CHAPTER VI

British Reactions and 's Gravesande's Reply

(1722-1729)

The discussions by Polenus and 's Gravesande sparked a series of counter experiments and arguments by British scientists in the years from 1722-1729.

The first of these papers (1722) was contributed by Henry Pemberton who later in 1728 published one of the three outstanding popular introductions to Newtonian science, a non-technical View of Sir Issac Newton's Philosophy. Although published after Newton's death (1727) this account, authorized by Newton himself, had been written while Newton was alive.

Pemberton's "Letter to Dr. Mead" appeared in the Royal Society's Philosophical Transactions for May, 1722.[1]

His introductory remarks stated his contention that Poleni's conclusions were wrong and that Leibniz's opinion was unreasonable:

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[1] Henry Pemberton, "A letter to Dr. Mead...concerning an Experiment, whereby it has been attempted to shew the falsity of the common Opinion in relation to the force of Bodies in Motion." April, May, 1722 issue of Phil. Trans., 32 (1724) 57-66.

> Perusing the Learned Polenus's Tract, De Castellis you were pleased to send me I have found in it several curious experiments among which I reckon that of letting globes of equal Magnitude but of different weights fall upon a yielding substance as Tallow, Wax, Clay or the like from the heights reciprocally proportional to the weights of the globes. This experiment engaged in particular my attention as it is brought with design to overturn one of the First Principles established in Natural Philosophy....I cannot by any means admit of the Deduction that is drawn from thence, that because the globes make in this experiment equal impressions in the yielding substance, therefore they strike upon it with equal force...On the contrary I think this very experiment proves the great unreasonableness of Mr. Leibniz's notion.[2]

The experiment of Poleni, he wrote, "Better informs us of the law by which these yielding substance resist the motion of bodies striking them, than to shew the forces with which Bodies strike". Using the Newtonian concept of action equals reaction, Pemberton changes the *vis viva* problem of free fall into a momentum problem. He treats Poleni's experiment essentially as an inelastic collision with the earth in which mv^2 is not conserved. His argument is as follows:

The opposition of the yielding substance to the globes of different weights entering equal distances into the substances is proportional to the time it takes them to move through the substance [$R_A / t_A = R_B/t_B$ where R_A, R_B = Resistance of yielding substance to globes A and B].

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[2] *Ibid.*, 57.

The opposition or momentaneous loss of force is therefore reciprocally proportional to the velocity of each globe. /$s = v_A t_A = V_B t_B$; hence $R_A v_A = R_B v_B$/ The whole force measured by the quantity of motion mv /where mass and weight are confused and taken as identical/ is likewise reciprocally proportional to the velocity of each globe. /Force of motion = mv/ The globes while penetrating equal distances into the substance lose parts of their force which bear the same proportion to the whole force. /$m_A V_A / R_A = m_B v_B / R_B$ where $R_A R_B$ = loss of force or opposition of tallow/ . Thus even if the velocities are proportional to the square root of the weights, as in the case of living forces, they are still proportional to the forces with which they press into the substance and will make equal indentations in it. "And therefore upon the Theory of Resistance here supposed, when the whole Force and Motion of both these Globes is entirely lost, they will be plunged into the substance at equal depths."[3] Pemberton concludes:

> But as I have asserted in the beginning of this letter that the very experiment of Polenus is not only reconcileable to the common Doctrine of Motion, as I have now demonstrated; but even that it does itself make manifest the great unreasonableness if not the absolute absurdity of Mr. Leibniz's Opinion.[4]

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[3] Ibid., 60.

[4] Ibid., 62.

Another member of the Royal Society, John Theophilus Desaguliers, known for his translation of 's Gravesande's work on physics, entered the discussion in 1723 with a paper entitled: "An Account of Some Experiments Made to Prove That the Force of Moving Bodies is Proportionable to Their Velocities: (Or Rather That the Momentum of Moving Bodies is to Be Found by Multiplying the Masses into the Velocities) In Answer to Such who Have Sometime Ago Affirmed That Force is Proportionable to be Square of the Velocity and to Those Who Still Defend the Same Opinion".[5]

Desaguliers had been curator of experiments at the Royal Society since 1713 and knew Newton intimately. He was widely known for the original and skillful demonstration experiments which he used in public lectures and courses on Newtonian science.

> He showed his hearers through their eyes what 'their reading had taught them only imperfectly' and 'of which they had only a superficial notion'. His experiments were completely convincing and without doubt nothing could have made a stronger impression upon the audience than the ocular proof of the Newtonian philosophy provided in his lectures.[6]

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[5] John Theophilus Desaguliers, "An Account of some Experiments made to prove that the Force of Moving Bodies is proportionable to their Velocities: (or rather that the Momentum of Moving Bodies is to be found by multiplying the Masses into the Velocities) In answer to such who have sometime ago affirmed that Force is proportionable to the Square of the Velocity and to those who still defend the same opinion." Jan.-Feb.,1723 issue of Phil. Trans., 32, (1724) 269-279.

[6] I. Bernard Cohen, Franklin and Newton, Philadelphia, 1956, 246.

Desaguliers begins his "Account of Some Experiments" with a summary of his view of the controversy:

> As far as I can learn Monsieur Leibniz was the first that opposed the received opinion concerning the Quantity of the Force of moving Bodies by saying that it was to be estimated by multiplying the Mass of the Bodies not by their velocity but by the square of it. But instead of shewing any Paralogism in the mathematical Demonstrations which are made up to Prove the Proposition or any mistakes in the Reasonings from the Experiment made to confirm it, he uses other Mediums to prove his assertions; and without any Regard to what others had said on that subject brings new Arguments which the Reverend and Learned Dr. Clarke has fully answered in his fifth letter to him. Messieurs John Bernoulli, Wolfius, Hermannus and others have followed and defended M. Leibniz's opinion and in the same manner so that what is answer to him is so to them. Polenus (Prof. at Padua) has acted after the same manner in the experimental way making some experiments to defend M. Leibniz's Opinion, without having shown those to be false which are made use of to prove the contrary...[7]

His first argument, designed to support the view that force is proportional to the mass multiplied by the velocity was:

> If a man with a certain Force can move a weight of fifty Pounds, through a Space of four Feet, in a determinate time; it is certain he must employ twice that Force to move one hundred Pounds Weight through the same Space in the same time.
>
> But if he uses but the same Force, he will move the one hundred Pounds Weight but two Feet in the same time. For as the one hundred Pounds Weight contains two fifty Pound Weights, if each of them has two degrees of velocity given to it, it will exactly require the same Force that would give one of them Four Degrees of Velocity; hence it appears that Force is proportionable to the Mass multiplied into the Velocity.[8]

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[7] Desaguliers, op. cit., 269-270.

[8] Ibid., 271.

This argument would be true only if there were twice as much friction or resistance operating on the 100 lb weight as on the 50 lb. weight./$\mu = \frac{F}{N}$; a) $\mu = \frac{F}{50}$, $mv = (50)(4)$; b) $\mu = \frac{F}{100}$, $mv = (100)(2)$/

The second argument is based on the law of the lever (see diagram, p. 194). Desaguliers, like Catalan and Papin erroneously used the principle of virtual velocities in arguing for momentum. A weight of 100 pounds having a lever arm of 1, balances a 25 pound weight with a lever arm four times as great. "It is known to all Mechanicians, that a Weight of one hundred pounds at A, will keep in Aequilibrio a Weight of twenty-five Pounds hanging at B, where it will have a Velocity four times greater than that of the Weight at A.... Whereas if the Forces were as the Mass multiplied into the Square of the Velocity, the twenty-five pound Weight should have been suspended only twice as far from the fulcrum as the 100 pound Weight."

This contrary argument of course was never claimed by Leibniz who asserted it as a case of dead force, mdv/dt or mdv since the times are equal, and not a case of mv^2. Desaguliers incorrectly cites a virtual velocities problem in support of momentum. The third argument of Desaguliers is the familiar one based on the time of descent of a heavy body: "As the Time of the fall through a space of four Foot is twice the Time of a fall through one Foot

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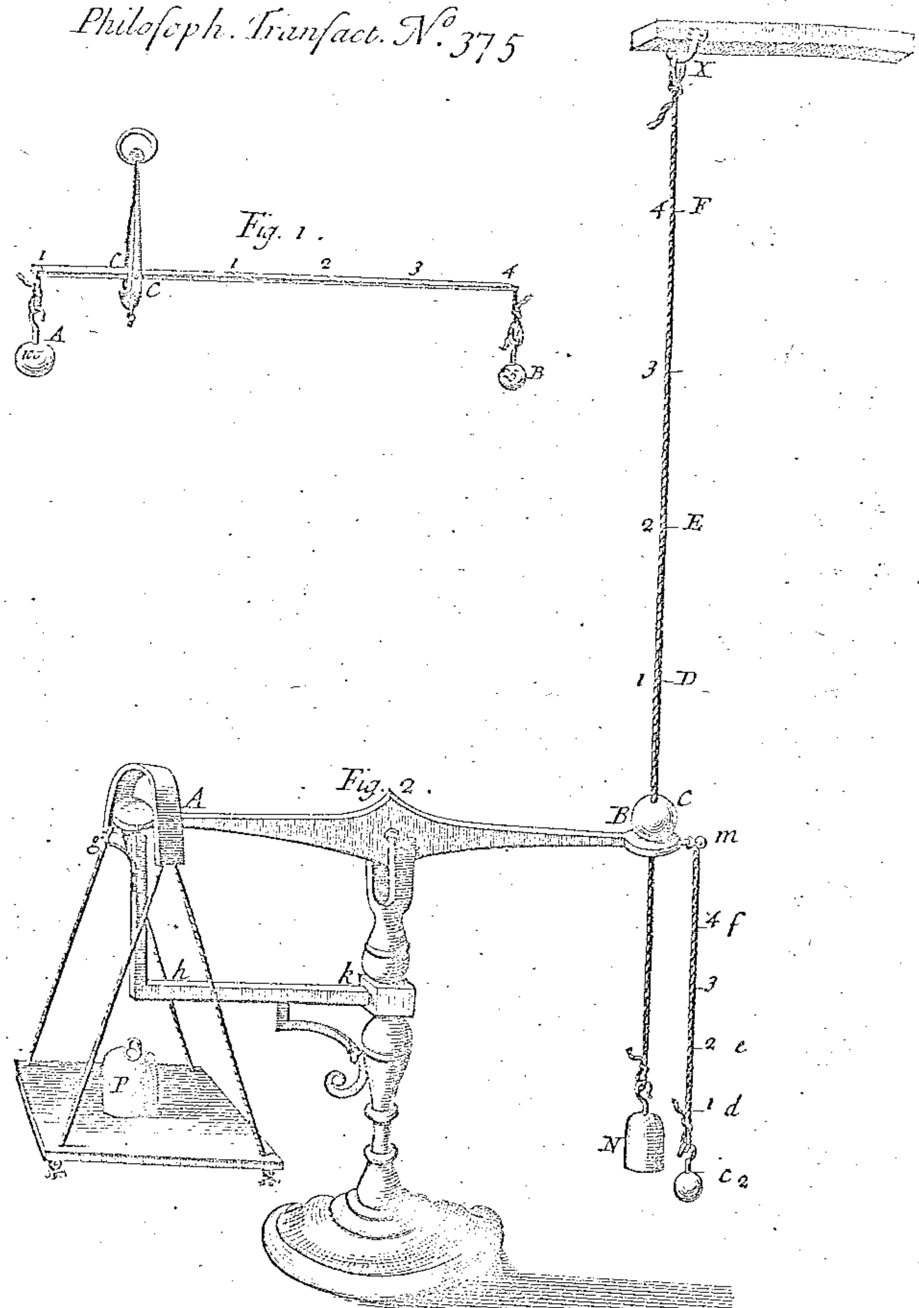

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$[s = \frac{1}{2}at^2]$, the Velocity in the latter Case is double that of the first $[Ft = mv]$, and consequently the Blow, that the Body will give, will be double."[9]

This defines the "Blow" as the body's momentum, *mv*. Here, unlike Leibniz's "Brief Demonstration", the comparison is made between different heights for the *same* falling weight. Although Desaguliers is attempting to refute Leibniz by referring to the times of fall rather than the distances, he actually has altered the problem by describing the momentum acquired by a falling body.

The experiments of 's Gravesande appearing in his *Mathematical Elements of Natural Philosophy* are repeated here by Desaguliers as confirmation that the "Congress of Elastic Bodies" shows that the "momentum of Bodies is in Proportion to the Mass multiplied into the Velocity...as demonstrated by Issac Newton in his *Principia*." [10]

A second paper by Desaguliers in the following issue of the *Transactions* criticized Poleni's experiment using bodies falling from different heights into soft clay.[11]

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[9] *Ibid.*, 272.

[10] *Ibid.*, 275-278.

[11] John Theophilus Desaguliers, "Animadversions upon some Experiments relating to the Force of Moving Bodies; with two new Experiments on the same subject." (March, April, 1723 issue of *Phil. Trans.* *32* (1724) 285-290.

According to Desaguliers the mistake made by Poleni was in estimating the force of the stroke of the falling balls by the depth of the impression in the yielding substance. Instead one must consider "That when two Bodies [of different weights] move with equal Forces [*mv*] but different velocities, that which moves the swiftest must make the deepest Impression..."[12]

An experiment was designed to illustrate this point as follows (see diagram, Fig. I, p. 197): An apparatus was constructed which consisted of a horizontal base on which stood two vertical parallel boards 4 inches apart. Between these boards, placed as horizontal shelves, were six evenly spaced wooden frames across each of which a paper diaphram (C) was extended. [These diaphrams served a function similar to the soft clay of Poleni.] From a support (F) a hollow ivory ball weighing 1½ ounces was suspended by a thread 4 feet above the first diaphragm. When the thread was cut the falling ball broke through 4 of the paper diaphragms. [$ws = \frac{3}{2}(4) = 6$; $mv = 3$]

The hollow ball was then filled with lead such that it weighed twice as much as before and was allowed to fall from a height of 1 foot. This time it broke only two diaphragms. [$ws = 3(1) = 3$; $mv = 3$, the forces, or *mv*'s thus

[12] *Ibid.*, 286.

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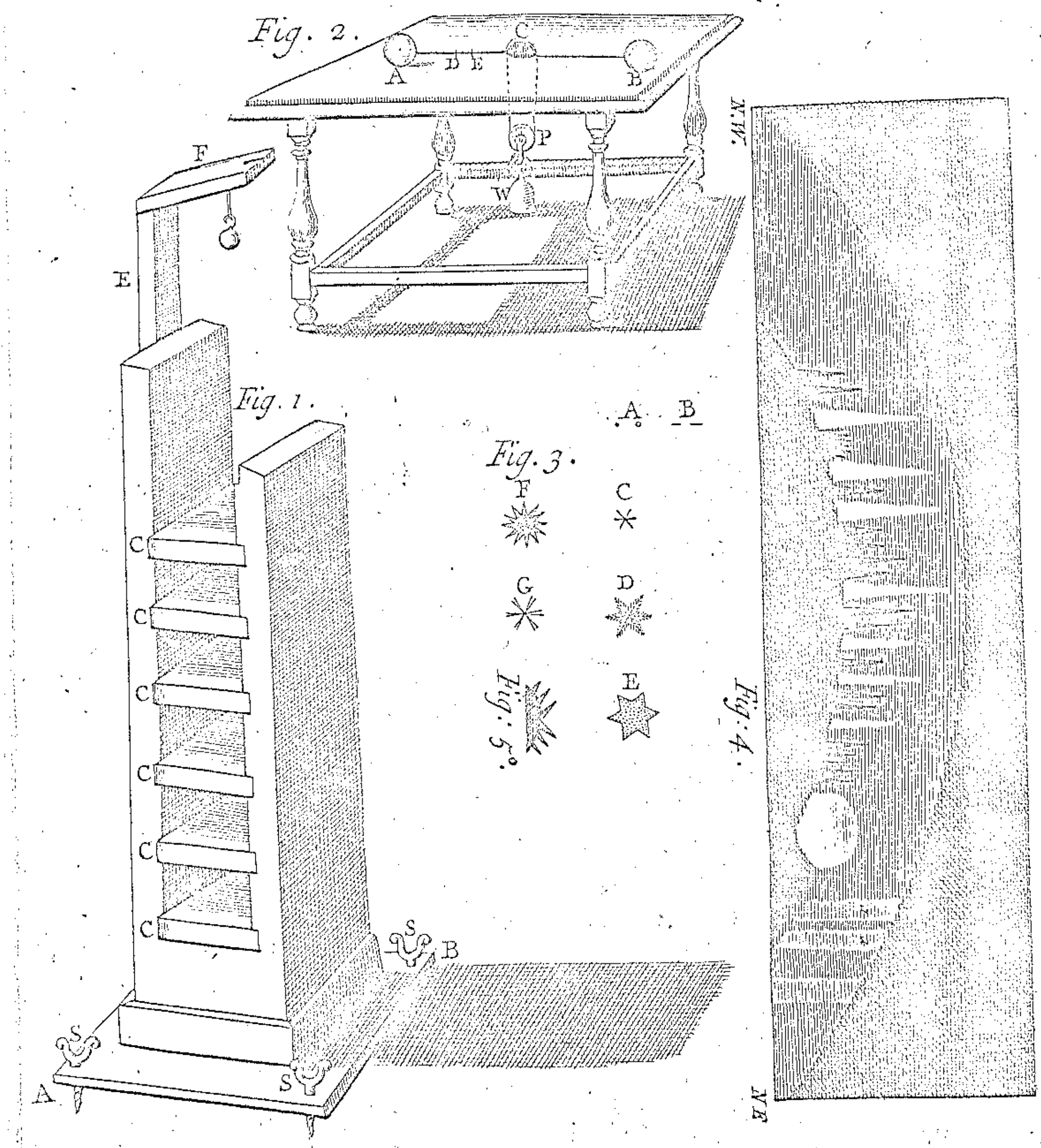

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being equal as stated above.] Upon repetition of the experiment using different heights whose proportion was 4 to 1, it was found that when the weight of the balls were in the ratio of 1 and 2, the heavy and slowest ball [$w = 3$; $v = 1$] broke through but half the number of papers.

In this experiment both balls, of weights in the ratio of 1:2 falling through heights in the ratio of 4:1, hit the first paper diaphragm with equal momenta [$mv = (1)(2) = (2)(1)$]. The *vis viva* of the lighter [$w = 1$; $mv^2 = 1(2)^2 = 4$] is double that of the heavier [$w = 2$; $mv^2 = 2(1)^2 = 2$]. Thus this experiment is not identical with that of Leibniz's "Brief Demonstration". The time of fall ($s = \frac{1}{2}gt^2$) of the lighter, falling through $s = 4$ is double that of the heavier falling through $s = 1$. The times of fall are independent of the weights but dependent on the heights. Thus the number of diaphragms broken will be in the same ratio as that of the times or 2:1, where the heights are in the ratio of 4:1. The lighter ball then has greater *vis viva* than the heavier, but they have equal momenta.

Desaguliers concludes as follows:

> Now tho' this Experiment does at first seem to confirm Polenus's Theory; yet; when duly weigh'd, it proves no such thing. For the lighter Ball does not break thro' more Papers, because it has more Force, or a greater Quantity of Motion, but because each Diaphragm has but half the time to resist the Ball that falls with a double Velocity, and therefore their Resistance being as the time, as many more of them must be broken by the swift Ball as by the Slow one.[13]

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[13] *Ibid.*, 288.

The import of Desaguliers' paper seems to be the following: If force is measured by mv, then if the mv of two falling bodies are equal, the depth of the impressions or the number of diaphragms broken are unequal. Thus the depth of the impression cannot be used as a measure of the body's force. (see p. 196.)

Here then the problem is one of definition. If force is defined as mv^2 (Leibniz) then the depth of the impressions are equal for equal forces because mv^2 depends on the height. But if force is defined as mv (Desaguliers) then for equal forces the impressions are not equal.

A contribution of 1726 by John Eames[14] reiterated the conclusion of the two preceding authors (Pemberton, 1722; Desaguliers, 1723) that the proper use of an experiment such as Poleni's was to discover the laws of resistance which soft or yielding substances make to bodies moving in them and not to discover the force itself of the moving bodies. Again this was an attempt to reduce a vis viva problem to a momentum problem.

Eames' paper was a refutation of a statement made by /Muschenbroek7 professor at Utrecht that force was as the proportion of the mass and the square of the velocity

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[14] John Eames, a "Remark upon the New Opinion relating to the Forces of moving Bodies, in the case of the Collision of Non-Elastic Bodies," November, December, 1726 issue of Phil. Trans. 34, (1728) 183-187.

for the case of non-elastic bodies:

> A variety of experiments have been made and reasoning used in England and France to prove the truth of the common opinion; but they do not entirely satisfy all the gentlemen on the other side of the question. The present ingenious Professor of Mathematics and Philosophy at Utrecht tells us in the Preface to his Epitome Elementorum Physico-Mathematicorum, published this year 1726.... Et quando experimenta a' Poleno et 's Gravesandio descripta examinantur et inspiciuntur, tam manifesto evincunt vires corporum percutientium esse in ratione composita ex quadrata velocitatum, et simplici massarum, ut illis subscribere teneamur, nisi apertissumis contradicere studeamus.
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> I beg leave to examine the truth of the new Opinion in the Case here proposed, viz, Vires corporum percutientium; and I shall endeavour to show from their own Principles, that it cannot be true in all the Cases of Non Elastic Bodies.[15]

Based on the rule for finding the common velocity of non-elastic bodies as stated by 's Gravesande, Eames finds the force, mv^2, for two cases of inelastic bodies A and B containing equal quantities of matter. In the first case B is at rest, and A moves toward it with 8 degrees of velocity. Using 's Gravesande's rule that the common velocity after the collision is found by dividing the sum of the quantities of motion by the sum of the quantities of matter, [$v' = \frac{Aa + Bb}{A + B}$][16] the common velocity is found to be 4. The force after the stroke would then be as the square of that velocity or 16.

[15] *Ibid.*, 184.

[16] See 's Gravesande, "New Theory on Collision," (1722) 39 and this dissertation, Ch. V, p. 177.

In the second case B moves forward with velocity 2 and A follows with velocity 10, thus retaining the same relative velocity, 8, as in case 1, and the same force of collision. The common velocity after collision will then be 6, or half the sum of the initial velocities. Now the force, mv^2, of B before the collision will be as its velocity squared or $2^2 = 4$ and the force after will be $6^2 = 36$. The force communicated by the collision will be the difference or $36-4 = 32$. This is double the effect communicated by the same force in case 1 and shows that if the force were as the mass times the velocity squared, "equal strokes would produce unequal Effects." A table was given showing additional cases to prove that inelastic collisions produced unequal effects.

If Eames' argument is interpreted to mean that living forces are not conserved in inelastic collisions, it is of course quite correct in its intent.

'S Gravesande however in his "New Theory of Collision" did not claim that forces, mv^2 were the same before and after an inelastic collision. The derivation of the expression, $v' = \frac{Aa + Bb}{A + B}$, had already taken into account the force lost in collision. It was simply an expression for the common velocity of the two bodies.

A second paper by Eames in the same issue was a discussion of a proof by John Bernoulli based on the composition and resolution of forces showing that forces are as the squares of the velocities.[17]

Here Eames shows that "far from proving that side of the Question for which it was brought, this demonstration will equally serve to prove the truth of the other, namely that the Forces of the same Body moving with different Velocities are as those Velocities".[18] His conclusion is: "Since therefore this Proof drawn from the Doctrine of Composition and Resolution of Forces equally proves both sides of the Question it proves too much, or in reality nothing at all; and is therefore far from deserving the Name of a Demonstration."[19]

In showing that both sides of the question could be proven valid Eames had hit upon a fruitful method of attack. Yet in merely concluding that the demonstration was not really a proof, he fell short of d' Alembert's insight that both measures of force were actually correct.

Also appearing in the Philosophical Transactions,

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[17] John Eames, "Remarks upon a supposed Demonstration, that the moving Forces of the same Body are not as the Velocities, but as the Squares of the Velocities," November, December, 1726 issue of Phil. Trans., 34 (1728), 188-191.

[18] Ibid., 190

[19] Ibid., 191.

1728, was a paper by Samuel Clarke[20] who eleven years before in 1716 had argued with Leibniz against the conservation of living force. This was one of the bitterest attacks the controversy produced. An interesting illustration of this purely polemical aspect of the controversy is provided by the following statement from the pen of Mr. Clarke:

> It has often been observed _in general_ that Learning does not give men _Understanding_; and that the absurdest things in the world have been asserted and maintained by persons whose education and studies should seem to have furnished them with the greatest extent of Science.
>
> That knowledge in many _languages_ and _Terms of Art_ and in the _History of Opinions_ and _Romantick Hypotheses_ of Philosophers, should sometimes be of no effect in correcting Men's _Judgment_, is not so much to be wondered at. But that in _Mathematicks_ themselves, which are a _real Science_, and founded in the _Necessary Nature_ of _Things_; men of very great abilities in abstract computations, when they come to _apply_ those computations to the _Nature of Things_, should persist in maintaining the most _palpable absurdities_, and in refusing to see some of the most _evident and obvious truths_; is very strange.
>
> An extraordinary instance of this, we have had of late years in very eminent Mathematicians, Mr. _Leibniz_, Mr. _Herman_, Mr. _'s Gravesande_, and Mr. _Bernoulli_; (who in order to raise a _Dust of Opposition_ against Sir _Issac Newton's_ philosophy, the glory of which is the _application of abstract Mathematics_ to the real _phenomena of Nature_,) have for some years insisted with great Eagerness, upon a principle which subverts all Science, and which may easily be made to appear... to be contrary to the _necessary and essential Nature of Things._

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[20] Samual Clarke, "A letter from the Rev. Dr. Samuel Clarke to Mr. Benjamin Hoadly, F.R.S. occasion'd by the present Controversy among Mathematicians, concerning the Proportion of Velocity and Force in Bodies in Motion," _Phil.Trans._, _35_ (1728), 381-389.

> What they contend for is, That the Force of any Body in Motion is proportional, not to its Velocity, but to the Square of its Velocity.
> The Absurdity of which Notion I shall first make appear and then shew what it is that has led these gentlemen into Errour.[21]

In this 1728 paper Clarke argued that "In the Nature of Things... every Effect must necessarily be proportionate to the cause of that Effect; that is to the Action of the Cause or the Power exerted at the Time when the Effect is produced. To suppose any Effect proportional to the Square or Cube of its Cause, is to suppose that an Effect arises partly from its Cause and partly from Nothing."[22]

With regard to a body in motion, the portion of the force arising from the quantity of matter as its cause is necessarily proportional to its quantity of matter, and the force arising from the velocity is proportional to its velocity. "If the Forces were as the Square of the velocity, all that part of the Force which was above the [simple] Proportion of the Velocity would arise either out of Nothing or (according to Mr. Leibniz's Philosophy) out of some living soul essentially belonging to every Particle of Matter".[23]

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[21] Ibid., 381, 382.

[22] Ibid., 383.

[23] Ibid., 383-384.

Clarke cites the importance of considering the time by saying that the space described by a body in motion is not as the force alone but as the force and the time taken together.[24] A body thrown upwards with *double* the force [i.e. $2v$] will be carried *four times* as high before its motion be stopped by the uniform Resistance of Gravity; because the *double* Force will carry it *twice as high* in the *same* Time and moreover require *twice the Time* for the uniform Resistance to destroy the Motion."[25] That is, if a body is thrown upwards with velocity v rising to height h in time t, then if its velocity were $2v$ in $2t$ it would rise not $2h$ but $4h$. "The space described must needs be as the *Force*, and as the *Time wherein* the Force operates." Thus the "Force" is in the proportion of the space applied to the time, or as $\frac{4}{2}$ to $\frac{1}{1}$ or 2 to 1. Here Clarke is comparing the momenta of the same body (or two bodies of the same mass) thrown upwards with different initial velocities: $(m)(2v) : (m)(v)$. Leibniz's argument would be to compare the *vires vivae* of the same body with these same initial velocities: $(m)(2v)^2 : (m)(v)^2$. Both would be correct depending on the meaning given to the word "force".

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[24] *Ibid.*, 384.

[25] *Ibid.*, 385.

The same argument was given in Clarke's fifth reply in his correspondence with Leibniz.[26]

> In this the Cartesians and other Philosophers and Mathematicians agree; all of them making the impulsive Forces of Bodies proportional to their Motions, and measuring their Motions by their Masses and Velocities together, and their velocities by the Spaces which they describe, applied to the Times in which they describe them. [$v = s/t$] If a body thrown upward does by doubling its Velocity, ascend four Times higher in twice the Time, its impulsive Force will be increased, not in the proportion of the Space described by its Ascent but in the Proportion of that Space applied to the Time, that is in the proportion of [s/t] $\frac{4}{2}$ to $\frac{1}{1}$ or 2 to 1.[27]

Thus Clarke in the above argument is again comparing the momenta of the two equal bodies, $2mv$ and mv. Leibniz's argument would compare the living forces, $4mv^2$ and mv^2. Clarke is using the relationship $mv = mgt$, while Leibniz's argument is based on the equation $mgs = [\frac{1}{2}]\, mv^2$.

A second and similar argument in this same "Fifth Reply" goes as follows:

> The Force required to make body B of four pounds rise up 1 yard will make body A of 1 pound weight rise up (not 4 yards as Mr. Leibniz represents) but 16 yards in quadruple the time. For the gravity of 4 pounds

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[26] Samuel Clarke, A Collection of Papers Which Passed Between the Late Learned Mr. Leibniz and Dr. Clarke, "Dr. Clarke's Fifth Reply", London, 1717, footnote to secs. 93-95, pp. 327-339. For a discussion of this see Alexander Koyré and I. Bernard Cohen, "Newton and the Leibniz-Clarke Correspondence", Archives internationales d'histoire des sciences, 15 (1962) 63-126.

[27] Clarke's "Fifth Reply," op. cit., 333, footnote.

> weight in one part of time acts as much as the gravity of one pound weight in four parts of time.[28]

Here Clarke is misrepresenting Leibniz's argument for the time in which the one pound weight would rise four yards is not quadruple the time but double the time. In quadruple the time it would, of course, rise 16 yards.

A puzzling result of the measure of force in this free fall problem, on the weight of the body, invalidated in the minds of Clarke and Newton, the Leibnizian argument originally derived from the free fall case. Clarke says that Hermann in his Phoronomia (p. 113) agreed with Leibniz and argued that the Cartesian idea that the forces of falling bodies are proportional to the times of falling was based on a false hypothesis. This false hypothesis was that bodies thrown upward received from the gravity resisting them an equal number of impulses in equal times. Clarke interpreted Hermann to mean

> that the swifter the motion of bodies is upwards, the more numerous are the impulses, because the Bodies meet /more of/ the (imaginary) Gravitating Particles. And thus the Weight of Bodies will be greater when they move upwards and less when they move downwards. And yet Mr. Leibniz and Mr. Hermann themselves allow that gravity in equal Times generates equal Velocities in descending Bodies and takes away equal Velocities in ascending Bodies; and therefore is Uniform. In its action upon bodies for generating Velocity they allow it to be uniform; in its action upon them of generating impulsive Force, they deny it to be uniform. And so are inconsistent with themselves.[29]

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[28]Clarke, "Fifth Reply", op. cit., 335, footnote.

[29]Ibid., 337.

What did Clarke mean by this? He showed how gravity would be non-uniform for Leibniz by following Galileo's treatment of falling bodies ($2gs = v^2 - v_o^2$).[30] But where Galileo uses the term velocity, Clarke uses the term "force". Then he equates "force" with the "action of gravity" which generates it in order to show that given Leibniz'd dependence of force, $\underline{mv}^2$, on space, that gravity varies non-uniformly as the velocity of the falling body. In the interchanging of the terms <u>velocity</u>, <u>force</u>, and <u>action of gravity</u> lies the speciousness of the argument, for the force of gravity does not vary with velocity. Here is Clarke's line of thought.

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[30] Galileo Galilei, <u>Two New Sciences</u>, trans. Henry Crew and Alfonso de Salvio, New York, 1914 edition, 161. "...we may, in a similar manner, through equal time intervals conceive additions of speed as taking place without complication; thus we may picture to our mind a motion as uniformly and continuously accelerated when during any equal intervals of time whatever, equal increments of speed are given to it. Thus if any equal intervals of time whatever have elapsed, counting from the time at which the moving body left its position of rest and began to descend, the amount of speed acquired during the first two time intervals will be double that acquired during the first time interval alone; so the amount added during three of these time intervals will be treble; and that in four quadruple that of the first time interval...a motion is said to be uniformly accelerated when starting from rest, it acquires during equal time-intervals, equal increments of speed."

If the "Force" of a falling body is as the space, as Leibniz says, then in the first part of a series of equal times, the body will gain one part of force, [v^2 = 1], in the first two parts of time it will gain four parts of force, [v^2 = 4], in the first three parts it will gain nine parts of force [v^2 = 9], etd. Consequently during the second moment it gains five parts of force, 4-1 = 3, in the third moment it gains five parts, [9-4 = 5] , in the fourth it gains seven parts [16-9 = 7] etc. Now if the <u>action of gravity</u> for generating these forces be supposed in the middle of the second, third, and fourth parts, be of three, five, and seven degrees etc. The force of gravity "will be proportional to the time and to the velocity acquired. And by Consequence in the Beginning of the Time it [the action of gravity] will be none at all and so the body for want of gravity will not fall down. And by the same way of arguing when a body is thrown upwards its gravity will decrease as its velocity decreases and cease when the body ceases to ascend, and then for want of gravity, it will rest in the Air, and fall down no more. So full of absurdities is the Notion of the Learned Author in this particular."[31]

It has been recently discovered by Koyré and Cohen that the above argument was drafted by Newton himself. Fragments were located in the Newton manuscript collection indicating that Newton played a far greater role in the argument with Leibniz than had heretofore been suspected. Much of the language used by Clarke in this particular argument comes verbatim from replies drafted by Newton.

Newton, for example, expressed the above argument in the following two fragments, like Clarke equating the terms weight and gravity with force or velocity, thereby making the weight of a body proportional to its time of fall.

> And upon these rules of ascending and descending Galileo demonstrated that projectiles would in spaces void of resistance describe parabolas. And all Mathematicians (not excepting Mr. Leibniz himself) unanimously agree that he was in the right. And yet Mr. Leibniz would have us measure the force imprest, not by the velocity generated to which it is proportional but by the space of ascent to which it is not proportional.[32]

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[31] *Ibid.*, 338-339, footnote.

[32] Koyré and Cohen, *op. cit.*, 118.

> The weight or gravity of the body which by its action impresses these impulsive forces upon the body acts with three times more force in the second part of the time than in the first and with five times more force in the third part of the time than in the first and with seven times more force in the fourth part of the time than in the first and so on. Which is as much as to say that the falling body grows heavier and heavier as it falls, and becomes three times heavier in the middle of the third part of the time than in the middle of the first and so on. Or that the weight of the body is proportional to the time of its falling. And by consequence that in the beginning of the first part of the time the body hath no weight at all. Which is contrary to the hypothesis of uniform gravity and to experience itself.[33]

Excerpts from another fragment in the Newton manuscript collection show where Clarke derived his argument that Leibniz's one pound weight would be thrown 16 times as high as his four pound weight by the same force.

> The reason of his inconsistency in this matter was his computing by a wonderful unphilosophical error the quantity of impulsive force acquired by a falling body from the quantity of its matter and of the space described by it in falling,[34] reckoning the force acquired to be in a compound ratio of the matter and the space together. Now matter is as the weight thereof, and the space described is as the square of the time of its falling and therefore according to Mr. Leibniz the force acquired in falling is in a compound ratio of the weight of the falling body and the square of the time of its falling.
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> ......

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[33] Koyré and Cohen, op. cit., 119.

[34] Cf. Clarke, "Fifth Reply," op. cit., 329. "The reason of his Inconsistency in the Matter, was his computing by a wonderfully unphilosophical Error, the Quantity of Impulsive force in an Ascending Body from the Quantity of its Matter and of the Space described by it in Ascending without considering the Time of its ascending."

...a body therefore of one pound weight is not (as Mr. Leibniz supposes in the Acta Eruditorum ad annum 1686 pag. 162) thrown in vacuo four times as high but sixteen times as high by the same quantity of impulsive force wherewith a body of four pound weight is thrown one foot high.[35]

Newton must have assumed that Leibniz's one pound weight fell in time four from height four rather than in time two. In following Newton, Clarke carried over the same error to his 1728 paper.

In his "Fifth Reply," Clarke like the Cartesians made the mistake of confusing the virtual velocities of a body, mdv, with quantity of motion or force as shown by this statement: "And if equal bodies librate upon the arms of a Balance, at various distance from the axis of the balance, the Forces of the Bodies will be in Proportion as the Arches described by them in librating, because they librate in the same times."[36]

In his 1728 paper Clarke also answered the experimental arguments of Poleni and 's Gravesande:

> When a Body projected with a double Velocity, enters deeper into snow or soft clay or into a heap of springy or elastic parts, than in proportion to its Velocity; t'is not because the Force is more proportional to the Velocity; but because the Depth it penetrates into a soft Medium, arises partly from the Degree of the Force or Velocity, and partly from the Time wherein the Force operates before it be spent. [$s = \bar{\bar{v}}t$; $F = mv$][37]

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[35] Koyré and Cohen, op. cit., 119.

[36] Ibid., 331.

[37] Clarke, 1728, op. cit., 387.

Clarke's final argument in his 1728 paper involves the conservation of force, (i.e. momentum) referring back to his 1716 letters to Leibniz. In the collision of hard inelastic bodies, he says, when a moving body hits another of the same size at rest, both will move on together dividing the motion between them. However if the balls are perfectly elastic the moving ball will communicate all its motion to the one at rest and will itself come to rest in the other's place.

> Were it now true that the *Force* of the *moving Ball* was as the *Square of its velocity*; these Experiments would then show (which is infinitely absurd) that the *Force* or *vis inertiae* in the *quiescent Ball*, the *dead* Force, was always proportional to the *Square of the Velocity* (which these gentlemen affect fantastically to call the living Force) of the *moving* Ball, whatever its velocity were. Or the *Force* in *Both* might just as reasonably be supposed to be as the Cube or the *quadratoquadrate* or any *other Power* of the Velocity of the *moving* Ball. Which is turning the *Nature of Things* into Ridicule. ...But from the Experiments now mentioned 'tis evident that if the *Force* of Bodies in Motion could be exalted even to the *infinit 'th Power* of their *Velocity*; yet since to answer the Phenomena of Nature with Regard to *Action* and *Re-Action*, the same *Force* must necessarily be allowed to all *quiescent* bodies likewise, it could be of *No Effect*.[38]

In other words if force were measured by mv^2 then in a collision each of the two bodies should retain their mv^2 as a measure of their force even if at rest.[39] Further to maintain the equality of action and reaction, a *resting*

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[38] *Ibid.*, 387-388. Italics Clarke's.

[39] This would be true if mv^2 were stored as heat or other energy.

body should have a force calculated by the same measure as a moving body, i.e. by mv^2. Now Clarke had maintained in his 1716 correspondance with Leibniz that two inelastic bodies colliding with equal forces lost all their motion, implying that this was an example of the diminution as opposed to the conservation of force in the universe. He refers to these letters in the above argument.[40] Leibniz had answered that in the collision of two soft or in-elastic bodies although quantity of motion did not remain the same (meaning $m|v|$) the forces (mv^2) were lost only in appearance. For "the wholes lose it with respect to their total motion, but their parts receive it, being shaken internally by the force of the concourse....The bodies do not lose their forces, but the case here is the same as when men change great money into small."[41]

To this Clarke had replied that the problem lay not with soft inelastic bodies but with hard inelastic bodies.

> But the question is; when two perfectly HARD unelastic bodies lose their whole motion by meeting together, what then becomes of the motion or active impulsive force? It cannot be dispersed among the parts, because the parts are capable of no tremulous motion for want of elasticity.[42]

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[40] Samual Clarke, "Clarke's Fourth Reply", The Leibniz-Clarke Correspondence, edited H. G. Alexander, Manchester, 1956, 52, sec. 38. For the correspondence see also Gerhardt, P.S. VII, 345-440.

[41] Leibniz, "Fifth Letter to Clarke,"Alexander, 87,sec.99.

[42] Clarke, "Fifth Reply," Alexander, 111, sec. 99.

In his "Fifth Reply" Clarke had also explained the vis inertiae of matter(ref. p. 213): "The very same force which is requisite to give any certain velocity to any certain quantity of matter at rest, is always exactly requisite to reduce the same degree of velocity to a state of rest again. This vis inertiae is always proportional to the quantity of matter; and therefore continues invariably the same...whether at rest or in motion; and is never transferred from one body to another.... So that properly and indeed all force in matter either at rest or in motion, all its action and reaction, all impulse and all resistance, is nothing but this vis inertiae in different circumstances."[43]

Active impulsive force is always proportional to the quantity of relative motion, that is, proportional to the quantity of matter and the velocity (not to the quantity of matter and the square of the velocity.) This active force, so defined,

> does naturally diminish continually in the material universe.[44] That this is no defect is evident; because 'tis only a consequence of matter being lifeless, void of motivity, unactive and inert. For the inertia of matter causeth...that solid and perfectly hard bodies void of elasticity, meeting together with equal and contrary forces, lose their whole motion and active force....and must depend upon some other cause for new motion.[45]

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[43] Ibid., Footnote to sec. 99. Alexander, 111,112.

[44] Although Clarke's measure of force, mv, agrees quantitatively with Descartes' quantity of motion, Clarke does not agree with Descartes that this force is conserved in the universe.

[45] These arguments were taken directly from Newton with whom Clarke had consulted during the correspondence and who

Although Clarke's final argument at the close of his 1727 paper is obscure, the issue seems to be that if the law of action and reaction were to be obeyed in collisions of bodies, elastic or hard inelastic, all resting bodies would have to have a force of $\underline{mv}^2$, if the force of the moving bodies was measured by $\underline{mv}^2$. But force is not conserved in hard inelastic collisions and hence is not

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had written in his Queries to Book III of the Optics:

> The Vis inertiae is a passive Principle by which Bodies persist in their Motion or Rest, receive Motion in proportion to the Force impressing it, and resist as much as they are resisted. By this Principle alone there never could have been any Motion in the World. Some other Principle was necessary for putting Bodies into Motion; and now they are in Motion. Some other principle is necessary for conserving the motion.* For from the various composition of two Motions, 'tis very certain that there is not always the same quantity of Motion in the World.... By reason of the Tenacity of Fluids, and Attrition of their Parts, and the Weakness of Elasticity in Solids, Motion is much more apt to be lost than got, and is always on the Decay. For Bodies which are either absolutely hard, or so soft as to be void of Elasticity will not rebound from one another. Impenetrability makes them only stop. If two equal bodies meet directly in vacuo, they will by the Laws of Motion, stop where they meet, and lose all their Motion and remain at rest, unless they are elastick, and receive new Motion from their Spring... Seeing therefore that the variety of Motion which we find in the World is always decreasing, there is a necessity of conserving and recruiting it by active Principles such as are the cause of Gravity, by which Planets and Comets keep their Motions in their Orbs, and Bodies acquire great Motion in falling, and the Cause of Fermentation..."

(Issac Newton, Opticks, New York, Dover Publication, 1952, 4th ed., 1730, 397-399, Book III, query 31. *Contrast Descartes, "Principia Philosophiae," Oeuvres, 8, op. cit., Principle 36, "God created matter along with motion and rest in the beginning; and now, merely by his ordinary cooperation, he preserves just the quantity of motion and rest in the material world that he put there in the beginning."

conserved in the universe except when God periodically adds new force to prevent it from running down. Hence $\underline{mv}^2$ is not the measure of force.

As already pointed out, Leibniz's reply maintained that the force of the universe was constant. But if this is the case it logically follows from Clarke's own argument that force should be measured by $\underline{mv}^2$.

In the Leibniz-Clarke Correspondence the dual between the Leibnizians and Cartesians becomes a threefold issue which now includes the Newtonians. Descartes' quantity of motion, mv, was the same quantitative measure of force as Newton's quantity of motion, mv. Newton's measure however took into account the sign of the velocity, and late in the seventeenth century was referred to as momentum, the word "force" in Newtonian mechanics, being reserved for the time rate of change of momentum. But Descartes held that mv was conserved in the universe while Clarke and Newton argued that the "force" of the universe was continually decreasing due to the loss of mv in hard inelastic collisions (and due to aberrations in the planetary motions). On the other hand Leibniz, like Descartes, argued for conservation but maintained that $\underline{mv}^2$ was the measure of force.

The Leibniz-Clarke Correspondence had originated with a similar issue. Leibniz had asserted that a God who did not have to intervene in the workings of the world after its creation was more powerful than one who did:

> Sir Issac Newton and his followers have also a very odd opinion concerning the work of God. According to their doctrine, God Almighty wants /i.e. needs/ to wind up his watch from time to time, otherwise it would cease to move. He had not it seems, sufficient foresight to make it a perpetual motion. Nay the machine of God's making is so imperfect according to these gentlemen that he is obliged to clean it now and then by an extraordinary concourse, and evem to mend it as a clockmaker mends his work...and to set it right. According to my opinion the same force and vigor remains always in the world and only passes from one part of matter to another, agreeably to the laws of nature and the beautiful pre-established order. And I hold that when God works miracles he does not do it in order to supply the wants of nature but those of grace. Whoever thinks otherwise must needs have a very mean notion of the wisdom and power of God.[46]

The basis of Clarke's reply is that God is more powerful if he is involved in the operation of the world; otherwise God is not necessary:

> And if God, or Man or Any Living or Active Power, ever influences any thing in the material world; and everything be not absolute Mechanism; there must be a continual Increase and Decrease of the whole Quantity of Motion in the Universe. Which this Learned Man frequently denies.[47]

Here then is a second similarity between Leibniz and Descartes. God's power for them lay in his creation and conservation of the world without day to day interference. The issue of conservation versus non-conservation

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[46] Leibniz, The Leibniz-Clarke Correspondence, "Leibniz's First Paper", Alexander, op. cit., 11, 12.

[47] Ibid., 327.

in addition to that of mv versus mv^2 will be seen in the arguments of the Leibnizians, Cartesians, and Newtonians as they appeared in the publications of the French Academy.

Clarke's 1728 paper had begun with a polemic against the defenders of mv^2. Among the names listed was Mr. 's Gravesande who was accused of "raising a dust of opposition against Sir Issac Newton" by insisting on "a principle which subverts all science".[48]

'S Gravesande replied to Clarke's attack on his intellectual honesty with a two part discussion in the Journal Literaire for 1729: "Remarks on the Force of Bodies in Motion and on Collision; preceded by some Reflections on the Manner of Writing of Doctor Samuel Clarke."[49]

"Most of those," he said, "who have attacked my writings have observed...all the rules of honesty" and their conduct has been most honorable...". I have worked in another manner in order to clarify the truth as much as I am capable of. In writing I have tried to propose my arguments in a manner which can best serve to resolve the difficulties; or better I have examined the difficulties

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[48] Clarke, 1728, op. cit., 382; see this chapter, p.

[49] William Jacob 's Gravesande, "Remarques sur la force des corps en mouvement et sur le choc; precedées de quelques reflexions sur la manier d'écrire de Monsieur le Docteur Samuel Clarke," Journal Literaire, La Haye, 13 (1729) pt. I, 189-197, and 13, pt. II, 407-430.

without responding directly to those who were the authors of them."[50]

Clarke had, he wrote, attacked his understanding, and accused hin of "the most palpable absurdities" and of refusing to admit "the most evident and obvious truths." Furthermore he, 's Gravesande, was accused of doing this in order to raise a "dust of opinion" against Newton.

'S Gravesande replied that "Monsieur Clarke had not preached morality all his life without knowing that to write not in order to clarify the truth but to obscure discoveries as beautiful as those of M. Newton, is not a behavior suitable to an honest man. It is not possible that such an accusation should fail to make an impression on the minds of those who know how great a reputation Clarke has acquired in the direction of theology and morality; and...it would be a pity to suggest that Clarke, (raising a "dust of opposition" to his own honesty,) had put forth something not in accord with the truth."[51] After all, "the point in question [that of living forces] was touched upon by Newton only in passing...Who could imagine that in writing something new on this subject one would wish to obscure the glory of M. Newton?"[52]

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[50] _Ibid._, 189,190.

[51] _Ibid._, 193, 194.

[52] _Ibid._, 194.

In Part II of this article in the Journal Literaire, 's Gravesande discusses the equivocation of the word "force".[53] "Most philosophers," he writes, "say that motion is the transporting of a body from one place to another or the successive application of the perimeter of a body to different parts of space (l'etendue). [i.e. as mv] ...Other philosophers regard the transport of a body as an effect of motion and not as motion itself. [i.e. as mv^2] and considering that whatever causes a body to be transported is the same as that which renders it capable of acting on an obstacle, Motion is confounded with the Ability to Act."[54]

"But in whatever manner one considers motion, everyone agrees that it is accompanied by what one calls "force". But here is a most equivocal word. What is Force?"[55]

One sees the effect of force in the meeting of two bodies and this effect is the action of one body on another. But the idea of action implies that of resistance or contrary action. Action and resistance express the same thing conceived differently. While one body acts, or resists so that it loses motion, the other body whose motion is augmented

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[53] William Jacob 's Gravesande, "Remarques sur la force des corps en mouvement et sur le choc; suite de l'Article XIII du Journal precedent," Journal Literaire, La Haye, 13(1729) pt. II, 407-432.

[54] Ibid., 407, 408.

[55] Ibid., 408.

resists less. Thus basically, increase and decrease of motion are only different ways of viewing the same change and this augmentation and diminution of motion is proportional to "force".[56]

A body in motion <u>acts</u> by its <u>force</u>. The same body having acquired motion, resists by its inertia (<u>inertie</u>). A body does not resist while resting but only while it is receiving motion; it never acts by force while it preserves its motion but only while its motion is diminishing. The word <u>force</u> which expresses this power to act (<u>Pouvoir d'agir</u>) is confused by some "philosophers" with motion itself (<u>Mouvement même</u>).[57]

Yet there is further confusion present in the literature. Another kind of action accompanies motion; an infinitely small motion or pressure (<u>pression</u>). This, writes 's Gravesande, had already been discussed in his 1722 paper. It has sometimes rendered the word "force" equivocal in the writings of some philosophers.[58]

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[56] <u>Ibid</u>., 409.

[57] <u>Ibid</u>., 411.

[58] <u>Ibid</u>., 411. See 's Gravesande, 1722, <u>op. cit</u>., p. 19, article V, "Differences entre Pression et Force". "1. La pression est infiniment petite en comparison de la force. 2. L'intensité de l'action d'une pression est determinée, et depend de la grandeur de la pression, l'intensité de l'action d'une force n'est point fixe, et elle dépend de la resistence que la force trouve, et qui peut etre plus or moins grande. 3. L'effet total d'une pression est indeterminé, et depend du tems pendant lequel elle agit. L'effet

'S Gravesande then goes on to explain the sense in which he uses the word "force". In an action carried out over a period of time, two things must be considered: (1) the size of the action in each infinitely small motion or "instantaneous action" (action instantanée) and (2) the size of the sum of all these small actions or the "total action" (action totale). "When the question is the measure of force, that is, the comparison of different forces, some people pay attention only to the "instantaneous action" and others [including 's Gravesande] consider the "total action".[59]

'S Gravesande explains his view of the measure of force as follows: "Total action is determined; a body having a certain degree of velocity...will lose it only in producing a determined effect which is always proportional to the square of the velocity"[60]... This can be proved directly

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total de l'action d'une force est determiné et est le même, quoi que le tems pendant lequel la force agit, soit plus ou moins étendu. 4. La pression étant un effort, il n'y a point de pression sans action contre un obstacle. La force est inherente au corps, quoi qu'il ne fasse point d'effort contre un obstacle, et elle demeure sans alteration aussi longtems qu'elle n'agit pas à surmonter quelque resistance. 5. La pression detruit souvent une pression contraire. La force ne detruit souvent une pression contraire. La force ne detruit jamais une force contraire, du moins immediatement.

[59] Ibid., 413.

[60] Ibid., 413.

by simple experiments. If action is proportional to effect, the total action is proportional to the total effect. If one calls force the total capacity to act, /la capacite totale d'agir7, that is, to produce an effect, then it cannot be denied that it is proportional to the square of the velocity multiplied by the mass.[61] If one uses for the definition of force, "total capacity to act" or to produce an effect, one sees that the capacity to produce an effect is proportional to that effect, and thus to mv^2.[62]

Those who have denied this measure of force, says 's Gravesande, have not paid attention to the sense which he, 's Gravesande, has given to the word "force". They have used it in another sense, saying that one must use the time during which the action lasts in determining the effect. It is in determining the "instantaneous action", that it is necessary to take the time into account. Therefore by "force", these critics really mean "instantaneous action" rather than "total action." They say that "force" is proportional to the velocity multiplied by the mass,

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[61] Cf. 's Gravesande, 1722, p. 20, "L'action de la force etant egale à la force que le corps perd par cette action, il est clair que les forces sont égales, dont les actions totales ne different pas; et en general que les forces sont en raison des actions, par lesquelles elles se consument entirement." ...."Prop. VIII, Dans les corps égaux les forces sont en raison des quarrez de leurs vitesses."

[62] Ibid., 413, 414.

and they add that a body of which the velocity is double and has the capacity to act during a double time, will produce a quadruple effect. If the velocity is triple, the body has the power to act during a triple time before having lost all its force. That is why the effect is increased ninefold.

However the two views can be reconciled. It is easy to see that the "total action" follows the proportion of the "instantaneous action" multiplied by the time during which it acts. $[mv\int_0^t dt]$ Thus if while increasing the velocity, one increases in the same ratio the instantaneous action $[mv]$ and the time $[t]$ that the action lasts, it is clear that one increases the total action in the "double ratio" of the velocity, $[mv^2]$. "Thus those who say that force when acting with equal masses is as the velocity, if by the word force they mean the capacity which produces "instantaneous action", say the same thing as those who maintain that force follows the square of the velocity [meaning] the <u>total capacity</u> (<u>le pouvoir total</u> $[mv^2]$ which produces the total action."[63]

"Instantaneous action" is infinitely variable; total action is determined. One can conceive the time during which the action lasts divided into an infinite number

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[63] *Ibid.*, 416.

of small parts each of which is multiplied by the "instantaneous action" during this moment. The total sum $[\Sigma\, mv\Delta t]$ of all these small products is the same, no matter in what way the "instantaneous force" has been varied. This sum is proportional to the "total force".[64]

In these passages 's Gravesande appears to be making a statement similar to that made by Leibniz in his "Specimen Dynamicum" (1695).[65] 'S Gravesande's "instantaneous action" is expressed quantitatively by mdv <u>mv</u>. Its integral over the time during which the action lasts gives the "total force", $\underline{mv}^2$ [i.e., $m\int_0^t \underline{vdt} = m\int_0^t \frac{ds}{dt}\,dt = \underline{ms}$ or $\underline{mv}^2$.]

'S Gravesande continues: "Those who wish to defend the ancient opinion (<u>l'ancien sentiment</u>) on the measure of

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[64] <u>Ibid</u>., 417.

[65] Cf. Leibniz, "Specimen Dynamicum" (1695) <u>op. cit</u>., Langley, 673. "Impetus is the product of the mass of the body into the velocity, and its quantity is so much that the Cartesians are wont to call it the quantity of motion, namely, the momentary increment (<u>momentaneam</u>), although, speaking more accurately, the quantity of motion itself, existing forsooth in time, arises from the aggregate of the impetuses (equal or unequal) existing in the moveable element in the given time multiplied in order into the time. We nevertheless in discussing with these have followed their fashion of speaking....as we may distinguish the present descent from the descent already made, which it increases; so we can discern and call Motion the momentary or instantaneous element of motion diffused by the motion itself through a period of time, and so that which is commonly ascribed to motion is called the <u>quantity of motion</u>." For a discussion of this see Ch. III, p. 94, and note 38.

force.... say that to have the "Force" of a body, it is necessary to multiply its mass by its velocity [mv] and that to have the total effect it is necessary to multiply the Force by the time that the action lasts [$mv\int_0^t dt$]."[66] It would appear in these passages that 's Gravesande is following Leibniz's statement that "the calculation of motion carried out through time is integrated from an infinite number of impetuses [mv]."[67] Although there is confusion between weight and mass in the equivalency of ms to mv^2, 's Gravesande seems to be attempting to take the sum or integral of dead forces through time, obtaining living force as a result. This procedure is not exactly legitimate. The expression ms should be mgs if it is to be equivalent to mv^2. Technically ms is equivalent to $\frac{1}{2}ft^2$.[68] If dead force is integrated over a space one obtains:

$$\int_0^s m \frac{dv}{dt}\, ds = \int_0^v mv\,dv = \frac{1}{2} mv^2$$

or living force. If dead force, mdv/dt, is integrated through time the result is: $\int_0^t m \frac{dv}{dt}\, dt = \int_0^v m\,dv = mv$ Thus it would appear that if 's Gravesande's "instantaneous force" is interpreted as dead force then its integral through time could be taken as living force when the confusion between

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[66] Ibid., 417.

[67] Loemker 2, 716.

[68] See Introduction, this dissertation, p. 9.

mass and weight is allowed for. 'S Gravesande at first glance seems to be distinguishing momentum from living force and saying that both measures of force have their own validity. On closer interpretation however he actually appears to be relating dead and living force while at the same time confusing dead force with the "ancient system of measuring force", mv.

Continuing his argument 'sGravesande writes that in the case of balls of masses (masses) in the inverse ratio of the squares of their velocities e.g. 4:1, when the velocities are 2:1 falling into soft clay, the times /of fall7 are as 2:1. But the body whose fall is the most rapid takes the least time to impress itself. Thus "when the impressions themselves are equal and similar, the times /needed for the balls to impress themselves7 are in the inverse ratio of the velocities." Multiplying each mass (4:1) by its velocity (1:2) one has 4 and 2. Again multiplying each mass by the time necessary for impression (2:1) the result is 8:2 or 4:1 for the ratio of the "effects".[69]

Again consider the case of two right cylinders moving along the direction of their axes; in colliding directly with soft clay, they lose their motion and are imbedded in it. If their masses are equal the times will be as their velocities. The total effects (les effets entiers) which

[69] Ibid., 418, 419.

are always as the squares of the velocities when the masses are equal will in this case be as the products of the velocities by the times, /vt7. In this particular case the "instantaneous action" is as the velocity, v. If this were the only case the dispute would be reduced to a dispute of words, /une dispute de mots7, Here those who say force is as the velocity would conform to experience just as would those who say that force is as the square of the velocity. One must only pay attention to the meaning given to the word "force".[70]

> Let us pass on to that regarding collision. I have spoken at length in the essay inserted in the twelvth volume of this Journal; I have treated it again at length in the second edition in 4 of my physics...but in the second edition in 8. which appeared last year I have added some new clarifications... /as to7 why in collision Force never destroys Force immediately; and that by the single examination of the nature of collision, one can demonstrate that it is contradictory that two unequal bodies having contrary motions rest quiescent after collision if their forces are not unequal... This obscurity is caused because in the Essay which I just mentioned, I have spoken as if there is a real distinction between force and inertia which is only relative. If one wishes to take the trouble to look at one of these writings one will see... that in regard to impact there is no more than a dispute of words (une dispute de mots) with those who say that force is proportional to mass multiplied by velocity. For if one pays a little attention to the method of reasoning on collisions, one will easily see that by the product of the mass and the velocity they understand the total force without paying attention to the instantaneous action.[71]

[70] Ibid., 419.

[71] Ibid., 427, 428.

"In beginning the writing of this essay," concludes 's Gravesande, "my purpose was not limited to the clarifications just seen: I proposed in my turn to attack the Defenders of the ancient System of measuring force [i.e. *mv*]; I call it ancient in opposition to the new, but I changed my mind. I am afraid I have created the occasion for new disputes and this is the style of writing which I naturally dislike."[72]

In this analysis, 's Gravesande has characterized the controversy as a "dispute over words" in which *mv* is a measure an "instantaneous force", and mv^2 is the effect of a force or its total capacity to act. Although he attempted to distinguish *mv* and mv^2 and to show that both had their own validity, he seems actually to be relating dead force to living force.

In 1733, 's Gravesande wrote an answer to an anonymous article, actually written by the Swiss mathematician Calandrin, who had criticized his impression experiment, by saying that the force lost was proportional to the change in momentum.[73] 'S Gravesande's reply[74] has been thoroughly

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[72] Ibid., 432.

[73] Anonymous (Jean-Louis Calandrini), "Dissertation sur la force des corps," J. Hist. Republ. Lettres, 2 (1733) 230.

[74] 'S Gravesande, "Nouvelles experiences sur la force des corps en mouvement," J. Hist. Republ. Lettres, 3(1733) 381.

discussed by Thomas Hankins in his "Eighteenth-Century Attempts to Resolve the *Vis Viva* Controversy", and will not be analysed here.[75] The issue concerned a cylinder striking a clay surface. Hankins writes:

> He ('s Gravesande) denied that equal amounts of "force" are consumed in equal times. The cylinder is moving faster when it first strikes the clay and consequently it pushes more clay out of the way during the first instant than during any later instant. If the resistance of the clay is likened to a series of strings that are broken by a moving object, more strings are broken per unit of time while the object is moving rapidly than in an equal unit of time when the object is moving more slowly. 'It can be seen that in order to compare the efforts of two pressures in equal times, it is necessary to take into account both the pressure and the speed of the points or surfaces being struck; and it is only by multiplying the intensity by this speed that one is able to determine the effort.'... In other words, to get the effect of the "force of motion" it is not enough to consider the force alone. It is also necessary to multiply it by the velocity with which the object moves, since the faster it moves the more obstacles it will encounter if the resisting medium is uniform.... 's Gravesande is saying that the "force of motion" is the "intensity of the pressure" multiplied by the increment of time and by the velocity, $pvdt$; but $vdt = dx$ (the increment of distance covered in time dt), so $\int pvdt = \int pdx = kv^2$.
> ...'s Gravesande was not dogmatic about his theory. He realized that those who measured the force of motion in another way were measuring a different thing and he concluded his article by saying again that the word "force" is ambiguous: 'Let someone give another sense to the word 'force'; Let him say that this other sense is more natural. I do not oppose that; all I wish to insist on is that what I call force ought to be measured by the product of the mass and the square of the velocity. By regarding force in another way, one can admit of another measure.'(Ibid., 396.)[77]

[75] Thomas Hankins, *op. cit.*, 289-290.

[76] Hankins, 290-291.

Here then is additional evidence that 's Gravesande believed that both measures of force had their own validity and sphere of applicability.

Conclusion

Since physics is an experimentally verifiable science one might expect that a controversy between two theoretical measures of force could be resolved by performing the proper experiments. Perhaps this would have been the case had only one of the two measures been valid. However since both mv and mv^2 each had their area of applicability, experimentation only served to enhance the dilemma and deepen the confusion.

The reason for this situation was that b aring other misconceptions over quantitative definitions, those experiments designed to justify mv^2 succeeded in so doing, while those performed to verify mv likewise proved the latter's validity. /For example, Poleni and 's Gravesande on free fall, and 's Gravesande's initial experiments on impact (1718)7. The real difficulties arose when the protagonists for one measure tried to interpret in their own terms, the experiments designed to prove the opposing case. Thus Poleni's and 's Gravesande's experiments allowing balls falling from different heights to strike soft clay were variously construed as an entirely different but equally valid experiment. For example it became a case of determining the resistance of soft substances to bodies moving through them (Eames) or a case proving the dependence of force on the time of fall whether through the air (Desaguliers) or through the soft clay medium (Pemberton).

The problem of inelastic collision and the conservation of _vis viva_ was still a problem despite 's Gravesande's ingenious method for finding the force lost. This was because 's Gravesande did not insist on _conservation_ of _vis viva_ but only on mv^2 as a _measure_ of force. Others however such as Eames and Clarke assumed that if mv^2 was the measure of force, it ought to be conserved in all cases including inelastic impact.

Samuel Clarke's and Issac Newton's contributions establish the position of the Newtonians in the controversy. This will be reflected in Maclaurin's contribution (1724, see Ch. VII) Their work raises the question of whether force, by whatever measure, is conserved at all in the universe. This argument had a religious foundation: was a God who intervened in the workings of the world more or less powerful than one who initially created a perfectly operating universe?

Clarke and Newton however brought additional confusion to the controversy by equating the variation of velocity and force over the space traversed in free fall, with a variation in the force of gravity. From this they deduced a variation of the body's weight as it rose or fell.

Finally out of the debris resulting in part from his change of mind, 's Gravesande arrived at the beginnings of a synthesis of the two points of view. He recognized the equivocal use of the word "force" and tried to show that $\underline{mv}$ and $\underline{mv}^2$ had different meanings and uses.

The controversy as it took place in England as a reaction to the work of Poleni and 's Gravesande is in keeping with the tendencies of British scientists toward an experimental and empirical approach to natural philosophy and with the philosophical empiricism developing in England during the eighteenth century. It further reflects the influence of Newtonian physical science and its fairly early teaching in British universities, and the interest of the Royal Society in experimental studies.