CHAPTER IV

The Defense of the Cartesian Hypothesis: Catalan, Arnauld, Malebranche, Papin, (1687-1691)

Leibniz's "Brief Demonstration" in French translation appeared in September 1686 in the _Nouvelles de la republique des lettres._[1] This paper was Leibniz's first serious appearance before the French literary public where Cartesianism had its most devoted adherents. His reputation in Paris heretofore had been as a scientist and a mathematician, established in part by his invention of the calculating machine. His attack on the Cartesian law of motion, made on the authority of his scientific reputation, was the beginning of his general refutation of Descartes' philosophy.[2]

The paper was immediately answered by the Cartesian,

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[1] Gottfried Wilhelm Leibniz, "Demonstration courte d'une erreur considérable de M. Descartes et de quelques autres touchant une loi de la nature selon laquelle ils soutiennent que Dieu conserve toujours dans la matiére la même quantité de mouvement, de quoi ils abusent même dans la méchanique" _Nouvelles de la republique des lettres_, _8_ (1686) 996-999.

[2] W. H. Barber, _Leibniz in France from Arnauld to Voltaire, A Study of French Reactions to Leibnizianism, 1670-1760_, Oxford, 1955, 32-34.

Abbé Catalan in a "Courte Remarque de M. l'Abbé D. C. où l'on montre à M.G.G. Leibnits le paralogisme contenu dans l'objection précédente". It appeared in the same issue, September, 1686.[3]

Catalan takes issue with Leibniz's claim that Descartes was lead astray by too great a faith in his own mind and his followers by too great a faith in the genius of others. He calls on the scientist to decide whether it is Descartes or Leibniz who is imbued with this usual fault of great men.

He interprets Leibniz as making the following claims: (1) Descartes asserts that God conserves the same quantity of motion, (2) Descartes takes as equivalent moving force [<u>force motrice</u>] and quantity of motion, [<u>quantité du mouvement</u>] (3) Several mathematicians have <u>estimated</u> the force of motion [<u>force mouvante</u>] by the quantity of motion, i.e. the product of the body [<u>corps</u>] by its velocity; but these two are different and Descartes' rule is false that the same quantity of motion is <u>conserved</u> in nature.[4]

It has been shown, writes Catalan, that two moving

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[3] Abbé Catalan, "Courte Remarque," <u>Nouvelles de la republique</u>, <u>8</u> (1686) 1000-1005.

[4] Catalan, "Courte Remarque", 1000-1001. This third point indicates that at least one contemporary interpreted Leibniz as inter-relating the two problems of the measure and the conservation of force. Catalan perceived their independence no more than did Leibniz.

bodies [*mobiles*] which are unequal in volume [*volume*] eg. 1 to 4, but equal in quantity of motion i.e. 4, have velocities proportional to the reciprocal ratio of their masses [*masses*] i.e. 4 to 1. Consequently they traverse [*parcourent*] in the same time spaces proportional to these velocities.

However Galileo showed that the spaces described by falling bodies are in the ratio of the squares of the times [$s = \frac{1}{2}at^2$]. Thus in the example of Leibniz in the "Brief Demonstration", the body of 1 pound [*livre*] rises to height 4 in time 2 and the body of 4 pounds rises to height 1 in time 1. If therefore the times are unequal, it is not surprising to find the quantities of motion unequal.

But, says Catalan, if the times are made equal by suspending them from the same balance at distances reciprocal to their bulk [*grosseur*], the quantities formed by the products of their masses [*masses*] and distances, or masses and velocities are equal.[5]

Catalan here has lumped together three separate problems as one; uniform traversal of space (momentum), free fall (*vis viva*) and the problem of the lever (virtual velocities). In the free fall problem, if the times were equal, the *mv*'s would be equal only for bodies of equal weight. If the times for unequal bodies are made equal by use of a lever,

[5] *Ibid.*, 1002, 1003.

the problem has been changed to a problem in statics where virtual work ($m \cdot ds$) or static force ($\frac{mdv}{dt}$) describes the situation. This is not the same as quantity of motion, mv. It shows that Leibniz was right in supposing, as he stated in his "Brief Demonstration" that certain Cartesians confused the measure of "force" in statics [mdv] with the measure of "force" in dynamics (mv according to Descartes, and mv^2 according to Leibniz).

During this same year, 1686, Leibniz continually sharpened his philosophical ideas as well as his arguments against Descartes' quantity of motion in his private correspondance with Antoine Arnauld. In a letter of September 28, 1686, Arnauld wrote to Leibniz that he had seen the article on Descartes' notable error and that some objections which might be raised by the Cartesians had occurred to him. He warned,

> Take care lest the Cartesians should reply that it brings nothing up against their position, because you posit something that they think is false--namely that a stone in descending, gives to its own self this greater velocity which it acquires as it descends. They will say that this acceleration comes from the corpuscles, which, in rising, cause everything that they find in their way to descend and impart to them a part of the motion which they had; and therefore there is no cause for surprise if the body B, four times the weight of A, has more motion when it has fallen one foot, than the body A when it has fallen four feet, because the corpuscles which have pressed on B have communicated to it a motion proportional to its mass and those which have pressed upon A, in proportion to its mass.[6]

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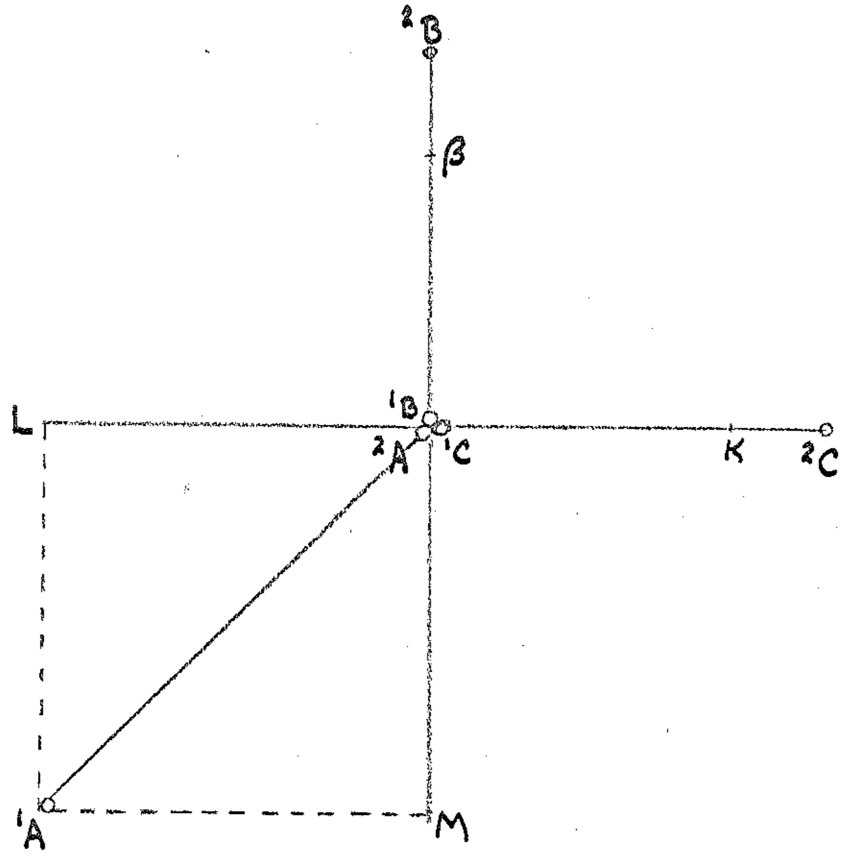
[6] Antoine Arnauld, "Correspondance with Arnauld", in Leibniz, Discourse on Metaphysics, Correspondance with Arnauld, Monadology, translated by George R. Montgomery LaSalle, 1957, 148. Gerhardt, P.S., 2, 67, Sept. 28.

Leibniz, in his reply to Arnauld, November 28-December 8, 1686, gives a second example to show that in distributing the motion between two bodies which come into <u>contact</u>, regard must be had not to the quantity of motion, ... but to the quantity of force; otherwise we should obtain perpetual motion in mechanics".[7] A horizontal two dimensional coordinate system is given in which two spheres of equal size, <u>B</u> and <u>C</u>, at the origin, are struck by a third sphere, <u>A</u>, of the same size traveling along the diagonal of the square formed by one quadrant, such that at the moment of contact the centers of the 3 spheres are found in an isoceles right triangle. (See diagram, p. 112).

Suppose, says Leibniz, that all the force of body <u>A</u> is <u>imparted</u> to bodies <u>B</u> and <u>C</u>, <u>A</u> remaining at rest after the contact. If <u>A</u> traversed the diagonal in one second, before the contact, then in one second after the contact, B will be found at <u>2B</u>, and <u>C</u> at <u>2C</u>, the question being what will be the lengths <u>1B2B</u> and <u>1C2C</u> representing the velocities? Now according to Leibniz, forces are proportional to the heights from which bodies would have to descend in order to acquire their velocities and these heights are as the squares of the velocities. The sum of the squares

[7] Leibniz, "Correspondance with Arnauld," <u>op, cit</u>., 164. Gerhardt, P.S. 2, 79, 28 Nov - 8 Dec. Italics added.



From Gerhardt, P.S., II, 79, 28 Nov-8 Dec. Leibniz's diagram.

of the sides (i.e. the velocities after collision) 1B2B and 1C2C equals the square on the hypoteneuse, i.e. the velocity 1A2A before contact. Thus there is as much "force" after as before the contact.

Using the Cartesian measure, the quantity of motion is estimated by the simple velocities. Before the contact the velocity is 1A2A, but after the contact it is the sum of the velocities 1B2B and 1C2C. But 1B2B plus 1C2C is greater than 1A2A. Thus if quantity of motion is the measure of force, the total value would have increased after the contact. If on the other hand the whole situation is reversed such that bodies B and C together impart all their motion to A at rest, then if the total quantity of motion is to remain equal, the total force, ws, would be augmented, and perpetual motion could be made to result. Although in this problem the quantities of motion are unequal before and after the contact, or as in the second part the Leibnizian force seems to have increased, this does not mean that perpetual motion would result. The impossibility of perpetual motion principle cannot be used to demonstrate the falsity of mv.

At the close of his reply to Arnauld, Leibniz outlines his answer to the objection of Abbé Catalan which he puts in print early the next year.

> I find in The News of the Republic of Letters for the month of September, of this year, that someone named Abbé D. C. of Paris, whom I do not know, has replied to my objection. The trouble is that he seems not sufficiently to have thought over the difficulty. While pretending to contradict me vehemently he grants me more than I wish and he limits the Cartesian principle to the single case of isochronous powers as he calls them, as in the five usual forms of machinery, and this is entirely against Descartes' intention. Besides this, he thinks that the reason why, in the case which I proposed, one of the bodies has quite as much force as the other, although it has a smaller quantity of motion, is the result of this body's having fallen for a longer period of time since it has come from a greater height. If this made any difference the Cartesian principle which he wishes to defend would be ruined by that very fact. This reason however is not valid, for the two bodies can descend from those different heights in the same time, according to the inclination which is given to the planes along which they descend; and my objection would still be entirely valid. I hope therefore that my objection may be examined by a Cartesian who shall be a Geometer and well versed in these matters.[8]

Here Leibniz points out the confusion between the mass and velocity relationship in the five simple machines and in the quantity of motion. Catalan, he says, tries to refute him by citing only the case of isochronous powers, thereby limiting the Cartesian principle to statics. This however says Leibniz is not Descartes' intention. (There is no evidence that Descartes himself confused these two principles by trying to extend the principle applying to statics into dynamics.) Leibniz adds here that the times

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[8] Leibniz, "Correspondance with Arnauld", op. cit., 168. Gerhardt, P.S., 4, 80, 81.

of fall are not crucial because these times can be altered by use of inclined planes. With respect to Leibniz's own *vis viva* principle this is true.

By the following February, 1687, Leibniz had issued a reply to Catalan in the *Nouvelles de la republique*: "Replique de M. L. à M. l'Abbé D. C. contenuë dans une lettre écrite à l'Auteur de ces Nouvelles le 9. de Janv. 1687, touchant ce qu'a dit M. Descartes que Dieu conserve toujours dans la nature la même quantité de mouvement."[9] In this reply Leibniz answers the objection of Abbé Catalan that since the two falling bodies acquire their forces in unequal times, the forces ought to be different.

If the force of a body of 4 pounds having a velocity of 1 degree is transferred [*transferer*] to a body of 1 pound, according to the Cartesians the second will receive a velocity of 4 degrees to preserve [*garder*] the same quantity of motion. But, argues Leibniz, this second body should only receive a velocity of 2. And in estimating the forces that the bodies have acquired no-one (except the Abbé D.C.) will measure whether they have acquired these forces in times long or short, equal or unequal. Time has nothing to do with the measure of force, [i.e. *vis viva*]. One can judge the present state without knowing the past. If there are

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[9]Leibniz, "Replique à M. l'Abbé D.C." in *Nouvelles de la republique des lettres*, 9 (1687) 131-144.

two perfectly equal and identical bodies having the same velocity, but that of the first is acquired in a collision, that of the second in a descent, can their forces be said to be different? This would be like saying a man is wealthier for taking more time to earn his money.[10]

Furthermore one can change at will the time of descent by changing the line of inclination of the descent, and in an infinite number of ways, two bodies can be made to descend from different heights in equal times. But a body descending from a certain height acquires the same velocity whether that descent is perpendicular and faster, or inclined and slower. Thus the distinction of time has nothing to do with the argument.[11]

Leibniz then goes on to prove that in the example given above, the velocity that the second body received from the transfer of force is 2, rather than 4 as the Cartesians would have maintained. The horizontal motions of the bodies can be converted to ascents by use of threads as in a pendulum. The weights /*poids*/ are in the reciprocal ratio of the heights to which they are able to rise in virtue of the velocities which they have, and these heights are proportional to the squares of the velocities. If the body

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[10] *Ibid.*, 133.

[11] *Ibid.*, 134.

receives a velocity of 2 it can rise by means of a pendulum or thread to a height of 4, but if it receives a velocity of 4, following Descartes, then it will rise to a height of 16 feet. In the latter case the effect (16 ft. times 1 lb.) would be quadruple the cause (4 lb. times 1 ft.). One will be able to establish as a law of nature, Leibniz adds, that "there is a perfect equation between the full cause and the complete effect."[12] This goes beyond saying that effects are merely proportional to their causes. For example, says Leibniz, consider Descartes' third rule of motion, and suppose that two bodies, B and C, each one pound, move toward each other, B with a velocity of 100 degrees and C with a velocity of one degree. Together their quantity of motion will be 101. Now C with its velocity of 1 can rise to 1 foot while B can rise to 10,000 feet. Thus the force of the two together will be able to elevate 1 pound to 10,001 feet. According to Descartes' rule after

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[12] Leibniz's "Replique", op. cit., 137. "C'est pourquoi je croi qu' au lieu du principe Cartesian on pourroit établir une autre loi de la nature que je tiens la plus universelle et la plus inviolable, savoir, qu'il y a toujours une parfaite equation entre la cause pleine et l'effet entier. Elle ne dit pas suelement que les effets sont proportionels aux causes, mais de plus que chaque effet entier est equivalent à sa cause... Mais pour faire mieux voir comment il s'en faut servir et pourquoi Descartes et d'autres s'en sont éloignez, considerons sa troisieme regle du mouvement pour servir d' exemple..." Italics Leibniz's.

the impact (<u>choc</u>) both move together with a speed of 50 1/2.[13] Multiplying this speed by two, the weight of both bodies together, the quantity of motion 101, is retained. However in this case the two pounds together can only be raised to the height of 2550 1/4 feet or equivalently, one pound can be raised to 5100 1/2 feet. Thus almost half the force is lost without any reason and without being used elsewhere.[14]

The reason that the Cartesian measure of force, <u>mv</u>, can be proven false in this case where a collision not a mere proportionality, actually is involved, is that Descartes' third rule itself is in error. Thus although the argument appears to be similar to the preceding ones, it is in reality quite different. Here Leibniz is initiating a new line of argument on which he relies in subsequent papers; that argument is that if the rules for colliding

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[13]Descartes' third rule states: "If /hard body7 <u>B</u> and /hard body7 <u>C</u> are equal in heaviness, but <u>B</u> moves with slightly greater speed than <u>C</u>, not only do both move to the left afterwards, but <u>B</u> also imparts to <u>C</u> half the difference of their initial speeds." See Appendix I to Chapter I.

[14]If the correct rule is used in which the velocity is taken as a vector and the bodies stick together, then the final speed is $49\frac{1}{2}$. ($mv + MV = 1(100) + 1(-1) = 99 = (m + M)v_f$; $2v_f = 99$; $v_f = 49\frac{1}{2}$).

bodies are shown to be false, then the principle upon which they are based, conservation of quantity of motion, must also be false.[15] By compounding Descartes' errors concerning quantity of motion, Leibniz is seeking to strengthen his own position.

This reply to Catalan in the Nouvelles de la republique concluded with Leibniz's criticism of the Recherche de la verité by Nicolas Malebranche (first published in 1674).

According to Leibniz, Malebranche had presupposed the truth of Descartes' maxim concerning the quantity of motion and believed that of the 7 rules of impact, 1, 2, 3, and 5 were correct. (Leibniz states that only rule 1 is correct.) Furthermore Malebranche argued that in these impacts involving hard bodies without elasticity there is rebound or separation after the collision only when they collide from contrary directions with velocities reciprocal to their magnitudes (grandeurs). In all other cases they are joined together after the collision retaining their original quantities of motion.[16]

To demonstrate the difficulty Leibniz proposes this example: If body B, magnitude 2, velocity 1, and body C,

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[15] Leibniz, "Replique", op. cit., 138. "Ce qui est aussi peu possible que ce que nous avons montré auparavant dans un autre cas, où en vertu du même principe Cartesien general on pourroit gagner le triple de la force dans aucune raison."

[16] Ibid., 139.

magnitude 1, velocity 2, collide from opposite directions, they will separate with their original velocities. But if one supposes the velocity or size of one body, say B, to be slightly augmented, so slightly that the former numbers could be retained, they should move together with a velocity of 4. B which arrived at the point of collision with a velocity of 1, now not only does not rebound, but continues forward with a velocity of 4. But it is impossible that for such a small change in the former conditions there results such a vast difference in the outcome: all tendency toward separation ceases, and B goes forward with a velocity of 3 units more than it had initially.

In a similar manner by pointing out the inequality between cause and effect, both Descartes' sixth rule and Malebranche's revision of it are criticized by Leibniz.

In closing his letter, Leibniz makes a point fundamental to his philosophy and around which his critique of Cartesianism is focused. There is a distinction between force and direction; or rather, he says, between the absolute force necessary to make an effect such as raising a given weight to a given height or compressing an elastic body, and the force of continued advancement in a given direction. A body of mass 2 and velocity 1 and a body of mass 1 and velocity 2, can mutually hinder each other from advancing, but the first can elevate a pound to only 2 feet, while the second can elevate a pound to 4 feet. To resolve the paradox it is necessary to admit in the body something other than extension and velocity, unless one wishes to refuse to the body all power to act. It is this power to act which is, according to Leibniz, rightly to be termed force.

April of 1687 saw in the same journal, the publication of a short extract of a letter of Malebranche to Abbé Catalan.[17] Here Malebranche admits that what Leibniz criticized about his treatment of the laws of motion in the Recherche de la verité appears to be accurate. He attempts to explain the cause of the incongruities while still retaining his conviction that God conserves an equal quantity of motion in the universe. The paradox, he claims, results from the fact that in nature we do not find bodies which are absolutely hard; hardness being possible only by the compression of subtle matter and not at all by the absolute rest of the body's parts as Descartes had claimed.[18]

In answer to Leibniz's difficulty concerning the problem of body B (magnitude 2, velocity 1,) and body C (magnitude 1, velocity 2,) Malebranche proposes that the results of collisions are arbitrary and depend on the volition of God. If the two bodies approaching each other,

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[17] Nicholas Malebranche, "Extrait d'une Lettre du P.M. a M. l'Abbé D. C." Nouvelles de la republique des lettres, 9 (1687) 448-450.

[18] Descartes, Principia Philosophiae, Pt. II, principles 54,55.

are joined together in the collision and finally proceed in the direction of B with a common velocity of 4, this has a very simple explanation. God causes a reciprocal permutation of velocities at the instant of collision, so that B would have a velocity of 2 and C a velocity of 1. Then they would proceed naturally in the direction of B, with a common velocity of 4, the apparent contradiction being resolved. By this permutation a weaker body can change the determination of a stronger body. It is experience which bears witness to the way in which the Creator acts. And this reciprocal permutation, claims Malebranche, is so simple that it is not surprising to find that what at first appeared unbelievable should actually happen.

In his reply to Malebranche,[19] July 1687, Leibniz points out that the incongruities between Descartes laws are too great to be explained either by the hypothesis that absolutely hard bodies are not found in nature or by the volition of God. Regarding the first explanation, perfect hardness may be conceived as infinitely prompt elasticity and this may be treated as a case of the true laws governing elastic bodies.

As to the second reason, Leibniz argues that the

[19]Leibniz, "Extrait d'une lettre de M. L. sur un principe général, utile à l'explication des loix de la nature, par la consideration de la sagesse divine; pour servir de repliqueàla reponse du R.P.M." Nouvelles de la Republique des lettres 10 (1687) 745-754. See also Loemker, 1, 538-543.

more one studies nature, the more geometrical it is found to be, and that a case where a small change can produce a large effect violates the General Principle of Order that "as the data are ordered, so the unknowns are ordered." (_datis ordinatis, etiam quaesita sunt ordinata_.)[20]

In other words both Malebranche and Descartes have violated the law of continuity formulated here by Leibniz as follows:

> _When the difference between two instances in a given series or that which is presupposed can be diminished until it becomes smaller than any given quantity whatever, the corresponding difference in what is sought or in their results must of necessity also be diminished or become less than any given quantity whatever._ Or to put it more commonly, _when instances or data approach each other continuously so that one at last passes over into the other, it is necessary for their consequences or results (or the unknown) to do so also._[21]

The relevant application of the principle in physics is that rest may be considered as an infinitely small velocity or as an infinite retardation. Each law of rest should be considered as a special case of the law for motion. If the result produces a great incongruity as in certain of Descartes' rules of motion, then the rules must be incorrect.

The outcome of this discussion with Malebranche is that Descartes' quantity of motion is incorrect on two different grounds. In the first place it would allow the production of a perpetual motion machine because the mathematical

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[20] _Nouvelles_, _10_, 747.

[21] _Ibid._, 747, Italics Leibniz's.

expression, mv, is incorrect. In the second place it violates the law of continuity when applied to cases of the impact of bodies. Thus Leibniz is publically making an attempt to undermine the Cartesian philosophy that the essence of nature may be explained as matter in motion, by showing that the mathematical formulation of that explanation is incorrect.

Of this attack, two things must be pointed out. Both these arguments of Leibniz have their justification and their shortcoming. The difficulty with the first argument has been indicated in the discussion of the "Brief Demonstration." Quantity of motion in its correct form, momentum, and its related laws form one picture of nature. Conservation of energy and its presuppositions are an alternate approach. To the extent which Leibniz means that mv-momentum is not conserved he is in error, but in the sense that m|v| is an incorrect measure of momentum, Leibniz is correct. This distinction was indeed perceived by Leibniz, as indicated above,(p. 83). That mv^2 is a measure of a quantity found in nature, is again correct. Concerning his second argument, that Descartes' rules of motion are incorrect as they stand, he is justified in his criticism. But the implication that the foundations of Descartes' description of nature (i.e. extended matter in motion) are in error because the 7 rules of impact are wrong, cannot be consistently maintained when conservation of momentum,

$\underline{mv}$, is substituted for conservation of $\underline{m|v|}$. Leibniz however thought of himself as substituting force, $\underline{mv}^2$ for $m|v|$, rather than $m\vec{v}$ for $m|v|$.

Returning to the discussion initiated by the Abbé Catalan, one finds that he replied only to that portion of Leibniz's paper (Feb., 1687) directed against himself.[22] He lists three different physical principles all of which he claims are true but regarding which two things entirely different are confounded by Leibniz.

"First, the principle of Descartes with respect to machines:

> As much force is necessary to lift a weight of 1 pound to a height of 4 ft. as to lift a weight of 4 pounds to a height of 1 foot. [That is, for the five simples machines, the weight and the distance through which it is moved are inversely proportional, $\underline{w_1 s_1} = \underline{w_2 s_2}$. This is a work equation.]
>
> Second the principle of Galileo on the fall of bodies: The spaces traversed by heavy bodies in their perpendicular fall towards the center of the earth are in the duplicate ratio of the times. [$\underline{s} = \frac{1}{2}\,\underline{at^2}$].
>
> Third the principle of Descartes on the quantity of motion: Quantity of motion is expressed by the product of the mass and velocity of a body. [$\underline{mv}$].[23]

Catalan procedes with a discussion of what he sees as Leibniz's confusion. He defines the force of a body as

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[22] Abbé Catalan, "Remarque sur la replique de M. L. touchant le principe méchanique de M. Descartes, contenuë dans l'article III de ces Nouvelles, mois de Février, 1687", Nouvelles de la republique des lettres, 10, 577-590.

[23] Ibid., 579.

its motion, or transportation with respect to the surrounding bodies. If one includes only the length of the space traversed in regarding the motion, one is speaking of velocity. But if one also includes the mass then quantity of motion is defined.[24]

But knowing that two equal bodies have traversed equal spaces, how does one know whether the motions are equal or unequal? One body may have used $\frac{1}{2}$ hour to complete the motion, the other $\frac{1}{4}$ hour. The second clearly has a greater quantity of motion. Thus to say that two motions are equal, the times of traversal must also be equal.

Here Catalan is talking about the momentum of equal bodies in equal times when they traverse a horizontal plane. But then in a continuous argument he goes on to discuss the "force" of unequal bodies in equal times. Here however without stating so explicitly he is referring to the mass times the virtual velocity or displacement of the simple machines. "Likewise when there is inequality between the quantities of motion, or the forces, and inequality between the bodies, the spaces traversed can only be reciprocal to the masses or proportional to the velocities if there is always unity or equality of the times. Thus again momentum is confused with static force.[25]

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[24] *Ibid.*, 580-581.

[25] *Ibid.*, 584.

Catalan continues to confuse the two in the following statement: There is no difference between lifting with equal force, the weight of 1 pound to a height of 4 feet and that of 4 pounds to a height of 1 foot [i.e. proposition 1] and between transporting with equal quantities of motion, the simple body 4 times the space and the quadruple body the simple space [proposition 3]. The contradiction of Leibniz, he says, is resolved if one considers the effect in a given duration. It is the disregard of duration in the meaning of effect which does not allow for any difference in the spaces traversed, and which entirely changes the question.[26] Descartes' rule speaks of moving forces, i.e. quantities of motion, in equal times; Galileo's rule compares forces applied in or motions acquired in unequal times and proportional to the square roots of the heights.[27]

The rebuttal of Leibniz that the times can be equalized by altering the line of descent of an inclined plane is useless, he says, because on an inclined plane, the force necessary to lift a body is less than that necessary to lift it perpendicularly to the same height.[28]

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[26] _Ibid._, 585-586.

[27] _Ibid._, 586.

[28] _Ibid._, 586, 587.

Here again two concepts are confused. Leibniz is discussing the fall of a weight through a vertical distance [mgs], or potential energy, where the time is irrelevant. Catalan really means the mechanical advantage of the inclined plane used as a simple machine, or force multiplying device. Here the effort or Newtonian force needed to push a body up the plane is less than that needed to lift it perpendicularly to the same vertical height.

Catalan closes his remarks with a brief pronouncement that he sees no absolutely necessary connection between the rules of motion of hard bodies and the fundamental proposition that God always conserves an equal quantity of motion in matter. Thus the remarks of Leibniz against Descartes' rules of motion cannot be applied against his own stand on quantity of motion. However he adds he would be willing to express his opinion on these rules of motion at another time.[29]

Leibniz's second reply[29] in the controversy with Catalan was in the main a reiteration of arguments previously evoked. Of an example in which all the force of a body of 4 lbs, velocity 1 is transferred to a second body of 1 lb., the velocity received by the second being 2, Leibniz states that the same quantity of force is pre-

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[29] *Ibid.*, 589. Gottfried Wilhelm Leibniz, "Reponse à la remarque de M. l'abbé D.C. contenuë dans l' article 1 de ces Nouvelles mois de Juin 1687 où il prétend soutenir une loi de la nature avancée par M. Descartes." *Nouvelles de la Republique des lettres*, 11(1687), 952-956.

served (garder) but the same quantity of motion is not. How this transferral of "force" is to be accomplished is not specified by Leibniz.

The Controversy with Denis Papin

Denis Papin entered the discussion on the quantity of motion in 1689 with a new argument in support of the Cartesian hypothesis, derived from his theory of the cause of gravity.[31]

He employs two principles, one taken from Galileo, the second from Huygens. Galileo demonstrated that falling weights add equal velocities in equal units of time. [$v = at$]. Huygens, claimed Papin, in providing a first principle upon which to base those of Galileo, assumed that "the power which is the cause of gravity has a speed unlimited in comparison with the velocity of a falling weight."[32]

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[30] Ibid., 954.

[31] Denis Papin, "De Gravitatis causa et proprietatibus observationes," Acta Eruditorum (1689) 183-188.

[32] Papin, "De Gravitatis causa", 184, "Potentia quae gravitatis causa est, celeritatem habet infinitam prae velocitatibus gravium cadentium..." This is a portion of Huygens' theory of the cause of gravitational force based on Descartes' hypothesis of a subtle matter filling all space providing for change by physical contact. Huygens' subtle matter was in very rapid motion and penetrated all parts of matter from every side. Huygens had attempted to work out an explanation of gravity as an effect of circular motion to substantiate Descartes' vortical theory of the motion of subtle matter.

See A. E. Bell, Christian Huygens and the Develop-

A weight, whether falling or at rest is thus always affected in the same manner by the force of gravity. Furthermore, an infinitely slow motion is not distinguishable from rest by any sensible effect. There is then no reason why the power which is the cause of gravity should not imprint the same quantity of motion in the second instant as it does in the first instant. The curve describing the motion is continued in the same uninterupted course in equal quantities of time [$v = at$; $mv = mat = Ft$].

From this follows his objection to Leibniz's estimate of force by ws. The quantities of motion are in the direct

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ment of Science in the Seventeenth Century, London, 1947, 161-164.

Papin in this paper (184, 185) describes an experment of Huygens to measure the speed of the circular motion of the subtle matter causing gravitational force. In this experiment a device was used to horizontally rotate two pendulum balls hanging on thin threads. The principle is the same as another of Huygens' experiments described by Mach, in The Science of Mechanics, op. cit., p. 200.

"In a closed vessel containing water Huygens placed small particles of sealing wax which are slightly heavier than water and hence touch the bottom of the vessel. If the vessel be rotated the particles of sealing wax will flock toward the outer rim of the vessel. If the vessel be then suddenly brought to rest, the water will continue to rotate while the particles of sealing wax which touch the bottom and are therefore more rapidly arrested in their movement, will now be impelled toward the axis of the vessel. In this process Huygens saw an exact replica of gravity... The detailed exposition of this kinetic theory of gravity is found in Huygens' tract On the Cause of Gravitation."

The existence of a subtle matter was also hypothesized to explain certain effects encountered in experiments with a vacuum pump which Huygens performed with Denis Papin in 1674 and described by Papin in Nouvelles Experiences du vide (1674). See Bell, 163.

ratio of the times of the motion /$m_1v_1/t_1 = m_2v_2/t_2 = F$/.
The force of a falling body is thus increased or diminished according as there is more or less time for the motion to occur.

Then from discussing the momentum acquired by a falling body, Papin leaps in a continuous argument to the situation of equal times. This case would hold true only for equal bodies falling during equal times, or in statics.

"If the times are equal no more or no less force can be added or subtracted by making the spaces traversed longer or shorter. Thus a measure of force estimated by the spaces traversed cannot be correct."

Papin is arguing that Leibniz' analysis is wrong because static or Newtonian force does not apply to Leibniz's free fall demonstration. Of course there is no reason for its application since Leibniz's "force" is an energy concept.

Papin closes his paper with the statement that although he has taken the part of the Cartesians in the matter of the measure of force, he would not defend in the same way the rules of motion.

Leibniz's reply ("On the Cause of Gravity and a Defense of his Opinion of the True Laws of Nature Against the Cartesians," May, 1690)[33], promulgated a new demonstration which

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[33]Gottfried Wilhelm Leibniz, "De Causa Gravitatis et Defension sententiae sua veris naturae legibus contra Cartesianos" Acta Erud. (1690) 228-239. Also in Gerhardt, M.S., /2/, 2, 193-203.

was intended to firmly establish ws as the measure of force. However it further points up the confusion between measure and conservation of "force". Papin's next reply will shrewdly attack this confusion though not clarify its existence.

Leibniz begins by clarifying the issue at stake, in order he says, to exclude all verbal misunderstanding. Anyone is at liberty to define force as he wishes, whether as quantity of motion or as motive force. The issue is to decide which is conserved (conservare) whether it be the product of weight /pondus/ and speed or the product of weight /pondus/ and height. This will be decided by whether or not perpetual motion can arise from the acceptance of either definition.[34]

Although the issue is clearly a verbal one, as Leibniz astutely points out, it cannot be decided by the production of perpetual motion.

The example which will demonstrate perpetual motion is as follows (Fig. 4, following page).[35]

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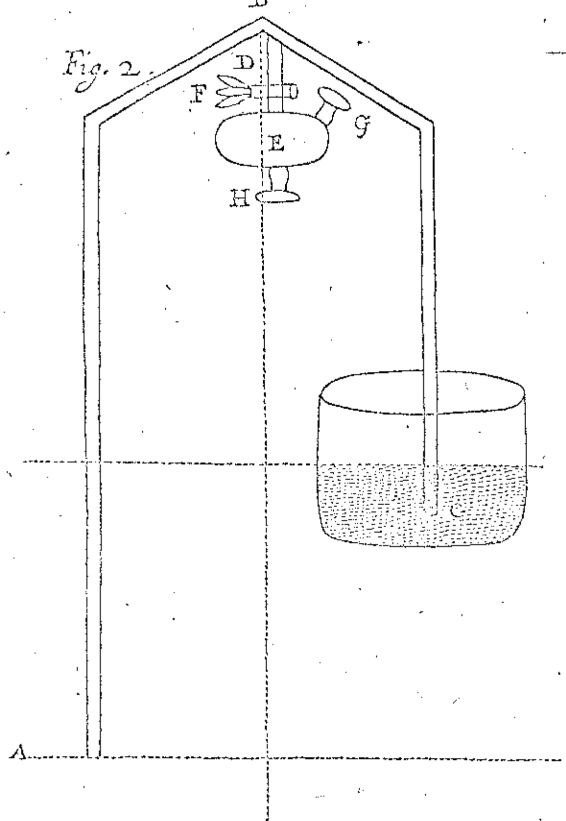
[34]Ibid., 234. "Sed ante omnia logomachiae excludena occasio est; erunt enim, qui sibi permissum dicent vim definire per quantitatem motus, et duplicata corpris dati celeritate, vim ejus duplicatam dicere; neque hanc ego libertatem cuiquam nego, quam mihimet concedi postulo. Sed cum controversia nobis sit realis, utrum scilicet motus conservetur, an vero potium eadẽ quantitas virium eo sensu, prout a me accipitur, id est in ratione composita non ponderis et celertatis, sed ponderis et altitudinis, per quam corpus ab agente vim habente attolli potest, facile de verbis transigemus."

[35]Ibid., 235-239.

TAB. VII. Ad A. 1690. pag. 218. sqq.

Fig. 2.



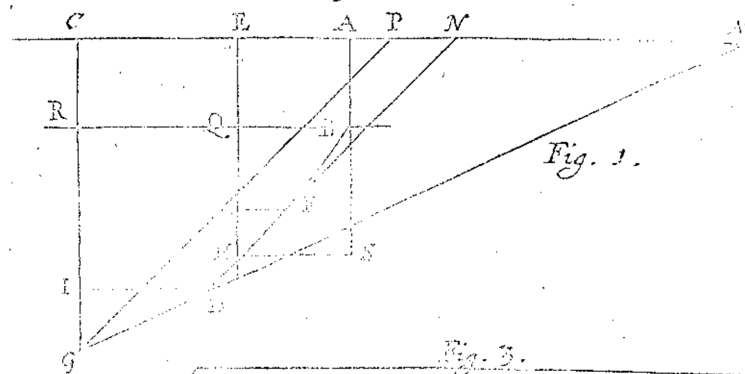


Fig. 1.

Fig. 3.

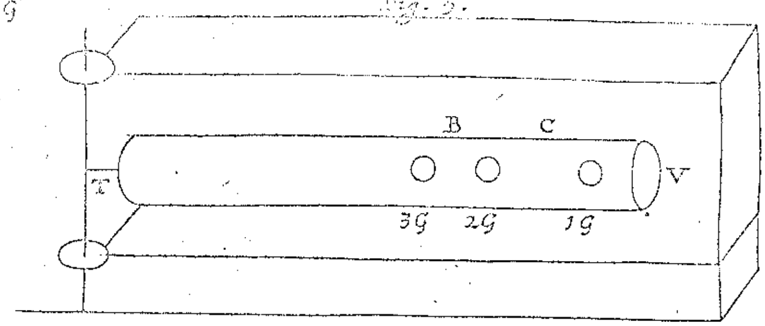
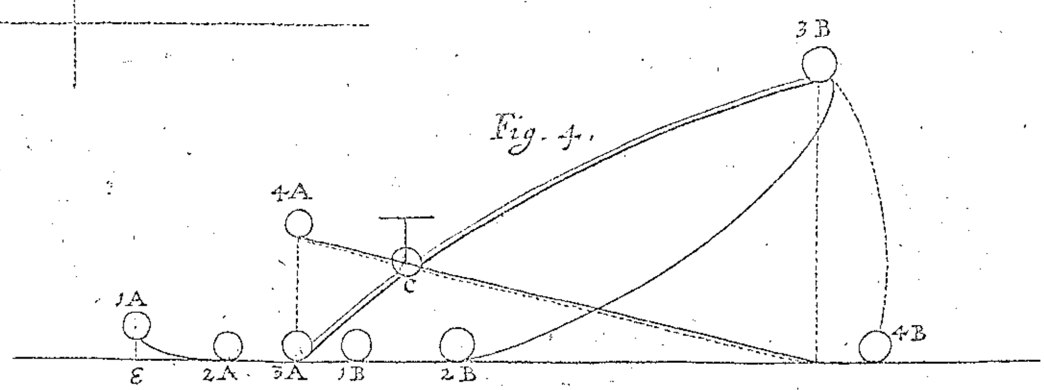


Fig. 4.



Acta Eruditorum, May, 1690.

A ball A of weight 4 descends from the height 1AE = 1 foot, by the inclined plane 1A2A until it arrives in the horizontal plane EF. There it travels from 2A to 3A with a velocity of 1 gained in the descent. In the same horizontal plane a second ball, B, of one pound rests at point 1B. Let all the force of ball A now be transferred to ball B such that A rests in the horizontal place 3A, and B alone is moved. How much speed should ball B receive so that it has only as much force as does ball A?

The Cartesian answer according to Leibniz will be a speed of 4; $m_A v_A = 4(1)$; $m_B v_B = 1(4)$. But if this is the case, perpetual motion will arise. For body B having weight 1, speed 4, traversing from 1B to 2B arrives at the incline 2B3B. It is then able to ascend to 3B, or the perpendicular height F3B = 16 feet, since the height is proportional to the velocity squared. Now perpetual motion, or an effect more powerful than its cause can arise, because B can be made to descend to the horizontal position 4B. There by means of a lever with fulcrum at C it is able to elevate ball A of weight 4 resting at 3A to a perpendicular height of nearly 4 feet, the lever arm C3B being a little greater than 4 times the lever arm C3A. This is absurd: in the initial state A was at a height of 1 foot and B rested in the horizontal plane; in the final state A is restored to a height of 4 feet, while B again rests in the horizontal plane. A can easily be returned from 4A to 1A and thus create perpetual motion by the force of its own descent. No new force has been contributed or absorbed by other agents or patients. We conclude therefore against the Cartesians that quantity of motion should not always be conserved.[36] If _ms_ _ws_ is accepted as correct, the quantity of motion _mv_ after the transfer has _decreased_ from 4 [_mv_ = 4(1); _ms_ = 4(1)] to 2 (_mv_ = 1(2); but _ms_ = 1(4)]. And other cases may be contrived in which the quantity of motion will be _increased_. Thus if the ratio of the bodies to the heights to which they are able to ascend, is used instead of _mv_, perpetual motion will not arise. Expressed mathematically, the relationship, _Ax_ + _Bz_ = _A_(_x_) + _B_(_z_) will allow the same "power" always to be preserved, where _x_ and _z_ and (_x_) and (_z_)represent the heights to which bodies _A_ and _B_ are able to ascend before and after the action. But the relationship _Ae_ + _By_ = _A_ (_e_) + _B_(_y_) where _e_ and _y_, (_e_) and (_y_) are the initial and final speeds, will not follow since it is not always possible to preserve the same quantity of motion.[37]

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[36] _Ibid._, 237. "Itaque eadem opera conclusimus contra Cartesianos, non semper debere conservari quantitatem motus.

[37] _Ibid._, 237. "Et generaliter si sit corpus _A_ praeditum initio celeritate _e_, corpus vero _B_ celeritate _y_; at post actionem sit corpus _A_ praeditum celeritate (_e_), corpus autem _B_ celeritate (_y_). Et similiter altitudines, ad quas corpora _A_ et _B_ ascendere poterant, ante actionem sint, (respective) _x_ et _z_, post actionem vero (_x_) et (_z_); ajo debere esse _Ax_ + _Bz_ = _A_(_x_) + _B_(_z_) ut eadem servetur potentia; unde utiq. sequitur, non semper posse esse _Ae_ + _By_ = _A_(_e_) + _B_(_y_), seu non posse eandem semper servari quantitatem motus.

Leibniz concedes that the speeds acquired or lost by freely ascending or descending bodies will be as the times, but it is the moving forces which are conserved and these are not estimated by the speeds.[38]

Papin, (January, 1691),[39] reasserted first the objection drawn from the time of ascent, saying that the ascent of body A, speed 1 (in the above example) should be through 1 unit of time and that of body B, speed 4 through 4 units of time. For each degree of time and speed, each body will overcome equal resistance of gravity.

His second objection to Leibniz's perpetual motion example is a crucial one. He concedes that perpetual motion is absurd and that if it could actually be demonstrated by the above example the Cartesian measure of force would be reduced to an absurdity. But he denies the possibility of actually transferring in nature all the "power" of body A to body B.[40]

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[38] Ibid., 237. "At ego VIRES MOTRICES, id est egs quae conservandae sunt, ostendi non esse aestimandas gradibus celeritatis."

[39] Denis Papin, "Mechanicorum de viribus motricibus sententia, asserta adversus cl. G.G.L. objectiones," Acta Erud., (1691), 6-13.

[40] Ibid., 9. "Ego autem motum perpetuum absurdum esse fateor, cl. Viri demonstrationem ex supposita translatione esse legitimam; sed Hypothesis ipsius possibilitatem, translationis nimirum totius potentiae ex corpore A in corpus B, pernego: manifestum autem est, quod si dicta translatio in rerum natura nullibi et nulla ratione fieri posset, qui ab ea sperabatur motus perpetuus remanet etiam impossibilis, neque Cartesiani ad illud absurdam rediquntur."

He promises publically that if any method can be indicated by which all the moving force of the greater body can be transferred directly to the smaller body at rest without the occurance of a miracle, he will either concede that effective perpetual motion is possible or will concede victory to Leibniz.[41]

Leibniz's final reply (Sept. 1691)[42] did not adequately meet the objections of Denis Papin. Leibniz claimed that the demonstration of the physical transfer of the total force from the larger to the smaller body was not at all difficult. First of all in the course of the experiment the law of continuity must be assumed, so that a leap from one value of force to another could not occur out of which perpetual motion might arise. Secondly denying the possibility of a leap, the equipollence of cause and effect must be assumed. For whereas many will agree that an effect cannot be more powerful than its cause, some will argue

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[41] Ibid., 9. "Possem hic substitere: hactenus enim validissimus demonstrationibus suffulta Mechanicorum sententia ab adversariorum tetis inconcussa remanet: Dominoque Leibn. incumbit probandum, vel quam nego translationem in natura esse posibilem; vel sattem ipsius possibilitatem sequi ex adversariorum suorum doctrina: mihi autem sufficeret palam hic promittere, quod si mihi indicet rationem aliquam quo tota vis motrix, sine miraculo, ex corpore majori transferri queat in corpus minus et quiescens: Ego vel motum perpetuum effectum, vel manus victas dabo."

[42] Gottfried Wilhelm Leibniz, "De Legibus naturae et vera aestimatione virium motricium contra Cartesianos responsio ad rationes a Dn. P. mese Januarii proximo in Actis hisce p. 6 propositas," Acta Erud., (1691), 439-447. Also in Gerhardt, M.S., /2/, 2, 204-211.

that the effect can be less than its cause. But Nature neither decays and decreases in perfection nor does it recuperate losses or attain higher perfection by a miracle. Further one cannot justify a decrease in "force" after a collision by arguing for the annihilation of the remainder of the cause over and above that used to obtain the effect.

Proceding under the assumption that Denis Papin is too sophisticated to fall into any of the above blunders, Leibniz offers two methods of transferring all the "force" from a larger body to a smaller one at rest, claiming that additional demonstrations have been left with a friend in Florence. The first method is to divide body A into 4 parts, all equal to the size of body B, the totality retaining the velocity of body A i.e. 1. The power of each of these smaller bodies is then transferred successively onto body B at rest.[43]

The physical impossibility of accomplishing this by successive collisions is obvious. The first collision will set body B in motion with the velocity of the first small part. But thereafter body B and the second small part of body A will be in motion with equal velocities.

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[43] *Ibid.*, 443. "Et quidem si concedatur, posse totam vim minoris transferri in majus sine motum sine quiescens, igitur A motum, majus B quiescente dividamus in partes ipso B minores, totius A velocitatem retinentes, et cuius libet deinde potentiam in B transferendo successive, tota ipsius A majoris potentia translata erit in B quiescens."

Leibniz's second method is to connect bodies _A_ and _B_ by a sufficiently long rigid line. On this is assumed an immovable point _H_ around which the compound is able to be rotated. Point _H_ is close enough to _A_ and sufficiently removed from _B_ that when _A_ with its initial force is attached it can be brought effectively to rest. Almost all its force will then be translated to _B_ at the other end, and when _A_ rests, _B_ is unbound.[44] The details of this method are obscure and it is not at all clear how such a device could by physically operated and still fulfill the conditions of _A_ having initial velocity 1 and _B_ having zero velocity.

Behind Papin's challenge to Leibniz lay the modern idea of conservative systems in mechanics. If the transfer of "force" is to be accomplished as a two-body interaction, then momentum would be conserved and mechanical perpetual motion could not result. If, however, external forces such as springs were used to transfer the force, then Leibniz's argument would still be valid for quantity of motion, or momentum, would not be conserved in compressing the spring.

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[44] _Ibid._, 443. "Aliter: Aet B connectantur linea rigida quantum satis est longa, et in ea sumatur puntum H quod sit tam vicinum ipsi A et tam remotum a B, ut celeritas quae inter circulandum competit ipsi A, sit quantum-vis parva. Ita potest A haberi pro quiescente, vel quasi, et tota quasi vis ejus soluto mox nexu seu linea rigida sublata, translata erit in B."

But even though momentum conservation would be violated, perpetual motion cannot occur because this principle applies only to conservation of vis viva, and here vis viva is conserved.

A text of Leibniz, written in 1692 was recently discovered, edited and discussed by Pierre Costabel.[45] In regard to content, this Léibniz text is very similar to the two papers written against the ideas of Denis Papin but presents the argument in the form of logical definitions, axioms and propositions. The example given is the same one discussed above and the principles upon which the conclusions are based are the impossibility of perpetual mechanical motion, the principle that the total cause must equal the complete effect and that the same quantity of force is conserved.[46]

Again all transfer of force is by substitution of a body in one state of motion and position for a body in another, equal to that of the first. The possibility of

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[45] Pierre Costabel, Leibniz et la dynamique, les texts de 1692, Paris, 1960.

[4646] Costabel, 98. "Definition 3: Le mouvement perpetual mécanique (qu' on demande en vain) est un mouvement ou les corps se trouvent dans un état violent, et agissant pour en sortir n'avancent pourtant point, et le tout se retrouve au bout de quelque temps dans un état non seulment autant violent que celui ou l'on etait au commencement, mais encore au dela, puisque outre que le premier état est restitué il faut que la machine puisse encore produire quelque éffet on usage mécanique sans qu'en tout cela aucune cause de dehors y contribue."

"Axiom I: La même quantité de la force se converve, ou bien, l'effet entier est égal à la cause totale."

physical transfer is not discussed except to say that one can imagine certain techniques for the execution of these transfers. Propositions identical with the conclusions in the other two papers are proved by use of the axioms and definitions. Proposition 8 reads: "When the forces are equal, the quantities of motion are not always equal and vice versa." While this proposition is true, the conditions for the validity of proposition 9 following it are not specified: "The same quantity of motion is not always conserved."[47] The similarity of this 1692 paper to Leibniz's 1690 paper against Papin is not mentioned by Costabel.

The paper does make one contribution to the development of Leibniz's physical thought, as pointed out by Costabel. He uses here the terms force vive and force morte, ordinarily considered by historians to have been introduced in 1695 in the essay "Specimen Dynamicam".[48] "Et il est a propos de considérer que l'equilibre consiste dans un simple effort (conatus) avant le mouvement, et c'est ce que J'appelle la force morte qui a la même raison a l'egard de la force vive (qui est dans le mouvement même) que le point à la ligne."[49]

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[47] Costabel, 103.

[48] These terms, however, were also used in his "Essay de dynamique" written about 1691. See this disertation Ch. III, p. 87.

[49] Costabel, 104. Concerning this Costabel writes, "La tradition attribue à Leibniz l'expression même de force vive. M. Guéroult indique que le terme de "vis viva" fait son apparition pour la première fois dans le Specimen dynamicum et note que Leibniz s'est plus souvent servi de l'expression

Although Leibniz made the final statement in the discussions with Denis Papin and although he himself concluded that he had satisfied all the contrary arguments of his "renowned antagonist,"[50] his case, as shown above, was not at all clear cut. The argument with Papin emphasizes the misapplication of the principle of the impossibility of perpetual motion and the confusion between the conservation and the measure of "force." It points up the fact that the situations in which momentum is and is not conserved were not clearly specified. Thus the question was not solely one of verbal definition as to the mathematical expression of force as d'Alembert was later to characterize the controversy. The papers exchanged between Papin and Leibniz as those between Catalan and Leibniz, point up a second major confusion in the controversy, that between the measure of a force acting through a distance, ($fs = mv^2$), the measure of a force acting through a time interval ($ft = mv$), and the force of statics (mdv/dt).

"potentia" pour designer mv^2. Il est aisé cependant de constater que dès 1686 l'expression de "potentia viva" par opposition a "potentia mortua" appartient au langage leibnizien. On lit en effet dans la Brevis demonstratio.... "Est autem potentia viva ad mortuam vel impetus ad conatum ut linea ad punctum vel ut planum ad lineam"....Les textes que nous venons de relever prouvent encore une fois que la notion de force vive, nettement conçue par opposition à la force morte, existe chez Leibniz des 1687 et l'Essay de 1692 temoigue que des que Leibniz a voulu traduire en francais, il a employé l'expression de "force vive." Costabel, 50,51.

[50]Leibniz, "De legibus Naturae," op. cit., 444.

Conclusion

Thus to summarize Leibniz's position regarding Descartes' quantity of motion: (1) He did not believe that _mv_ was the mathematical measure of "_force_". (2) He did not believe that quantity of motion, as defined by Descartes, was conserved because a) if correct it would give rise to perpetual motion allowing inequality between total cause and effect, and b) it gave rise to false rules for the motion of colliding bodies. (3) He was _not_ objecting to Descartes' quantity of motion solely on the basis of Descartes' neglect of the vectorial aspect of the velocity, for he knew the work of Wallis, Wren, and Huygens on the conservation of _mv_, and indicated this as early as 1686. Any objection of Leibniz on this point would be correct. (4) The real crux of his objection to the quantity of motion was that he was convinced that an absolute and not a relative quantity was conserved in nature and that _mv_ did not meet the qualifications for such a quantity. This is closely tied to the philosophical system he was developing at the time, as has already been indicated.

LEIBNIZ'S ARGUMENTS AGAINST QUANTITY OF MOTION

(Carolyn Iltis)
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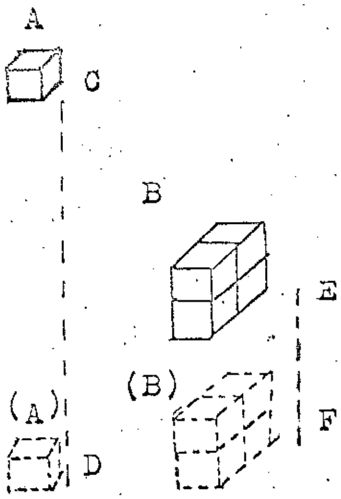


Fig. 1
Diagram Leibniz's

1. Argument in the Acta Eruditorum, March, 1686

Assumptions:

(1) A body falling from a certain height acquires the same force necessary to lift it back to its original height if nothing external interferes. (This defines force as quantity of matter times distance, or ms.)

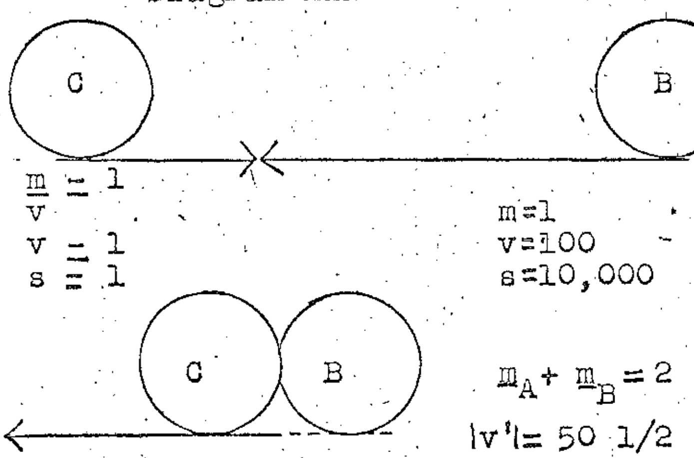
(2) The same quantity of force is necessaryto raise body A (Fig. I) having mass, $m=1$, to a height $s=4$, as is necessary to raise body B, $m=4$ to height $s=1$.

Conclusions:

(1) Body A, $m=1$, in falling a distance $s=4$, will acquire the same force as body B, $m=4$, falling $s=1$. $[(ms)_A = 4 = (ms)_B]$

(2) The quantities of motion are not equal. $\left[(mv)_A = (1)(2) = 2;\ (mv)_B = (4)(1) = 4\right]$

Fig. 2
Diagram mine



2. Argument in the Nouvelles de la republique des lettres, 9:138 (February, 1687)

Descartes' third rule for colliding bodies: "If [hard] body B and [hard] body C are equal in heaviness, but B moves with slightly greater speed than C, not only do both move to the left afterwards, but B also imparts to C half the difference of their original speeds," i.e.,

$$|v'| = |v_C| + \left[\frac{|v_B| - |v_C|}{2}\right]$$

Before collision:

$$m_C(|v|_C) + m_B(|v|_B) = (1)(1) + 1(100) = 101$$

$$m_C s_C + m_B s_B = (1)(1) + (1)(10{,}000) = 10{,}000$$

Thus a 1 lb. weight would be elevated to 10,000 feet.

After collision:

$$|v'| = |v_C| + \left[\frac{|v_B| - |v_C|}{2}\right] = 1 + \left[\frac{100-1}{2}\right]$$

$$= 1 + 49\ 1/2 = 50\ 1/2$$

$$m_{B+C}(|v'|) = 2(50\tfrac{1}{2}) = 101$$

$$m_{B+C}(s') = (2)(50\ 1/2)^2 = 5100\ 1/2$$

Thus a 1 lb. weight would be elevated to 5100 1/2.

3. Argument in the *Acta Eruditorum*, May, 1690

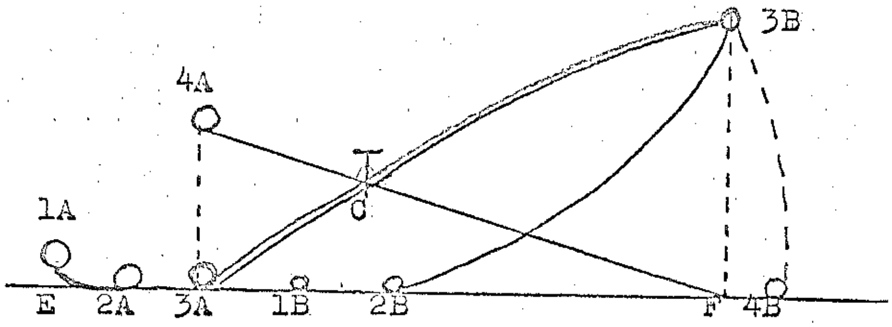


Fig.3 Leibniz's diagram

A ball A of weight 4 descends from the height 1AE = 1 foot, by the inclined plane 1A2A until it arrives in the horizontal plane EF. There it travels from 2A to 3A with a velocity of 1 gained in the descent. In the same horizontal plane a second ball, B, weight 1, rests at point 1B. Let all the force of ball A now be transferred to ball B, such that A rests in the horizontal place 3A, and B alone is moved. How much speed should ball B receive so that it has only as much force as does ball A?

The Cartesian answer (according to Leibniz) will be a speed of 4; $m_A v_A = 4(1)$; $m_B v_B = 1(4)$. But if this is the case, perpetual motion will arise. For body B having weight 1, speed 4, traversing from 1B to 2B arrives at the incline 2B3B. It is then able to ascend to 3B or the perpendicular height F3B = 16 feet, since height is proportional to velocity squared. Now perpetual motion or an effect more powerful than its cause can arise, because B can be made to descend to the horizontal position 4B, where by means of a lever with fulcrum at C it is able to elevate ball A of weight 4 resting at 3A to a perpendicular height of nearly 4 feet, the lever arm C3B beigg a little greater than 4 times the lever arm C3A. This is absurd: in the initial state A was at height 1 foot and B rested in the horizontal plane; in the final state A is restored to a height of 4 feet while B again rests in the horizontal plane. A can easily be returned from 4A to 1A and thus create perpetual motion by the force of its own descent. No new force has been contributed or absorbed by other agents or patients. We conclude therefore against the Cartesians that quantity of motion should not always be conserved.