mere mathematical dispute as to the measure of force, but a fundamental disagreement as to the very nature of force itself.

The "Discourse on Metaphysics" also indicates that Leibniz was well aware that Descartes had not taken the direction into consideration in his law of conservation of m<sub>1</sub>v<sub>1</sub>. He writes, "the decree of the divine wisdom to conserve always the same total force and the same total direction has provided for this /i.e. that rules not be accepted con-

---

<sup>18</sup> Leibniz, "Discours de Metaphysique", Loemker, 1, 434. Gerhardt, P.S., 4, 443.

trary to the formation of a system<sup>7</sup>." <sup>19</sup>

The correct statement of the conservation of momentum was known by Leibniz and discussed in his own notes as early as 1669 following its formulation by Wallis, Wren and Huygens. <sup>20</sup> His purpose then cannot be construed as an implied criticism of Descartes for not conserving direction, for he mentions this only in passing and in a separate context. Rather as

---

<sup>19</sup> Leibniz, "Discourse on Metaphysics," Loemker, 1, 487. The full discussion reads: "If there were nothing in bodies but extended mass, and nothing in motion but change of place, and if everything should and could be deduced solely from the definitions by geometric necessity, it would follow, as I have elsewhere shown, that the smallest body in colliding with the greatest body at rest, would impart to it its own velocity, without losing any of this velocity itself; and it would be necessary to accept a number of other such rules which are entirely contrary to the formation of a system. But the decree of the divine wisdom to conserve always the same total force and the same total direction has provided for this."

<sup>20</sup> Gueroult, op. cit., 86. "From 1668 Leibniz had knowledge of the communications of Wallis, Wren and Huygens. He had a résumé translated into latin with a discussion in "De rationibus motus," Manuscripts de Hanovre, Math. 10, ed. Kabitz, 1909, 24-27.

Leibniz gives credit to Wallis, Wren and Huygens in his "Specimen Dynamicum" (1695) Loemker 2, 719. "So far as I know Huygens whose brilliant discoveries have enlightened our age was also the first to arrive at the pure and transparent truth in this matter, /of the falsity of the quantity of motion<sup>7</sup>, and to free this doctrine from fallacies, by formulating certain rules which were published long ago. Almost the same rules were obtained by Wren, Wallis and Mariotte, all excellent men in this field though in differing measure."

According to Loemker, note 101, "It was the laws of motion of Huygens and Wren formulated in response to an invitation of the Royal Society which occasioned Leibniz's own efforts to develop laws of motion in 1669. (See Kabitz, Phil. des jungen Leibniz, pp. 65-68, 135-148.)"



will be borne out by his subsequent writings his objection seems to have been that a conserved quantity of force with a positive unchanging total value for the universe cannot be measured by  $m|v|$ , the quantity of absolute external motion. Leibniz is in reality initiating an ambitious attack on the foundation of Descartes' explanation of the universe as extended matter in motion. This will be amplified in his arguments with the Cartesians, 1686-1691. (See chapter IV.) His own explanation of the world in terms of the nature of force as the substance of reality as expounded in his later papers, 1691-1716 must now be considered.

The first of these papers entitled, "Essay on Dynamics on the Laws of Motion, in which It Is Shown That Not the Same Quantity of Motion Is Preserved, But the Same Absolute Force, or Rather the Same Quantity of Moving Action," was thought by Gerhardt to be written in about the year 1691 at the termination of Leibniz's correspondence with the Cartesian, Denis Papin. However, it was found by Gerhardt among some of Leibniz's unpublished papers and was first published in 1860 in Leibnizen's Mathematische Schriften, edited by Gerhardt.<sup>21</sup>

---

<sup>21</sup> Gottfried Wilhelm Leibniz, "Essay de dynamique sur les loix du mouvement, ou il est montré, qu'il ne se conserve pas la même quantité de mouvement, mais la même force absolue, ou bien la même quantité de l'action motrice," Mathematische Schriften, ed. G. I. Gerhardt, Halle, 1860, 27 2, 215-231. Gerhardt's discussion, ibid., 14. In view of a recent discovery of P. Costabel, the paper probably should not be placed as written before 1692. A translation of the essay appears in the appendix to the New Essays Concerning Human Understanding, translated by A. S. Langley, La Salle, Ill. 1949, 657-670.

Leibniz writes that "after some philosophers, abandoned the opinion that quantity of motion  $\sum miv$  is preserved in the concourse of bodies,<sup>22</sup> they did not recognize the conservation of anything absolute to hold in its place. However, our minds look for such a conservation and many find themselves unable to give up the axiom without finding another to which to ascribe."<sup>23</sup>

In an effort to prove the existence of an absolute quantity which is conserved, Leibniz discusses the conservation of three different quantities in order to establish which one can be called absolute. These are all presented with reference to the impact of two bodies proceeding along a straight path. The first conservation principle is that of the same relative velocity /la même vitesse respective/ between two interacting bodies. This had already been discovered by Huygens.

If one of two bodies is at rest, or if both are in motion and proceed the one against the other, or in the same direction, there is a relative velocity with which they approach or depart the one from the other; and we find that this relative velocity remains the same, so that the bodies depart after the impact with the velocity with which they were approaching before the impact. But this

---

22

Leibniz here indicates the author of the Search after Truth, i.e., Malebranche. The reference is to a supplement to this work entitled "Lois generales de la communication des Mouvements," published 1692.

23

Langley, 658.

relative velocity can remain the same although the true velocities and absolute forces of the bodies change in an infinite number of ways so that this conservation does not concern that which is absolute in bodies.<sup>24</sup>

The second principle is called the conservation of total progress /Quantité du progrès/. Progress is defined as the quantity of motion,  $mv$ , with which a body proceeds in a certain direction. If the body proceeded in a direction contrary to the given one, the progress would be negative. Now if two bodies proceed in the same direction, the total progress is the sum of the progresses of each. However, if they proceed in opposite directions, the progress of one of them will be negative and must therefore be subtracted from the other to obtain the total progress. The conservation of progress of two colliding bodies means therefore that the progress in a given direction is the same before and after impact. This concept is identical with Descartes' conservation of quantity of motion with the sign of the velocity taken into consideration. Of this quantity Leibniz comments,

But it is also plain that this conservation does not correspond to that which is demanded of something absolute. For it may happen that the velocity, quantity of motion and force of bodies being very considerable, their progress is null. This occurs when the two opposed bodies have their quantities of motion equal. In such case, according to the sense given, there is no total progress at all.<sup>25</sup>

---

<sup>24</sup> Leibniz, "Essay on Dynamics", ed. Langley, 658; GM /27 2, 217.

<sup>25</sup> Langley, 658; GM, /27 2, 217.

Proceeding then to develop the conservation of a truly absolute quantity, Force absolue, Leibniz derives by a new method, the conservation of  $mv^2$ , formerly referred to as motive force, ws, which he calls by its subsequently famous name, living force (vis viva) or force vive). Living force is estimated from the violent effect /effect violent/ which it can produce and which entirely consumes the "force of the agent", /force de l'agent/ as occurs when the horizontal motion of a body is converted to a vertical ascent. The mathematical estimate of this effect is ws /le produit de la masse ou de la pesanteur multipliée par la hauteur/.

Only in the case of equilibrium however are the heights proportional to the velocities: "The products of the weights /poids/ by the velocities are as the products of the weights /poids/ by the heights." This special case,

happens only in the case of dead force /Force morte/, or of the infinitely small motion which I am accustomed to call solicitation, which takes place when a heavy body tries to commence movement, and has not yet conceived any impetuosity; and this happens precisely when bodies are in equilibrium, and trying to descend, are mutually hindered. But when a heavy body has made some progress in descending freely and has conceived some impetuosity or living force, /force vive/ then the heights to which this body might attain are not proportional to the velocities, but to the squares of the velocities. And it is for this reason that in the case of living force the forces are not as the quantities of motion or as the products of the masses /masses/ by the velocities.<sup>26</sup>

---

<sup>26</sup> Langley, 660., G.M. /27 2, 218, 219.

Here again Leibniz has referred to the product of the weight and the virtual velocity which applies to statics, and which he calls dead force /vis mortua or force morte<sup>7</sup> and the quantity of motion, mv, as being different from the living force of a body in motion, mv<sup>2</sup>. However here Leibniz does not distinguish the velocity in quantity of motion from the virtual velocity in dead force.

In his "Essay on Dynamics" Leibniz goes on to derive the conservation of mv<sup>2</sup> from a principle called the conservation of moving action /action motrice<sup>7</sup> which says,

there must always be in the concurring bodies between themselves alone, the same quantity of moving action in one and the same interval of time.<sup>27</sup>

Moving action is estimated from a quantity defined as the formal effect /l'effect formel<sup>7</sup>. The formal effect which moving action produces is the mass of the body multiplied by the distance it traverses with uniform motion, i.e., ms. Moving action is then defined as the formal effect times the velocity which produces the action (i.e., msv). Since the motion is uniform, velocity is here defined at s/t. Moving action is therefore ms(s) or m(vt) (v). Now, since the conservation of moving action always occurs in equal times, the times may always be taken as equal to one.

---

<sup>27</sup>Langley, 661.

Conservation of moving action is therefore the conservation of  $mv^2$  or living force.<sup>28</sup> This derivation shows us that living force is conserved in collisions of bodies moving with uniform velocity along a horizontal plane.

Having established these three conservation principles Leibniz gives the mathematical equations representing them and indicates how they may be used together in the solution of impact problems. He writes

I shall reduce the whole to three equations very simple and beautiful, and which contain all that concerns the central concourse of two bodies in one and the same straight line.<sup>29</sup>

The method below of solving impact problems was not generally utilized or appreciated until early in the 19th century. Until then only conservation of momentum was recognized as giving a valid solution. The symbols used in the equations are as follows; the velocity of body a before impact is v, after impact x, the velocity of body b before impact is y, after impact z. These velocities may be positive or negative.

The first equation Leibniz calls a lineal equation [Equation lineale]; it expresses the conservation of relative velocity:

$$\underline{v} - \underline{y} = \underline{z} - \underline{x}$$

---

<sup>28</sup>Langley, 665-66.

<sup>29</sup>Langley, 666.

The second is the plane equation representing conservation of total progress, or in modern terms conservation of momentum:

$$\underline{av} + \underline{by} = \underline{ax} + \underline{bz}$$

The third is the conservation of absolute living force or of moving action:

$$\underline{avv} + \underline{byy} = \underline{axx} + \underline{bzz}$$

This equation always expresses an absolute quantity since the square masks the sign of the velocity, whereas the others depend on relative velocities.<sup>30</sup>

-y and y have the same square, +yy, so that all these different directions of y produce nothing more. And it is also for that reason that this equation gives something, absolute, independent of the relative velocities, or of the progressions from a certain side. The question here concerns only the estimating of masses and velocities, without troubling ourselves from what side these velocities arise. And this it is which satisfies at the same time the rigor of the mathematicians and the wish of the philosophers. - the experiments and reasons drawn from different principles.<sup>31</sup>

Leibniz indicates the use of these equations in the solution of percussion problems and their mutual dependence.

Although I put together these three equations for the sake of beauty and harmony, nevertheless two of them might suffice for our needs. For taking any two of these equations we can infer the remaining one.<sup>32</sup>

---

<sup>30</sup>Langley, 666-668.

<sup>31</sup>Langley, 668.

<sup>32</sup>Langley, 668.

In closing his essay Leibniz shows that he was quite aware that  $mv^2$ , now (as  $\frac{1}{2}mv^2$ ) called kinetic energy, is conserved only in totally elastic impacts.

Now when the parts of the bodies absorb the force of the impact as a whole as when two pieces of rich earth or clay come into collision, or in part as when two wooden balls meet, which are much less elastic than two globes of jasper or tempered steel; when I say some force is absorbed in the parts, it is as good as lost for the absolute force and for the respective velocity, that is to say for the third and the first equation which do not succeed, since that which remains after the impact has become less than what it was before the impact, by reason of a part of the force being turned elsewhere. But the quantity of progress or rather the second equation is not concerned therein... But in the semi-elastics, as two wooden balls, it happens still further that the bodies mutually depart after the impact, although with a weakening of the first equation, following this force of the impact which has not been absorbed... But this loss of the total force, or this failure of the third equation, does not detract from the inviolable truth of the law of the conservation of the same force in the world. For that which is absorbed by the minute parts is not absolutely lost for the universe, although it is lost for the total force of the concurrent bodies.<sup>33</sup>

---

<sup>33</sup>Langley, 670.

For an evaluation of this see Erwin Hiebert, Historical Roots of the Principle of Conservation of Energy, Madison, 1962, 88-90. "In these passages Leibniz apparently postulated an inner force of motion for the invisible smallest parts of bodies. These smallest parts were thought to acquire the kinetic force lost by bodies for inelastic deformable collisions. Leibniz also assumed this inner force to be equivalent to the external force of motion, since he stated that the total force remains unchanged for the universe even for inelastic collisions. There is I believe no statement in Leibniz which would lead one to credit him with either observation or knowledge of the fact that this phenomenon is accompanied by heat changes. Nevertheless by this time it was common belief especially among philosophers that heat was due to or synonymous with the motion of the smallest parts of matter."



Here then is a significant restatement of the problem of the controversy in terms of the search for an estimate of force as a mathematically absolute or positive quantity which can never be taken as zero or negative in the impact of elastic bodies. Further it draws together the conservation principles of momentum,  $mv$ , and living force,  $mv^2$ , presenting the solution of all impact problems as the solution of these two simultaneous equations. Although this paper remained unpublished until 1860, Jean Bernoulli had knowledge of its ideas, expressing the same equations in his paper of 1727 for the French Academy. See this dissertation pp.288-289 ).

An essay of 1692, "Critical Thoughts on the Principles of Descartes"<sup>34</sup> summarizes and reiterates the criticisms of Descartes' 7 laws for colliding bodies showing in detail how they violate the principle of continuity. Descartes' general conservation law and his three secondary laws are likewise subjected to criticism.

Because the points made in this essay relevant to the controversy have already been fully elaborated, further discussion of it will be eliminated.

---

<sup>34</sup>Gottfried Wilhelm Leibniz, "Animadversiones in partem generalem principiorum cartesianorum," in Gerhardt, P. S., 4, 350-392. Loemker, 2, 630-676.

Leibniz's "Specimen Dynamicum" of 1695,<sup>35</sup> presents a mature synthesis of his concept of force, drawing together the observations and opinions expressed since 1686 in his papers on dynamics and incorporating philosophical views developed concurrently with this work in physics. It gives an interpretation of Leibniz's "force" as the very foundation of our understanding of both the physical and spiritual universe.

What is real in the universe is activity; the essence of substance is action, not extension as Descartes had insisted. This activity is constituted by a primitive force or a striving toward change; it is the innermost nature of a body. Leibniz was later to call the basic indivisible substances whose essence is a continual tendency toward action, monads (cf. "The Monadology," 1714). But force has a passive as well as an active character, and this too is a characteristic of the monad. In the monad passivity is reflected by the way in which the

---

<sup>35</sup>Gottfried Wilhelm Leibniz, "Specimen Dynamicum pro admirandis naturae legibus circa corporum vires et mutuas actiones detegendis et ad suas causas revocandis," Gerhardt, M.S. / 27 2, 234-246, Part I. A translation appears in Loemker, *op. cit.*, 2, 711-738. The original part I appeared in the *Acta Eruditorum* (1695). A similar but later essay written in 1702 will be found in Gerhardt, *Leibniz, Math. Schrift.*, / 27 2, 98-106; Gerhardt, *Leibniz Philos. Schrift.*, 4, 393-400. A translation is found in the appendix to the *New Essays*, ed. Langley, *op. cit.*, 699-706.

changes in one monad are adapted to the changes in all the others. In corporeal bodies this passivity is a primitive force of "resisting" or "laziness" which shows itself as the inertia or "repugnance of matter to be set in motion."<sup>36</sup>

While the reciprocity of activity and passivity in primitive substance is entirely metaphysical, it shows itself quantitatively in the phenomenal world as conatus, impetus and vis viva. Each of these three mathematical measures of primitive force is composed of two aspects, the passive inertial aspect provided by matter or mass, and the active dynamic aspect, represented by motion or the tendency toward motion. A moving body has a velocity at any moment of time. This velocity together with direction is called conatus.<sup>37</sup> Impetus is the product of the mass [molis]<sup>7</sup> of the body and its velocity. The Cartesians call this the quantity of motion. More accurately however "quantity of motion is the integral of the impetuses in the moving body through a given interval of time."<sup>38</sup>

---

<sup>36</sup>Loemker, 2, 714, 715. GM /27, 2, 236, 237.

<sup>37</sup>"Velocitas sumta cum directione conatus appellatur." G.M. /27 2, 237.

<sup>38</sup>"Impetus autem est factum ex mole corporis in velocitatem, ejusque adeo quantitas est, quod Cartesii appellare solunt quantitatem motus, scilicet momentaneum, tametsi accuratius loquendo ipsius motus, quippe in tempore existentis, quantitas ex aggregato impetuum durante tempore in mobili existentium (aequalium inaequaliumve) in tempus

There are two kinds of force: dead force (vis mortua or force morte) and living force (vis viva or force vive). Dead force is the force of "statics", a pressure or tension. It is a solicitation or striving toward motion existing before motion actually arises. It occurs when there is a potential for motion but something hinders its accomplishment. As examples Leibniz gives a stone held in a sling, bodies in equilibrium, and a body prevented from falling by some obstacle.

---

ordinatum ductorum nascatur." G.M. 27 2, 237.

"Motion is the continuous change of place and thus requires time. But as the moving body has its motion in time so it has a velocity at every moment of time, a velocity which is the greater in the degree that more space is passed through in less expenditure of time. This velocity along with direction is called conatus. Impetus, however, consists in the product of the mass /molis/ of the body by its velocity, and so its quantity is that which the Cartesians usually call the quantity of motion, that is, the momentaneous quantity, although speaking more accurately, the quantity of motion, having an existence in time, is an integral of the impetuses (whether equal or unequal) existing in the moving body through an interval of time. In our debate with the Cartesians, however, we have followed their way of speaking." Loemker, 2, 715.

In an explanatory footnote to this section Loemker writes: "Every body has a velocity at a particular moment of time:  $\underline{v} = (ds/dt)$ . The product of the mass by this velocity is here  $\underline{mv} = (mds/dt)$ , while the quantity of motion over a period of time would be the integral:

$$\underline{m} \int_0^t \underline{v} dt = \underline{m} \int_0^t \frac{ds}{dt} dt = \underline{ms} \text{ or } \underline{mv}^2$$

But since distance is proportional to  $\underline{v}^2$ , this is Leibniz's own quantity of force." Loemker, 2, n. 35.

Living force arises from an infinite number of continuous impressions of dead force. It is the measure of the force of a body which has been falling for some time. Although Leibniz did not use the term work, nor did he make explicit the work-energy relation which lay at the basis of his concept of living force, we can interpret his concept of force in modern terminology. We can generalize his relationship between the weight of a body falling through a distance,  $mg s$ , and living force,  $\frac{mv^2}{2}$ , to the work-energy relation:

$$\frac{mv^2}{2} = F \cdot ds$$

That is, living force, or more accurately, one half its value, kinetic energy, and work, or force acting through a distance are equivalent. Differentiating this equation with respect to time for a constant force and constant mass, and substituting the relation  $v = ds/dt$ , we obtain the equation for static or dead force:

$$\frac{2mvdv}{2 dt} = F \cdot \frac{ds}{dt}$$

$$\frac{m ds}{dt} \cdot \frac{dv}{dt} = F \cdot \frac{ds}{dt}$$

$$\frac{mdv}{dt} = F \text{ i.e. Dead Force}$$

For the case of gravity as dead force, we would have  $F = mg$ , or weight. If the distance of fall is taken into account,

ings, we would have the potential energy concept which lay at the basis of Leibniz's "Brief Demonstration." For the case of equilibrium, as in the lever, the force on one arm balances the force on the other:  $m_1 \frac{dv_1}{dt_1} = m_2 \frac{dv_2}{dt_2}$ ,

and  $\frac{dv}{dt} = \frac{ds}{dt}$ , the virtual displacement. Here the times are equal, i.e.  $\frac{dt_1}{dt_2} = 1$ , so  $\frac{ds}{dt} = \frac{dv}{dt}$ . Thus  $m_1 \frac{dv_1}{dt_1} = m_2 \frac{dv_2}{dt_2} = m_1 \frac{ds_1}{dt_1} = m_2 \frac{ds_2}{dt_2}$ . Since dead force is proportional to the body's

mass and the /differential or virtual/ velocity /quantitate ex ductu molis in velocitatem facta/ Leibniz again attributes the Cartesians' error to their generalization of this static measure to force in general /vim in universum/. Leibniz writes:

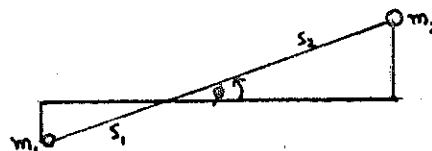
So far as we know, the ancients had a knowledge of dead force only, and it is this which is commonly called mechanics, which deals with the lever, the pulley, the inclined plane (applicable to the wedge and screw), the equilibrium of liquids, and similar matters concerned only with the primary conatus of bodies in itself, before they take on an impetus through action. Although the laws of dead force can be carried over, in a certain way, to living force, yet great caution is necessary, for it is at this point that those who confused force in general with the quantity resulting from the product of mass by velocity were misled because they saw that dead force is proportional to factors. As we pointed out long ago this happens for a special reason, namely that when, for example different heavy bodies fall, the descent itself or the quantities of space passed through in the descent are at the very beginning of motion while they remain infinitely small or elementary, proportional to the velocities or to the conatuses of descent.

But when some progress has been made and living force has developed the acquired velocities are no longer proportional to the spaces already passed through in the descent but only to their elements.<sup>39</sup>

In "Specimen Dynamicum" Leibniz again attempts refutation of the Cartesian principle of force. No reference is made here to conservation, but rather the demonstration is given in terms of two principles, the principle that the "whole effect is always equal to the full cause,"<sup>40</sup>

<sup>39</sup> Leibniz, "Specimen Dynamicum," Loemker, 2, 717, 718, GM /27 2, 239, italics added. On the background of "dead force" and its meaning, see Taliaferro, op. cit., 26. "The origin of Descartes' error, Leibniz claims lies in his generalizing of the law of the lever by differential displacements (or as one would say later, virtual displacements or velocities):

$$m_1 v_1 = m_2 v_2, \quad m_1 s_1 \sin \phi = m_2 s_2 \sin \phi$$



to actual speeds. This is a false leap; these virtual speeds are what Leibniz calls dead forces as opposed to the live forces which produce, not the tendency to motion but the actual motion, and which are the integral of these:

$$\int m v dv = \frac{1}{2} m v^2 + C, \text{ the } \frac{1}{2} \text{ usually being omitted.}^2$$

According to Erwin Hiebert, op. cit., 49, Descartes knew that it was the "commencement of movement which has to be taken into account at each instant. He says 'notez que ie dis commencer à descendre, non pas simplement descendre'. Here Descartes has emphasized the importance of stating the relationship in terms of infinitesimal virtual displacements, for he recognized that the force to sustain a weight on the arm of the lever, exerted in the direction perpendicular to the lever, varies continuously with changes in position of the lever, and so the ratio of displacements will also vary continuously. See Descartes, Oeuvres, 2, 222-246."

<sup>40</sup> Loemker 2, 726.

and the principle that equipollent bodies may be substituted for each other in our calculations as if the substitution had actually been carried out in nature.<sup>41</sup>

The example, drawn from the mechanics of the pendulum, envisions two pendula both of the same length but with bodies of different weight. Body A has a magnitude of 2 and falls through a vertical height (A<sub>2</sub>H) of 1 ft. (See diagram, p. 100). When it reaches the perpendicular, A<sub>1</sub> it has acquired a velocity of 1. At this level A<sub>1</sub> a second body C, is substituted for body A which the Cartesians would say is equipollent to body A because it has a magnitude 1 and a velocity of 2. But the supposed equipollent, C, with velocity 2 will rise to a height of 4. "Thus", writes Leibniz, "merely by the descent of a 2 pound weight, A, from the height of 1 foot A<sub>2</sub>H, we have, by substituting its supposed equipollential, brought about a rise of 1 pound to 4 feet which is double the former effect. Therefore we have gained this much force or achieved a perpetual mechanical motion, which is absurd."<sup>42</sup>

The question raised by this thought experiment is, how can Leibniz show that perpetual motion will arise from the acceptance of momentum as a measure of "force"?

---

<sup>41</sup>Loemker 2, 726, 729.

<sup>42</sup>Loemker, 2, 727.



Since perpetual motion cannot occur in nature and since mv is a legitimate concept, where is the difficulty in Leibniz's proof?

If the weight of a body multiplied by its distance of fall is taken to be the measure of force and as the basis for equipollence, there is no reason to expect the equipollent bodies to have equal mv's and well as mv<sup>2</sup>.

The impossibility of perpetual motion is another way of stating the conservation of kinetic energy since a body cannot rise to a greater height than that from which it fell without the aid of an external force. This however is not the basis for the conservation of momentum law which applies to systems of interacting bodies. There is no reason to expect perpetual motion to arise from the substitution of a body of a certain mv, for perpetual motion and momentum have nothing to do with one another. Leibniz is confusing the issue by applying the perpetual motion principle to the question of the validity of quantity of motion.

Leibniz's ideas on force together with his critique of Descartes discussed above, lead directly into his well-known concept of the monad. The monad is a simple substance which can neither be created nor destroyed naturally. All monads must begin simultaneously and be annihilated at once. A monad is a being capable of action whose changes in state occur from an internal principle. This prin-

ciple of change is a striving or conatus toward a future state, a primitive force.<sup>43</sup> Thus the conservation of substance and force form the very basis of Leibniz's philosophical viewpoint.

Every monad changes continuously and spontaneously following a certain pre-established order in such a way that God sees its previous and future states in its present one. The changes in one monad according to the pre-established order are in exact agreement and harmony with those occurring in the lives of every other monad throughout the universe and eternity. Certain stages or states in the lives of all monads are said to exist simultaneously. The order present in all co-existing things is Leibniz's definition of space. Space therefore is a relation between simultaneous states of real things, that is, between monads. These simultaneous states determine the phenomena apparent to us. Time is an order in the existence of states which are not simultaneous but prior of posterior. Space and time are therefore neither realities nor substances, but merely relations. Duration is the magnitude of time; extension is the magnitude of space. Thus motion which is the continuous change in both space and time is likewise only a relation. What is real in motion is force, a momentary state which carries

---

<sup>43</sup> Leibniz, "The Monadology," (1714) Loemker, 1044-1046, and "The Principles of Nature and of Grace," (1714) Ibid., 1033, 1034. Gerhardt, P.S., 6, 607-623 and 598-606.

with it a striving toward a future state. When these momentary states are considered over a period of time and over a portion of space, a relation or order among the states is perceived and this is called motion.<sup>44</sup>

---

<sup>44</sup>Loemker, 2, 806. In these discussions on physics, Leibniz conformed the language of his philosophical system to that of ordinary speaking. Thus all these points on the level of physics have a counterpart in Leibniz's system of monads or souls in which there is no real space or motion, and in which there is no real communication of motion. For each case of impact in the world of phenomena there is a counterpart in the real world of monads which consists in the heightening and diminution of the states of perception of infinite numbers of monads. All of this takes place in accordance with the system of pre-established harmony. For Leibniz's discussion of this problem see, The Monadology and The Correspondance With Arnauld, op. cit. Thus in the Correspondance with Arnauld Leibniz writes: (p. 153) "Thus the souls change nothing in the ordering of the body nor do the bodies effect changes in the ordering of souls (and it is for this reason that forms should not be employed to explain the phenomena of nature). One soul changes nothing in the sequence of thought of another soul, and in general one particular substance has no physical influence upon another; such influence would besides be useless since each substance is a complete being which suffices of itself to determine by virtue of its own nature all that must happen to it." (p. 163): "...all the phenomena of the body can be explained mechanically or by the corpuscular philosophy in accordance with certain assumed mechanical principles without troubling oneself whether there are souls or not. In the ultimate analysis of the principles of physics and mechanics however, it is found that these assumed principles cannot be explained solely by the modifications of extension, and the very nature of force calls for something else. (p. 182) "Nevertheless, we have the right to say that one body pushes another; that is to say, that one body never begins to have a certain tendency excepting when another which touches it loses proportionally, according to the constant laws which we observe in phenomena; and since movements are rather real phenomena than beings, a movement as a phenomenon is in my mind the immediate consequence of effect of another phenomenon, and the same is true in the mind of others. The condition of one substance, however is not the immediate consequence of the condition of another particular substance."

It is clear therefore why motion and extension cannot be the essence of reality for Leibniz as they were for Descartes.

We are thus able to view Leibniz's attack on the Cartesian measure of force as primarily an attempt to establish his own philosophical system based on the conservation of force, and to place less emphasis on a simple attempt to substitute the mathematical formula  $mv^2$  for the formula  $mv$ . This latter aim is encompassed in the more general purpose of the former. Perhaps his insight into a universe which was fundamentally energistic led him to make assumptions about the possibility of transferring that energy and to implicitly identify the conservation and the measure of force, the establishment of both being an integral part of his ultimate aim.

### Conclusion

In this chapter some reasons have been suggested for the initiation of the controversy over what quantity was to be the measure of force. First Leibniz showed that the "force" of a body in motion could be measured by the quantity  $mv^2$  (vis viva) which was dependent upon the distance through which a body is moved. But in so doing he claimed that the quantity  $mv$  was not a legitimate measure of force. However momentum,  $mv$ , is actually a different

quantity, its measure being dependent on the time during which the body moves.

In making his claim Leibniz tried to show that  $\underline{mv}$  was not conserved. This argument was not legitimate because in some cases the bodies did not interact and hence the question of conservation could not even arise. He also tried to show that perpetual motion would occur if  $\underline{mv}$  were the measure of force. However the perpetual motion argument applies only to vis viva conservation and not to momentum cases.

Leibniz also claimed that the Cartesians confused the measure,  $\underline{mdv}$ , in statics with the measure  $\underline{mv}$ , a concept applicable in dynamics, calling both  $\underline{mv}$ . Although Descartes did not make this error, his followers actually did as will be shown in chapters IV and VI. Leibniz shrewdly attacked the error and at the same time showed how  $\underline{mdv}$  or dead force is related to its integral, living force,  $\underline{mv}^2$ ,  $\int \underline{mdv} = \frac{1}{2}\underline{mv}^2 + C/$ .

In solving impact problems Leibniz used the simultaneous application of the equations, conservation of  $\underline{mv}^2$  and conservation of  $\underline{mv}$ . He called this latter equation conservation of total progress which was equivalent to Descartes' quantity of motion with the velocity taken as a vector. Leibniz thought of  $m(v)$  as being in error because  $\underline{mv}^2$  was the true measure of force. He did not think of himself as correcting  $m(v)$  to its accurate expression  $\underline{m\vec{v}}$ .

Behind his challenge to Descartes and his followers lay a deep-seated philosophical conviction that force and not motion was conserved in the universe. Motion is only relative whereas force is the dynamic principle behind the simultaneous unfolding of the lives of all the monads. The observed motions of bodies are merely phenomena reflecting pre-established changes in the ordering of monads.