

## CHAPTER VII

### Prize Winning Essays of the French Academy (1720-1726)

The controversy over living force as it unfolded in France in the 1720's centered around different problems than had the experimental phase in England. In England the vis viva produced in free fall and its experimental evidence had been a central issue. Here in France the problem turned to the laws obeyed by bodies moved under the action of elastic springs, while the problem of the impact of bodies continued to be of prime importance. The view of the composition of matter, whether of "hard" atoms or elastic bodies determined the stand of some participants. Related to this position was the question of the conservation of  $mv$  or  $mv^2$  in a universe which was running down or periodically fluctuating. Here in France, the home of Descartes' philosophy, the champions of  $mv^2$  were in a minority. A series of papers written for competitions sponsored by the French Academy produced only one defender of Leibniz, Jean Bernoulli, who was not a Frenchman (See Ch. VIII). A second series of essays at the end of the decade resulted in reaction to that one paper (see Ch. VIII).

A non-technical contribution to the argument was made by the Swiss professor of philosophy and mathematics at Lausanne, J. P. de Crousaz. A vociferous opponent of Leibniz's philosophy, Crousaz had rebuked Leibniz for errors in his attacks on Descartes. In a paper entitled "Discourse on the Principle, Nature and Communication of Motion"<sup>1</sup> which won the prize of the French Academy in 1720, he accepts the Cartesian measure of force, mv.

In leading up to a definition of the quantity of motion, Crousaz first discusses the nature and origin of motion.

"I see bodies at rest after having perceived them <sup>move</sup> in motion and I see them/after having been at rest. From that I conclude that by its nature a body is indifferent to one or the other of these states." It has been determined by some cause to be in one or the other state. A body at rest can be put in motion by the impulsion of another, but since the second could have been at rest before being in motion one must ask whence came the motion with which it could push the first.<sup>2</sup>

To understand the nature and origin of motion it is first necessary to understand the nature of body. Following Descartes, Crousaz states that it is necessary to agree that the extended is a substance. "The extended" being a substance, the extended, and extended substance

<sup>1</sup>J. P. de Crousaz, "Discours sur le principe, la nature et la communication du mouvement," Pièces qui ont remporté les deux prix de l'Académie Royale des Sciences, (1720), 1-67.

<sup>2</sup>Ibid., 6.

are synonymous terms."<sup>3</sup>

One sees that body (corps) and extension (étendue) are the same thing and "no portion of the extended can draw its own motion from itself since of itself it does not...pass from a state of rest to one of motion." A body "is indifferent to one or the other of these two states and is equally susceptible to one and the other.... Consequently some exterior cause (cause extérieure) is needed to determine one state rather than the other."<sup>4</sup>

It is necessary to look for an intelligent substance to which to attribute the first motion of the universe. It is in the power and will of this substance that one looks for the first origin of motion. The eternal intelligence can produce all that it wishes and produce it with infinite ease.<sup>5</sup>

In attributing the original source of motion to an external cause and in considering particular motions as states of the moving body, Grousaz is following Descartes. Later in the essay he says: "It is the true cause of motion which is created anew and of which the collision of bodies is simply the occasion."<sup>6</sup>

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<sup>3</sup>Ibid., 12.

<sup>4</sup>Ibid., 16-17.

<sup>5</sup>Ibid., 17.

<sup>6</sup>Ibid., 56.

This would seem to carry Crousaz beyond Descartes to the occasionalist theory of motion (see footnote 7).

For Descartes and Newton motion was generated and continued by a cause, God, outside of matter. For Leibniz motion was not real but the force which phenomenally caused the action of bodies permeated the world. In the sense that the actions of material bodies occur in harmony with the pre-established lives of the monads, motion, for Leibniz, can be said to be based on an internal principle.<sup>7</sup>

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<sup>7</sup>On the background of the various theories of the nature of motion held up until the time of Crousaz see Edward Jan Dijksterhuis, The Mechanization of the World Picture, trans., C. Dikshoorn, Oxford, 1961; on Aristotle see p. 21: "For Aristotle, motion in the most general sense of the word is the term used to denote any transition from potential to actual being (generatio) or decay (corruptio), whether it undergoes changes in bulk or quantity (augmentatio and diminutio), or whether it is going to occupy a different place (motus localis) ...The original Aristotelian definition... reads: motion is the actuality of that which is potentially, viewed from the standpoint of potential being." p.24: "In contrast with atomism in which the perpetual motion of the atoms was postulated without need for causal explanation being felt, Aristotelian physics is based on the axiom that every motion (motus) presupposes a mover (motor). This motor must either be present in the mobile (moving body) or be in direct contact with it; Actio in distans (action at a distance) is excluded as inconceivable; a motor must always be a motor conjunctus (connected with the mobile). On Occam and the Paris Terminists (Buridan) see pp. 175-176: "From the conception motus localis est mobile quod movetur (local motion is the body that moves) Occam drew the conclusion that the word motus is no more than a word, a vox, which designates two positive things, to wit the mobile and the places successively occupied, and one negative thing, namely the fact that no two places are occupied simultaneously...The opposition which this extreme nominalist position aroused in the fourteenth century among the Paris terminists, for instance,...raised the question of whether

by the forma fluens conception of which it was the consequence the nature of local motion is expressed sufficiently, or whether it is not rather at the same time a physical reality existing outside the mind, a fluxus formae. Motion was defined by the Paris terminists as an intrinsece aliter et aliter se habere (an intrinsically different behavior on each occasion); it is real state distinct from the mobile, which cannot be classed with one of the categories; its nature is not susceptible of any further determination, but on the ground of experience, it simply had to be assumed as existing. ...For the Terminists...motion was something absolute: according to them it could be predicated of a body that moves without its relations to other bodies being taken into consideration."

On Descartes, "Principia Philosophiae," op. cit. Pt. II, Prin. 25. "Further I understand that it /motion/ is a mode of the mobile thing and not a substance, just as figure is a mode of the figured thing and repose of that which is at rest." Prin. 27: "What is in question at present is not the activity conceived to exist in the object that produces or arrests motion, but simply translation, and absence of translation or rest. Plainly this translation can have no being outside the moving body; and the body is in a different condition when it is being transferred and when it is not being transferred or is at rest. Thus motion and rest are simply two different states (modi) of a body."

On the occasionalist theory of motion see Beatrice K. Rome, The Philosophy of Malbranche, Chicago, 1963, p. 211: "When Malbranche talks of God as alone the Creator he equates this with the claim that God alone is motor. Since God alone can create he alone can move." p. 226: "The essence of matter is its immutable and quantitative relations; but its existence is its process or movement: matter acts only by the efficacy of the movement that I impress on it. Again Malbranche declares: "Matter is essentially movable. It has by its nature, a passive capacity for movement. But... it has no active capacity; it is actually moved only by the continual action of the creator." pp. 227-228: "We may say that the sun is the cause of many admirable effects, because its force is the movement that animates it. However God is the true cause of the movement, and hence the sun is an occasional or natural cause, for its active power comes from God's laws.... Occasional causes do move for Malbranche, they do communicate and impart motion; and they do so because God, through His laws which are but expressions of His will, empowers them to act thus.... Natural causes...are not true causes at all. They are only occasional causes which act solely by the force and efficacy of the will of God. Thus the occasional cause remains active but its activity depends

To understand the nature of this first motion, Crousaz continues, it is necessary to suppose all parts of the universe in perfect rest. One begins with the most simple suppositions: rest is infinite in comparison to motion. A body at rest is always in the same state and constantly conserves the same relations.

To show how motion could have been initiated by an intelligence, Crousaz conceives a sphere six feet in radius in which nothing is changing and which is completely at rest. Its convex surface at rest is perfectly polished and immediately touched at all points by the concavity encircling it and which is also perfectly polished.

"Imagine that the supreme intelligence wishes that this sphere apply successively the convex surface which encloses it to the concave surface which immediately embraces it. This wish will be immediately carried out and the sphere will put itself in motion." Following this "all its parts will apply successively the convex surfaces which enclose them all, to the concavity which touches them" and all parts will be set in motion.<sup>8</sup>

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on God's creative and conserving power."

On Leibniz see this dissertation ch. III, pp. 102-103, n. 44. and Leibniz, The Discourse on Metaphysics, op. cit., 32: "...motion if we regard its exact and formal meaning, that is, change of place, is not something entirely real, and when several bodies change their places reciprocally it is not possible to determine by considering the bodies alone to which among them movement or repose is to be attributed... But the force or the proximate cause of these changes is something more real and there are sufficient grounds for attributing it to one body rather than to another..."

Ibid., 20.

The way in which this is accomplished is not clarified by Crousaz.

From this it can be seen that "motion is a state of a body which successviely applies its surface to the extension immediately touching it."<sup>9</sup> This is the first essential property of motion. A second characteristic no less essential is that there is no part in this sphere that changes without ceasing to be in the same place with respect to the parts of the concavity to which they are compared. All the parts change their situation in the successive applications of the convex surface.<sup>10</sup>

The entire assemblage which composes the sphere in applying its surface successively and in changing its situation, traverses a space."<sup>11</sup> This is the third essential property of motion. "The concavity which the sphere in motion embraces is the extremity of an extended body. It is necessarily of a certain capacity, and...that which encloses it is also an extended body..., this concavity is of a determined capacity."<sup>12</sup> When a sphere moves itself around its center a certain portion of the concavity, after having been traversed successively by a certain part of

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<sup>9</sup>

Ibid., 20.

<sup>10</sup>

Ibid., 21.

<sup>11</sup>

Ibid., 22.

<sup>12</sup>

Ibid., 23.

convexity of the moving body, is afterwards traversed by another in the same way. A third succeeds the second, traversing the same part, and thus without interruption.

"This idea of motion conceived as a state of a body which traverses a space, or which traverses a concavity of a determined capacity...is what one calls the quantity of motion. All physicists...define this as being the product of the weight (pesanteur) and the velocity (vitesse)."<sup>13</sup> When more space is traversed there is more motion than when less space is traversed, or when the concavity traversed is of lesser capacity, there is less motion. One velocity is to another as the length of the path that one of the moving bodies traverses is to the length of that which the other traverses in the same time. When one talks about quantity of motion, one never thinks of the motion of only a single body but one always compares the motion of two bodies.<sup>14</sup>

"Motion being the state of a body which traverses a space and the quantity of motion being always proportional to that space, one sees that motion does not differ from its quantity."<sup>15</sup>

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<sup>13</sup>Ibid., 23.

<sup>14</sup>Ibid., 23-25.

<sup>15</sup>Ibid., 27. In saying that motion does not differ from its quantity, Crousaz goes further than Descartes. Descartes said that space (or matter) does not differ from its quantity i.e. extension. See "Principa Philosophiae," op. cit., Prin. 8 "For quantity differs from extended substance

"A body only has force by its motion; the force of motion is the motion itself....The force of motion and its quantity are again the same thing."<sup>16</sup>

A moving body of one ounce which traverses six feet in one minute has the same quantity of motion and consequently the same force as a moving body of three ounces which traverses two feet in the same time. Thus if a body of two ounces at rest resists collision with one of these, it will resist the other, since they are of the same strength.<sup>17</sup>

Like other Cartesians, Crousaz emphasized that one of the essential properties of time was to be the measure of motion. If this is not recognized one finds oneself in a vicious circle in comparing two unequal velocities. One must compare the distances traversed in equal times. Equal times are those during which equal distances are traversed

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not in reality but only in our conception." Prin.11:" ...The same extension which constitutes the nature of body constitutes the nature of space, nor do the two mutually differ, excepting as the nature of the genus or species differs from the nature of the individual..." Concerning motion Descartes said only that it was a mode of the moving body and had a definite quantity, but did not identify motion and its quantity. See Prin 36. "Motion is indeed only a state (modus) of the moving body; but it has a certain definite quantity and it is readily conceived that this quantity may be constant in the universe as a whole, while varying in any given part."

<sup>16</sup> Ibid., 27. The Cartesians characteristically admitted only force of motion in their analysis. The Leibnizians also included force of position, or "dead force".

<sup>17</sup> Ibid., 38.

by equal velocities. The existence of motion in a body is the existence of time in this body, and the time and motion of a body are the same thing.<sup>18</sup>

Although Crousaz does not directly attack Leibniz in this highly metaphysical prize winning paper, he recalled to the attention of the French literary public, the Cartesian concept of motion as a state of a moving body rather than as a phenomenon manifesting forces inherent to matter (Leibniz). He agreed with the Cartesian measure of motion as the weight times the velocity, equating it with the idea of force acting through time.

The second paper to appear in the French Academy's series of prize winning papers on motion likewise sided with the measure of force as mv. It did so not from the Cartesian point of view already familiar and accepted in France, but from the Newtonian. It came from the brilliant Scottish professor at Aberdeen, Colin Maclaurin.

For the year 1724, prompted by concern over the problem of the communication of motion and the question of "hard" bodies, the French Academy of Science had announced that a prize of 2500 pounds would be given to whomever could resolve the following question: "What are the laws according to which a perfectly hard body, put into motion,

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<sup>18</sup>Ibid., 48, 50.

moves another of the same nature which it encounters whether at rest or in motion and whether in a void or in a plenum?<sup>19</sup>

The winner of the prize was Colin Maclaurin for his essay, Demonstration of the Laws of Collision of Bodies.<sup>20</sup>

Honorable mention was given to Jean Bernoulli. In the notice to the reader at the head of Maclaurin's essay the Academy explained that it had asked for the laws of collision of perfectly hard bodies without asking whether they exist. Some authors they said (meaning Bernoulli) had submitted essays on the laws for elastic bodies which are not the same as those for hard bodies.

Although the question was strictly on the laws for hard body collisions, the competition became part of the vis viva controversy. For Maclaurin (1724) submitted laws in terms of  $mv$ , at the same time criticizing the possibility of using  $mv^2$ . Bernoulli (1727) however denied the existence of hard bodies and submitted an analysis based on conservation of  $mv^2$ . Mazière (1726) derived the laws of collision for perfectly elastic bodies in terms of  $mv$ .

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<sup>19</sup> For this statement concerning the details of the competition see Jean Bernoulli, "Discours sur les loix de la communication du mouvement," Recueil des pièces qui a remporté les prix de l'academie royale des sciences, 2 (1727)4, separate pagination.

<sup>20</sup> Colin Maclaurin, "Demonstration des loix du choc des corps," Recueil des pièces qui a remporté les prix de l'academie royale des sciences 1 (1724)1-24, separate pagination.

Thus the question of impact became of prime importance in the controversy over  $mv^2$ . The papers of Maclaurin and Mazière will be discussed in this chapter; that of Bernoulli in the next chapter.

Colin Maclaurin (1698-1746) has been considered one of the three greatest disciples of Newton (the other two being David Gregory and Roger Cotes). He had become acquainted with the geometry and natural philosophy of Newton while in college, before meeting him in 1719. It was Newton who later helped him attain the chair of mathematics at Edinburgh with the recommendation: "not only because you are my friend but principally because of your abilities, you being acquainted as well with the new improvements of mathematics as with the former state of these societies."<sup>21</sup>

Maclaurin taught the Principia for 20 years at Edinburgh and in 1748 wrote An Account of Sir Issac Newton's Philosophy.

His contribution to the French Academy's 1724 contest was written while he was in Lorraine during a three year extended leave of absence from Aberdeen where he had held a teaching position. In Lorraine from 1722-1725 he had been engaged as a tutor for the son of Lord Polworth.

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<sup>21</sup> H. A. Turnbull, "Bicentenary of the Death of Colin Maclaurin, Aberdeen, 1951, 6.

Maclaurin's essay on percussion finds its foundation in the third law of Newton, that action and reaction are equal.

Before discussing the laws for colliding bodies, however, Maclaurin thought it essential to examine the hypothesis of Leibniz and 's Gravesande that the forces of bodies are as the product of their masses (masses) and the squares of their velocities. He writes,

As it is absolutely necessary to know how to determine the proportions of forces of bodies in motion, before looking for the laws of their collisions, and as it is contested that the forces of bodies are as the rectangles or the products of their masses by their velocities, it appears to me essential to clarify this matter and to examine with attention the opinion of M. Leibniz, explained and lately supported by M. 's Gravesande, in an essay which he published on the collision of bodies. This is the fundamental problem to be treated in connection with the impact of bodies; that is why I will elaborate particularly on its discussion.

Messieurs Leibniz and 's Gravesande maintain that the forces of bodies are as the products of their masses and the squares of their velocities. For example if the velocities of two equal bodies are as 10:8, the forces should be 100:64.<sup>22</sup>

Using these numbers as an example, suppose that two persons, one on a ship which advances with uniform motion with a velocity of 2 and the other at rest on the shore, throw 2 equal bodies A and B with equal efforts in the direction of motion of the ship. Suppose that the body B which was at rest

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<sup>22</sup>Maclaurin, op. cit., 7.

gains a velocity of 8. Body A advances on the ship with a velocity of 8, but advances in the air with a velocity of 10, the sum of the boat's velocity and its own. According to Leibniz the force of body A before it was thrown forward was 4, the square of the boat's velocity. Its increase of force after being thrown is  $8^2$  or 64 making its total force,  $64+4=68$ . But since its total velocity after being thrown is  $8+2=10$ , its force ought to be 100. This is contradictory and therefore forces cannot be proportional to the squares of the velocities.<sup>23</sup>

Maclaurin is making an algebraic error. The total force of body A after being thrown is  $(\Delta v + v_o)^2 = \Delta v^2 + 2v_o\Delta v + v_o^2 \neq \Delta v^2 + v_o^2$ , as Maclaurin calculated. Thus  $(8 + 2)^2 = 64 + 2(32) + 4 = 100$ . Maclaurin's figure of 68 is based on the incorrect equation,  $\Delta v^2 + v_o^2$ .

To further show the absurdity of this notion, continues Maclaurin, suppose that someone throws forward body C equal again in mass to A and B with the same effort, 8, on a ship advancing with velocity 4. The total velocity of C will then be  $8 + 4 = 12$ , and its force will be  $12^2 = 144$ . If  $4^2$  or 16, its force before it was thrown, is subtracted from 144, the result, 128, is the force added to the body

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<sup>23</sup>Ibid., 8.

by the same effort of throwing which added 96 or 100 - 4 degrees of force to body B and 64 degrees to body A. But these increases should be equal.<sup>24</sup>

Again Maclaurin is making the same error. The force of body C after being thrown is:  $v_c^2 = (\Delta v_c + v_o)^2$  or  $v_c^2 = (8 + 4)^2$ ;  $v_c^2 = 64 + 2(32) + 16 = 144$ . In this problem the increase in velocity is needed:  $v_c^2 - v_o^2 = \Delta v_c^2 + 2v_o \Delta v$ ;  $144 - 4^2 = 128 = \Delta v_c^2 + 2(4\Delta v)$ ;  $\Delta v_c^2 + 8\Delta v_c - 128 = 0$ ;  $(\Delta v_c + 16)(\Delta v_c - 8) = 0$ . Thus  $\Delta v_c = 8$ , not 128 as Maclaurin calculated. For body B the effort of throwing,  $\Delta v_B$ , is:  $v_B^2 = \Delta v_B^2 + 2\Delta v_B v_o + v_o^2$ ;  $100 = \Delta v_B^2 + 2\Delta v_B(2) + 2^2$ ;  $\Delta v_B^2 + 4\Delta v_B - 96 = 0$ ;  $(\Delta v_B - 8)(\Delta v_B + 12) = 0$ . Thus  $\Delta v_B = 8$  and not 96 as Maclaurin erroneously calculated.

Suppose now that two bodies, A and B hit two "hard" immovable obstacles, one on the shore and one on the ship. They should lose equal quantities of force. But body B will lose 64 degrees of force which is all it had received. Body A in losing 64 would have  $100 - 64 = 36$ . But body A actually loses all its velocity, except the two degrees it has in common with the ship. Its final force then is  $2^2 = 4$ . Again these results are contradictory.<sup>25</sup>

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<sup>24</sup>Ibid., 8.

<sup>25</sup>Ibid., 8.

The same error on Maclaurin's part accounts for the supposed contradiction. Solving for  $\Delta v$ , one obtains:

$$168 = \Delta v_A^2 + 2 \Delta v_A v_0 + 2^2; \quad \Delta v_A^2 + 4 \Delta v_A - 96 = 0;$$

$$(\Delta v_A + 12) (\Delta v_A - 8) = 0. \quad \Delta v_A = 8 \text{ and } \Delta v_A^2 = 64. \text{ Thus}$$

body A loses 64 degrees of force, but its loss of velocity is 8 which is all but the 2 degrees contributed by the ship.

Consequently if this system of Leibniz and 's Gravesande were true, the motions and collisions of bodies contained in a space which advances uniformly will be different from the motions and collisions in a space at rest relative to the first. Thus it will be possible to distinguish relative from absolute motions which is usually considered impossible in physics.<sup>26</sup>

A similar argument can be drawn from collisions of elastic bodies. Suppose there are two equal elastic bodies A and B moving in the same direction with velocities 10 and 5. If they had no elasticity they would stick together and proceed after collision with a velocity of  $7\frac{1}{2}$ . But since they are perfectly elastic they exchange their velocities and body A will now have 5, and body B, 10 degrees of velocity. 'S Gravesande, says Maclaurin, agrees that

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<sup>26</sup> Ibid., 9.

they separate with a difference of 5 degrees of velocity and that during the collision the elasticity of the bodies imprints  $2\frac{1}{2}$  degrees of velocity to each of  $\frac{25}{4}$  degrees of force. Without the action of elasticity the force of body A would be  $(7\frac{1}{2})^2$  or  $\frac{225}{4}$ . Thus  $\frac{25}{4}$  degrees of force are removed since there is no elasticity. There remains in body A,  $\frac{225-25}{4} = 50$  degrees of force. But since the common velocity of body A is only 5 its force can be no higher than 25. Thus there is a contradiction.<sup>27</sup>

An interesting case of collision discussed by many of the antagonists in the controversy was that of two bodies having velocities of 1 and 3 in the inverse ratio of their masses colliding from opposite directions. In this case if they have no elasticity the bodies rest motionless after hitting each other. According to 's Gravesande, says Mac-laurin, the forces ( $mv^2$ ) are in the ratio of  $1(3)^2 + 3(-1)^2$  i.e. 9 to 3 or 3 to 1. According to Maclaurin, however, the forces ( $mv$ ) are as 3 to 3 or 1 to 1.

This is an inelastic case, hence  $mv^2$  is not conserved,  $(1)(3)^2 + 3(-1)^2 = 12$ . The  $mv^2$  of the bodies before and after collision would not have to be equal. Since momentum

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<sup>27</sup> Ibid., 9. The same error on Maclaurin's part accounts for the apparent contradiction.

is conserved however, the mv's  $(1)(3)+3(-1) = 0$ . But Maclaurin argued that in 's Gravesande's system this means that one force should stop another force of which it is only a third. The proponents of this, he said, argue that the larger force loses its advantage in compressing the parts of the other body and the forces mutually consume themselves. But since the actions are mutual and contrary and begin and finish in the same time, it is repugnant to experience that these unequal forces should consume themselves entirely and both end at rest. In the case of the forces being equal, as in the system of Descartes and Maclaurin, there is no problem in seeing how the bodies can come to rest after collision due to the equality of action and reaction, i.e. Newton's third law. In the Leibnizian's Gravesande system, to resolve the difficulty one must say that the larger body resists the other, not only by its force, but also by its inertia in order to explain the phenomenon. The greater force of the smaller body is consumed by compressing the more numerous parts of the larger body.<sup>28</sup>

It would be necessary said Maclaurin, to change entirely the Newtonian ideas on force and motion in order to concede to 's Gravesande his explanation in

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<sup>28</sup> Ibid., 10-11.

terms of the inertia and resistance of bodies.

'S Gravesande's explanation of the inelastic bodies resting after collision in terms of inertia and resistance is not correct; the kinetic energies of the inelastic bodies are changed into heat energy as the bodies are deformed in collision. However both the Newtonian measure of force, i.e. the time rate of change of momentum, and the equal but opposite momenta of the two bodies are measures of other properties of the body. 'S Gravesande's explanation that the bodies rest motionless after collision could not be given in terms of the force defined as  $\underline{ma}$  or  $\underline{mv}$ . Thus Maclaurin's statement is true, but he does not recognize that two different concepts are at issue. Here the problem is truly a word dispute.

Maclaurin objects to 's Gravesande's 14th proposition which forms the basis for his treatment of inelastic and ultimately of elastic collisions: "The force lost in the collisions of two non-elastic bodies is the same as the absolute velocities of the two bodies if their respective velocity is the same."

Maclaurin then challenges 's Gravesande's 19th proposition in which he speaks of the force lost in a non-elastic collision in terms of the sum of the absolute forces of the two bodies  $\frac{ABdd}{A+B}$ . But according to Maclaurin the absolute motion lost in the collision of two non-elastic bodies

moving from opposite directions is double the force  $\frac{mv}{7}$  of the smaller body. In this example the non-elastic bodies are perfectly hard bodies, not soft bodies. This is Maclaurin's solution to the problem posed by the academy. To clarify this statement Maclaurin supposes that two bodies A and B have velocities V and u and that the sum of their absolute forces before the collision is  $|AV| + |Bu|$ . If the force of body A is the greatest and they travel toward each other from opposite directions, the force after the collision will be  $|AV - Bu|$  and the differences between these two forces before and after collision will be the force lost or  $(AV + Bu) - (AV - Bu) = 2 Bu$ . This loss then is equal to double the smallest force.<sup>29</sup>

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<sup>29</sup> Ibid., 13. In explaining the reasoning behind Maclaurin's statement, Wilson Scott, *op. cit.*, 20-22, gives the following analysis: "In contrasting the difference of elastic bodies' behavior to that of hard bodies, Maclaurin points out that the former undergo no net loss of force upon Choc regardless of whether their prior motion is in the same or in opposite directions. Moreover he explains why in a way that makes the entire problem quite clear:

The action of spring in the impact of perfectly elastic bodies doubles in the exchanges of the forces which would have been produced in the bodies if they had had no spring. The parts of the elastic bodies are depressed by the impact and always give way until the two bodies are advancing with a common velocity--just as if there were no elasticity, the respective velocities which compressed their springiness no longer acting. Then the altered parts rebound, and regaining their shapes by the same steps and with the same forces by which they were depressed produce the same effects in the bodies, which become separated at the respective speeds equal to those with which they approached each other before impact. There is therefore a

'S Gravesande, writes Maclaurin, says that forces /defined as  $mv^2$  never mutually destroy themselves but consume themselves in compressing the parts of the opposing body which sustains itself by its contrary force. One deduces from this that a force cannot lose very much in compressing the parts of a body if that body is not sustained by a contrary force or held by another obstacle. It appears reasonable to believe then that the force lost by the collision of bodies from opposite directions should

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double augmentation produced in the Force of the body which gains by the impact and a double diminution in the forces of the body which loses by the impact.

Scott gives the following example of L. W. Taylor by way of additional clarification (L. W. Taylor, Physics, Cambridge, 1941, 209). "Imagine a man standing on ice. If he catches a ball he will be pushed backwards with a certain velocity. Upon returning the ball he will be pushed backwards with the same velocity again. The first represents inelastic impact, the second elastic. The double augmentation has the effect of reversing the relative velocity. Thus if a hard body should strike a stationary wall the body would come to a stop, losing a velocity of say 88'/sec. But an elastic body would lose double 88'/sec. which would cause it to end up with -88'/sec. Now take similarly two elastic bodies of equal mass approaching at 88'/sec., the velocity of each turned through 180°. In this latter case however there is no net loss of speed or momentum." "To sum up: The mathematics developed by Maclaurin demonstrates that the momentum lost by one hard body is gained by another hard body when they are both moving in the same direction. However hard bodies moving in contrary directions suffer a loss of momentum equal to  $2m_2v_2$ .  $(m_1v_1 + m_2m_2) - (m_1v_1 - m_2v_2) = 2m_2v_2$  Upon doubling the loss of the first body and doubling the gain of the second body a process which takes place in the case of elastic bodies, there is no gain or loss whatsoever." For a mathematical explanation of this statement see appendix to this chapter containing Wilson Scott's calculation for the momenta of hard body collisions in opposite directions." (Wilson L. Scott, op. cit., appendix.)

be greater than when one of the two, with a velocity equal to the sum of their velocities, hits the other at rest. However, the respective velocity is equal in the two cases. If the respective velocity remains unchanged, the forces of the bodies do not change and consequently their resistances to each other, the compressions of the parts and the force lost cannot vary. This contradicts Maclaurin's own conclusion that the force lost would be greater when the bodies proceeded from opposite sides. The whole system of 's Gravesande, concludes Maclaurin, is thus subject to question because he can never reconcile his principle with the laws of collision established by experience.<sup>30</sup>

Maclaurin's argument here rests on the different meanings given to the words force and in particular to the force lost. 'S Gravesande's expression for the force lost in an inelastic collision is in terms of the difference of the living forces of the two bodies before and after collision i.e.  $\frac{ABdd}{A+B}$ . Maclaurin's expression is for the momentum lost in a hard body inelastic collision. Here again the disagreement is a dispute over the meaning given to the word "force".

Maclaurin's paper shows that the loss of force, or momentum is suffered by "hard" bodies meeting in collision

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<sup>30</sup> Ibid., 13-14.

from contrary directions. In other words in this special case, momentum is not conserved.

Maclaurin's arguments like those of the English experimentalists were mainly based on Newton's 3rd law of the equality of action and reaction. Since Newton defined force to be the quantity of motion or momentum produced in a unit of time, it follows that bodies acting on each other communicate in equal units of time, equal and opposite quantities of motion,  $mv$ . They receive velocities reciprocally proportional to their masses and opposite in sign.<sup>31</sup> This was the basis for the Newtonian-Cartesian position on the communication of motion.

Because of the fact that Jean Bernoulli had submitted an essay on elastic collision the French Academy had announced a second contest for 1726 on "the laws of collision of bodies having perfect or imperfect elasticity." The winner this time was Father Mazière for his title, "The Laws of Impact of Bodies Perfectly or Imperfectly Elastic, Deduced from a Probable Explanation of the Physical Cause of Elasticity."<sup>32</sup>

Mazière's essay adhered closely to the Cartesian explanation of the universe and brilliantly explained the

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<sup>31</sup> See Ernst Mach, The Science of Mechanics, op. cit., 243.

<sup>32</sup> Pierre Mazière, "Les loix du choc des corps a ressort parfait ou imparfait, deduites d'une explication probable de la cause physique du ressort," Recueil des pièces qui a remporté les prix de l'academie royale des sciences, 1(1727) 1-57, separate pagination.

laws of impact on the basis of mv. This may be the reason it was chosen for the prize, since Cartesianism was still the accepted philosophy in France.

"The bodies which surround us, began Mazière, are in continual agitation and communicate motions following uniform rules called the laws of collision. By these laws God produces the infinite varieties which are the object of admiration of all men..."<sup>33</sup> Between the two extreme cases of perfect and imperfect elasticity range all bodies.

The following three principles are presupposed by Mazière in his essay:

I. The spaces traversed are in a ratio composed of the velocities of the bodies and the times which they use in traversing them uniformly.

II. The forces of bodies or their movements are in a ratio composed of their masses and the velocities which they have at the instant they are under consideration. From this second principle follow three propositions:

A. The forces of bodies are in the ratio of their velocities when their masses are equal and in the ratio of their masses when their velocities are equal.

B. The forces of bodies are equal when their velocities are in the inverse ratio of their masses.

C. The velocity of a body is equal to its force divided by its mass.

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<sup>33</sup> Ibid., l.

III. The centrifugal forces of bodies are as the squares of their velocities, divided by the diameters of the circles which they describe by uniform motion.<sup>34</sup>

Mazière divided his memoir into two parts. The first contained a probable explanation of the physical cause of elasticity. The second contained the laws of the collisions of bodies with perfect or with imperfect elasticity exemplified in problems.

Mazière tried to explain the physical cause of elasticity in the Cartesian tradition by use of a theory similar to Descartes' vortex theory. Two bodies colliding with equal forces rebound only if they have elasticity. This unknown force of which the cause is sought is called elastic virtue (vertu elastique) or elasticity (ressort). Soft bodies remain at rest after collision as do perfectly hard or inflexible bodies since no new cause of motion occurs. Elastic bodies rebound only because their elastic parts are pressed together during the moment of compression and restored during the moment of restitution by that unknown force of which the cause is sought.<sup>35</sup>

Because the cause sought is physical it cannot be an intelligence but must be a body. Since rest has no

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<sup>34</sup>Ibid., 1-4. This third principle was necessary in giving a mathematical basis to the vortex theory of matter discussed in the latter portion of Mazière's paper.

<sup>35</sup>Ibid., 6-8.

force, the cause must be a body in motion. This in turn cannot be the solid parts of a body since these are fastened and integrated with one another. These solid parts cannot cause motion since by themselves they cannot return to their original position during the second or restitution part of the collision. They are at rest at the instant compression begins and at the instant it ends, and rest does not produce motion.

The cause of elasticity must therefore be the corpuscles of fluid which fill the pores of a body. It is not the air but rather a subtle ether penetrating all bodies. This subtle matter has an infinite force given to it by the Creator so that it can cause perfect rebounding by restoring the primitive forces of bodies after collision. "All men before Descartes have ignored its existence."<sup>36</sup>

Likewise it is a perfect fluid. Should a solid body receive a blow, it transmits the vibrations through the fluid. Thus the parts of bodies can change and re-establish their original positions in the smallest instants of times. It flows through all bodies with extreme facility and leaves no void in the immense spaces that it occupies. One corpuscle of air, for example, can contain a million corpuscles of subtle matter. The corpuscles of subtle matter are ordinarily spherical, but when a change occurs in a

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<sup>36</sup> Ibid., 8-12.

body whose pores it occupies, they can divide themselves into smaller parts, as does for example mercury, or incorporate with other corpuscles, or change their figure to ellipses.<sup>37</sup>

Subtle matter is infinitely compressible and responds to the force which compresses it with all the power of that which is compressing it. It is composed of an infinity of vortices (tourbillons) which turn about their centers with an extreme rapidity, counterbalancing one another as do the large vortices described by Descartes in his Principles of Philosophy. One cannot admit the large vortices of Descartes and the principles on which they depend, without at the same time admitting the small vortices described by Malebranche. These vortices whether large or small counterbalance each other by their centrifugal forces. This dynamic balance prevents them from moving away. Their centrifugal forces are in the inverse ratio of their diameters, increasing when their diameters decrease. Thus the centrifugal force of the infinitely small vortices is infinitely greater with respect to the infinitely large vortices. And therefore the centrifugal force of the small vortices is infinite. It is this property which is the physical cause of elasticity.<sup>38</sup>

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<sup>37</sup> Ibid., 13-15.

<sup>38</sup> Ibid., 16-21.

Mazière's idea of an elastic body is as follows. It is filled with an infinity of pores through which by its circular motion moves the subtle matter. Each pore contains one or more little vortices, giving by their centrifugal force, stability to solid bodies. The smaller the vortices the greater the elasticity of the body, because the greater is their centrifugal force uniting the parts of a solid and repelling the external forces which tend to separate them.

Suppose now that two bodies hit each other directly with equal and opposite forces. In successive instants of time they use their primitive forces to mutually compress one another. The subtle matter which by its nature never resists motion partially leaves the pores in the direction toward which it is pushed. Motion is communicated successively through the first pores to the others. The pores are flattened, assuming elliptical configurations, and continue to be flattened up to the precise instant that the bodies have exhausted all their primitive force in mutual compression.

The centrifugal forces of the vortices outside the two bodies are equal to what they were before the collision, when the exterior and interior vortices were in equilibrium. But the centrifugal forces of the vortices remaining inside are augmented because their diameters are diminished. At the end of compression the interior vortices have increased

their centrifugal forces whereas those outside have not. Thus the exterior vortices do not have centrifugal forces capable of stopping the action by which the interior vortices tend to enlarge the pores. They continue to enlarge the pores until the point where the compressed parts have been re-established. Thus bodies having perfect elasticity expand with velocities equal to those with which they were compressed, due to the infinite force of the little vortices.

Having explained the cause of elasticity to his satisfaction, Mazière then proceeds to describe the laws of collision for elastic bodies.

If one supposes that the colliding bodies have integral homogeneous parts, the effects of collision can be reduced to uniform laws and can be expressed by very simple formulas. If experiments are conducted with an instrument similar to Mariotte's, one will find that these laws extend to all bodies homogeneous or heterogeneous, to elastic bodies perfect or imperfect, fast or slow.<sup>39</sup>

Four laws of collision are set down.

- I. In the instant that compression ceases, the two bodies have an equal velocity whether their motions are contrary before collision or they came from the same direction.

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<sup>39</sup> Ibid., 25.

The subtle matter ceases to act only when the attacking body is no longer acting on the other body. After having communicated part of its motion, it retains such a quantity that it can accompany the other body without compressing it. At the instant the subtle matter ceases to act or that compression ceases, the two bodies have an equal velocity.

- II. At the instant that compression ceases, the attacking body and the body hit have lost an equal quantity of their primitive forces, when their motions are contrary. Up to that instant the two bodies have mutually compressed one another, and in these mutual compressions have employed equal forces. The forces they have used have been lost.
- III. In the instant that compression ceases, the attacking body has lost as much force as the body hit has gained when their motions are from the same side.

In this case as in the preceding, the compression is mutual; but the attacking body which has more velocity than the body hit should have lost in this instant, a part of its velocity, or a part of its force. The force that this body loses, the other gains. (Cf. Maclaurin, n.29)

- IV. The elastic ratio or constant (rapport elastique) is the same in bodies of the same nature.

If in a collision the force with which the elasticity

is restored in two bodies is to that with which they were compressed eg. 15 to 16, then in all other collisions of these two bodies, or of two others of the same nature, these two forces will always be as 15 to 16. That is why if one knows the elastic constant,  $\underline{r}$ , and the force lost or gained by one of the two bodies in the time of compression, one can obtain that force lost or gained in the time of restoration, by multiplying the force lost during compression by the elastic constant  $\underline{r}$ . This constant is equal to unity when the elasticity is perfect and less than unity when the elasticity is imperfect.<sup>40</sup>

Mazière then proceeds to the solution of several fundamental problems. In the first the masses  $\underline{A}$  and  $\underline{B}$  of two bodies, their velocities  $\underline{a}$  and  $\underline{b}$  before collision, and their elastic constant  $\underline{r}$  are given. Their motions after collision are to be found.

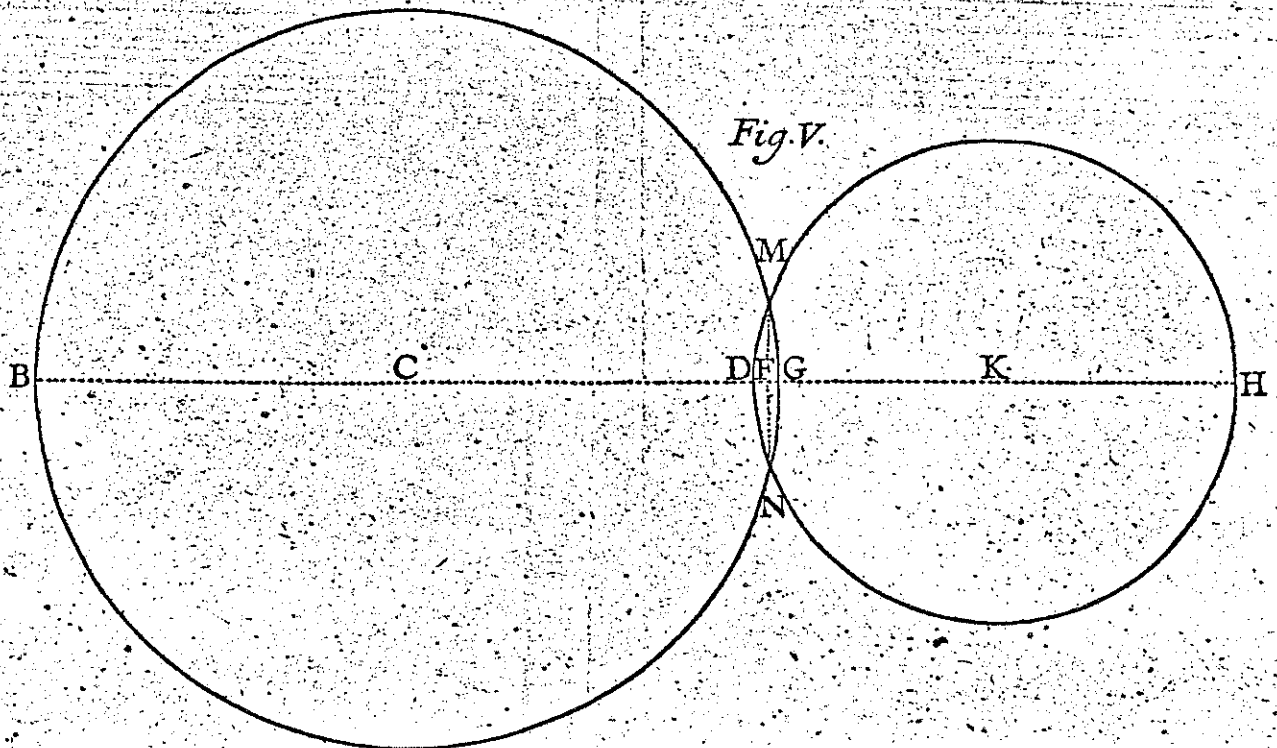
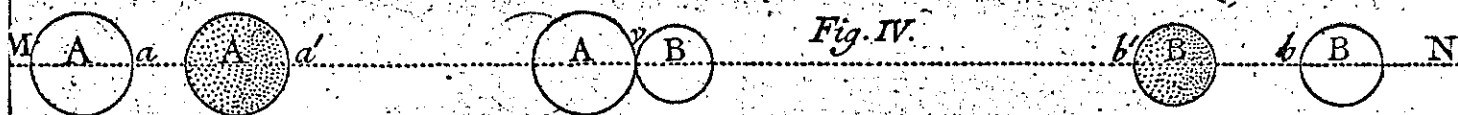
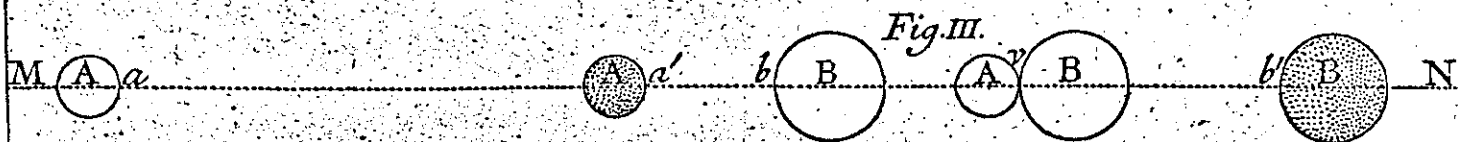
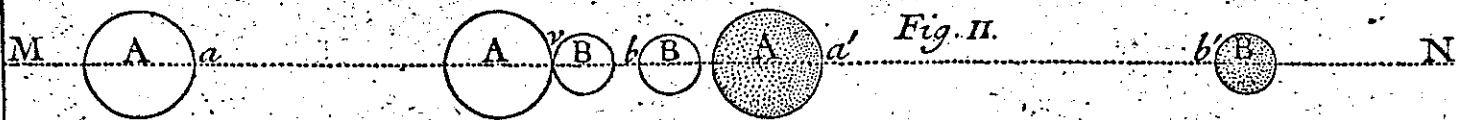
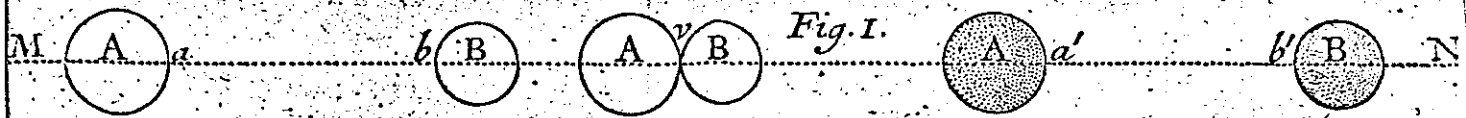
This problem is reduced to two principle cases. The first is when the motions are from the same side before collision as in figures I and III. The second is when the motions are contrary before collision as in figures II and IV. (See p. 266.)

Case I. When the motions are from the same side, the attacking body  $\underline{A}$  having a greater velocity than  $\underline{B}$ , loses one portion of force in the first moment of col-

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<sup>40</sup> Ibid., 28, 29.

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lision, and another portion in the second. Calling x the velocity which it loses in the first moment of collision, the force that it loses in this time is Ax and that which it loses in the second is rAx. Its force before collision is Aa and after collision Aa'. Thus one has this equation

$$Aa' = Aa - Ax - rAx$$

$$\text{or} \quad Aa' = Aa - A(r+1)x$$

But in the first moment of collision the body B gains as much force as is lost by attacking body A. Thus it gains the force +Ax in the first moment and thence the force +rAx in the second. Its force before the collision is +Bb and after is +Bb'. One thus has the second equation:

$$Bb' = Bb + Ax + rAx$$

$$\text{or} \quad Bb' = Bb + A(r+1)x$$

In the instant that compression ceases the force of the body hit is Bb+Ax. Consequently its velocity is  $\frac{Bb+Ax}{B}$ . The velocity of attacking body A in this same instant is a-x. But in this instant the velocity of the body hit is equal to that of the body attacked. One thus has the equation

$$\frac{Bb+Ax}{B} = a-x$$

$$\text{or} \quad x = B \frac{a-b}{A+B}$$

By placing this value of  $x$  in the two equations preceding, the following two equations are obtained resolving the first case of the problem:

$$Aa' = Aa - AB \left[ \frac{(r+1)(a-b)}{A+B} \right]; Bb' = Bb + AB \left[ \frac{(r+1)(a-b)}{A+B} \right]$$

Case II. When the motions are contrary the attacking body which is supposed to have more force than that attacked, loses a part  $Ax$  in the first moment of collision and another part  $rAx$  in the second. Thus in this second case as in the first, one has the equation:

$$Aa' = Aa - A(r+1)x$$

In the first moment of collision body  $B$  when hit loses as much negative force as the attacking body  $A$  loses of its positive force. Thus the body hit gains the force  $+Ax$  in the first moment and consequently the force  $+rAx$  in the second moment. Its primitive force which is negative is  $-Bb$ . Thus one has the second equation:

$$Bb' = -Bb + A(r+1)x$$

In the instant that the compression ceases the force of body  $B$  which is hit is  $-Bb + Ax$ , and its velocity is  $\frac{-Bb + Ax}{B}$ .

The velocity of the attacking body is  $a - x$ . In this instant the velocity of the body hit equals that of the attacking body. That is, one has:

$$a - x = \frac{-Bb + Ax}{B}$$

or

$$x = B \frac{a+b}{A+B}$$

By putting this value of  $x$  in the two preceding equations one has:

$$Ad = Aa - AB \left[ \frac{(r+1)(a+b)}{A+B} \right], \quad Bb' = -Bb + AB \left[ \frac{(r+1)(a+b)}{A+B} \right]$$

These formulas differ from the first case only in the sign of velocity  $b$ . This is natural since the direction of velocity  $b$  is from the positive direction in the first case and negative in the second.<sup>41</sup>

The general formula for the laws of colliding bodies in terms of the velocity after collision then is:

$$a' = a - B \left[ \frac{(r+1)(a-b)}{A+B} \right], \quad b' = b + A \left[ \frac{(r+1)(a-b)}{A+B} \right]$$

Thus the velocity of a body after collision has two parts. The first is the primitive velocity  $a$  which is always positive, or the primitive velocity  $b$  which is positive when the movements are in the same direction and negative when they are in opposite directions. The second part is the total velocity that each body gains or loses by the compression and restoration in the two moments of collision. That of the attacking body is always negative; that of the body hit is positive.<sup>42</sup>

When the bodies have perfect elasticity, the elastic constant is equal to unity so that  $r+1=2$ . Thus 2 appears in the general formula. When the bodies are not elastic

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<sup>41</sup>Ibid., 30-32.

<sup>42</sup>Ibid., 33, 34.

and are supposed perfectly soft, the elastic constant will be 0, hence  $r+1=1$ . When the elastic constant is equal to the ratio of the mass of the attacking body to the body hit, one has  $r=A/B$  and consequently  $r+1=\frac{A+B}{B}$ .

Special cases of collision substituted in the general equation check with the previously known facts. When the elasticity is perfect and the masses, A and B, are equal, the general equations come down to  $a' = b$ ,  $b' = a$ . Thus the bodies exchange their velocities. For the case of body B at rest before the collision one has for perfect elasticity:

$$a' = \frac{Aa - Ba}{A+B}, \quad b' = \frac{2Aa}{A+B}$$

For bodies without elasticity:

$$a' = b' = \frac{Aa}{A+B} \quad \text{showing that the bodies stick}$$

together and proceed with a common velocity.

When the body hit is at rest and the velocity of the attacking body is equal to the sum of the masses,  $a=A+B$ , the formula becomes:

$$a' = A - rB, \quad b' = (r+1)A$$

from which is deduced:

$$r+1 = \frac{b'}{A}$$

This gives an easy method of determining in experiments the value of  $r+1$  and consequently the proper value

given to two bodies with which one is going to experiment, or to two other bodies of the same nature. In this case for perfect elasticity one has  $a' = A - B$ ,  $b' = 2A$ ; for no elasticity,  $a' = A$ ,  $b' = A$ .

For the case of body B at rest and infinitely greater than body A attacking, one supposes  $A = 0$ . One obtains  $a' = -ra$ . Thus ie, this is the case of direct reflexion, the body A rebounding with its primitive velocity multiplied by its elastic constant. It rebounds with a velocity less than its primitive velocity when elasticity is imperfect and with a velocity equal to its primitive velocity when the elasticity is perfect.<sup>43</sup>

The remainder of Mazière's essay is the solution of several problems for the purpose of illustrating the generality of the foregoing solution and to show how all of the questions regarding the laws of collision have been resolved.

This very general and beautiful solution of the problem of impact solely in terms of momentum conservation merits great respect.

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<sup>43</sup>Ibid., 35-38.

### Conclusion

The papers of Crousaz (1720), Maclaurin, (1724) and Mazière (1726), written in response to the interest of the French Academy in the question of the nature and communication of motion, presented a combination of Cartesian and Newtonian arguments in support of mv as the measure of force.

The paper of Crousaz presented a non-mathematical argument along Cartesian lines, accepting quantity of motion, mv, as identical with force. Since bodies are susceptible to either motion or rest an exterior cause is necessary to set a body in motion.

Maclaurin's prize-winning paper presented the rules for collision of absolutely hard bodies in terms of mv. When two hard bodies collide from opposite directions, mv is not conserved and the loss of "force" in the collision is double the smaller momentum. Maclaurin added confusion to the controversy in two ways. He made a trivial mathematical error in criticizing 's Gravesande, which seemed to show that given 's Gravesande's measure of force, it would be possible to distinguish relative motion from absolute motion. Secondly he gave a different meaning to the word "force" and to "inelastic collisions" in criticizing 's Gravesande's expression for the force lost in collision. 'S Gravesande's analysis had been given in terms of the

difference between the living forces of inelastic bodies before and after collision. Maclaurin gave an expression for the difference between the momenta of absolutely "hard" bodies before and after collision. The two men were in reality arguing about different concepts.

Mazière presented a complete solution for inelastic and elastic collisions in terms of mv. His explanation of the nature of matter, upon which his solution was based, was cast in terms of the Cartesian vortex theory and was supported by the concept of centrifugal force.

Thus the Cartesian argument for force was given in terms of the conservation of mv in a plenum of vortex motion. The Newtonian explanation here was in terms of a special case of the non-conservation of mv in a universe of "hard" atoms.

From Wilson L. Scott, "The Significance of Hard Bodies in the History of Scientific Thought", Doctoral Dissertation, John Hopkins University 1960.

HARD BODIES MOVING IN OPPOSITE DIRECTION

(Calculation b)



$$M_1 V_1 - M_2 V_2 = (M_1 + M_2) V_3 \quad V_3 = \frac{M_1 V_1 - M_2 V_2}{M_1 + M_2} \quad \text{Common Velocity After Choc}$$

$$F_{M_1} = \frac{(M_1)^2 V_1 - M_1 M_2 V_2}{M_1 + M_2}, \quad F_{M_2} = \frac{M_2 M_1 V_1 - (M_2)^2 V_2}{M_1 + M_2}$$

$$\begin{aligned} \text{Force Lost by } M_1 &= M_1 V_1 - \frac{(M_1)^2 V_1 - M_1 M_2 V_2}{M_1 + M_2} = \frac{(M_1)^2 V_1 + M_1 M_2 V_1 - (M_1)^2 V_1 + M_1 M_2 V_2}{M_1 + M_2} \\ &= \frac{M_1 M_2}{M_1 + M_2} (V_1 + V_2) \end{aligned}$$

$$\begin{aligned} \text{Force Lost By } M_2 &= M_2 V_2 - \frac{M_2 M_1 V_1 - (M_2)^2 V_2}{M_1 + M_2} = \frac{M_2 M_1 V_1 + (M_2)^2 V_2 - M_2 M_1 V_1 + (M_2)^2 V_2}{M_1 + M_2} \\ &= \frac{M_2 M_1 V_2 + M_2 M_1 V_1 + 2(M_2)^2 V_2}{M_1 + M_2} \end{aligned}$$

$$\text{Adding} \quad \frac{M_1 M_2 V_1 + M_1 M_2 V_2 + M_2 M_1 V_1 + M_2 M_1 V_2 - M_2 M_1 V_1 + 2(M_2)^2 V_2}{M_1 + M_2} =$$

$$= \frac{2M_1 M_2 V_2 + 2(M_2)^2 V_2}{M_1 + M_2} = \frac{2M_2 V_2 \sqrt{M_1 + M_2}}{M_1 + M_2} = 2M_2 V_2 \quad \text{Total Force Lost in Choc}$$