GENERAL CONCLUSIONS

The controversy over living force was determined by philosophical issues to the extent to which each participant was predisposed by the accepted "natural philosophy" of his country or his own natural inclination. But far more importantly, the controversy occurred because the meaning and the limits of validity of the two measures of "force", momentum and kinetic energy, were particularly difficult to define with precision.

In retrospect the main issue was the discovery of which of the two concepts should be used to describe the physical situations under discussion and solve related problems? A concept can only be defined by its use and by the limitations on its generality. The controversy may be viewed then as the unfolding and unraveling of the definitions of $\underline{mv}$ of $\underline{mv}^2$.

Let us briefly re-examine the chronology of these various physical problems as they occurred during the controversy. Descartes had defined quantity of motion as an absolute quantity, $\underline{m|v|}$, the total value of which was conserved in the universe. He had used this quantity incorrectly to solve impact problems. As a result of this inaccuracy, Wallis, Wren, and Huygens had correctly set down the laws of impact taking into account the sign of the velocity, $\underline{mv}$. Wallis and Wren gave the law for momentum. Huygens gave the laws for impact using what we now take to be conservation of momentum and kinetic energy. Leibniz took issue with Descartes' measure of $\underline{mv}$ as force. He thought of himself not as correcting $\underline{m|v|}$ for its sign to $\underline{m\vec{v}}$ but rather as correcting $\underline{m|v|}$ to $\underline{mv}^2$ because $\underline{m|v|}$ was not a measure of force as he defined it (which was true) and because $m|v|$ was not conserved (which was not always true since in many of his examples conservation did not occur.) Leibniz also argued that perpetual motion would arise if $\underline{mv}$ were the measure of force. The fact that a body will rise to the same height from which it fell if nothing external interferes is the statement of vis viva conservation. This perpetual motion argument however does not even apply to momentum conservation since momentum is not dependent upon distance. Leibniz mainly defined the meaning of $\underline{mv}^2$ by cases of free fall (or retarded motion), but also used it in impact problems. He accepted the conceptual solutions of Wallis, Wren, and Huygens for these collision problems, but added, the algebraic equations to be solved simultaneously.

With the death of Leibniz, the controversy took on a different character. The issue of conservation, so important to Leibniz's philosophy faded somewhat. The problem of which concept was to be the measure of force in different physical situations became important. Of prime importance was the validity of each measure in impact problems. Defenders of <u>mv</u> solved all collisions problems by use of conservation of <u>mv</u>. This included 's Gravesande's early text on <u>The Mathematical Elements of Natural Philosophy</u> and Mazière's general solution for both elastic and inelastic collisions. Defender's of mv^2, mainly Bernoulli, followed the solution of Huygens and Leibniz using simultaneously, conservation of <u>mv</u> (which was not recognized as being identical with the opposing side's measure of force) and conservation of mv^2. Both approaches are correct but each side refused to recognize the validity of the other. Confusion was wrought in this issue by those who still followed Descartes in not taking into account the sign of the velocity: these included Malebranche, Voltaire and Hermann (cited by du Châtelet). Further confusion was caused by comparing the equality of momenta and <u>vis viva</u> in inelastic impact, eg. Maclaurin. 'S Gravesande tried to solve inelastic problems by a device using mv^2. Inelastic impact was not discussed by several of the defenders of mv^2: Bernoulli (1727) du Châtelet, d'Alembert. What became of the <u>vis viva</u> lost in inelastic impact was a significant drawback for those holding a belief in conservation of living force. Leibniz had stated that it went into the small parts of matter, to be released at another time. Bernoulli later (1735) indicated agreement. 'S Gravesande asserted that the force lost in collision was used to flatten the other body and found a mathematical expression for the lost quantity. Eames challenged 's Gravesande's method showing that the $\underline{mv}^2$ is not the same before and after an inelastic collision. Clarke argued against $\underline{mv}^2$ on the basis of the "force" not being conserved in hard inelastic collisions and hence not in the universe.

Secondly there was the problem of free fall or retarded motion. Here $\underline{mv}^2$ was certainly a correct definition since it was the measure of a body falling through space. This argument was cited by all defenders of $\underline{mv}^2$. However, the proponents of mv tried to find ways of changing the problem of free fall into one of momentum. Thus the English experimentalists, Pemberton, Desaguliers, Eames changed it into a problem of inelastic impact with the clay ground where momentum holds, or a problem of the time of motion through the clay, an mv problem. The French scientists, Louville and Mairan, Deidier and Voltaire reduced it to a problem of uniform motion where momentum is a valid measure.

Considerable confusion arose in this controversy over the relationship of dead force to momentum. Most of the advocates of mv (excepting Descartes himself) confused the produce of the mass of a body and its virtual velocity (mdv) with the body's momentum, its mass times its actual velocity. These men included the Cartesians, Catalan, Papin, and the Newtonians, Clarke and Desaguliers. The Leibnizians (Leibniz, Bernoulli, and Camus) pointed out this confusion, confining dead force to its use in statics. This then was a third problem.

Fourthly there was the problem of bodies set in motion by the expansion of elastic springs. 'S Gravesande used the expansion of springs to show that the effort necessary to increase the velocity of a body by an infinitely small degree is in the ratio of the velocity that the body already has. Bernoulli used springs as a device to prove that mv^2 was the measure of force. Camus related the force of expanding springs to the force of ascending bodies. Louville, however, in showing that mv was acquired by bodies from the action of compressed springs thought he had successfully demonstrated that they do not also give living force to a body. In describing the impulse of a force, Ft, he thought he had refuted the Leibnizian measure of dead forces. Deidier compared the times of the motion of the bodies pushed by springs, thus finding their momentum, and thereby arguing against living force.

Finally, there was the important question of mv as the measure of a force acting through time, versus mv^2 as the measure of a force acting through a distance. This very central issue was cited by almost every participant according as it pertained to his side of the controversy. Although this one question was clarified by d'Alembert the controversy was not completely resolved because he did not, at the same time, discuss the other places where each measure is valid, eg. impact problems and the elastic spring problems were not discussed in this regard. Furthermore the generality of d'Alembert's solution as characterizing two independent approaches to mechanics, called the Huygenian ($\underline{mv}^2$) and the Newtonian ($\underline{mv}$) by Mach, did not become clear immediately.

Several theoretical or philosophical concerns influenced many of the participants in their choice of positions in the controversy. The issue of conservation of force in a stable universe was of prime importance to Descartes and Leibnitz, while to Clarke and many of the followers of Newton, it was necessary for God to periodically renew the force of the universe as though "winding up his watch from time to time." Momentum conservation applied to both elastic and inelastic impact problems whereas _vis viva_ was conserved only in elastic impact. The Newtonians used this non-conservation of _vis viva_ to refute the conservation argument of the opposition. For Descartes, however, who originally had discussed conservation of $\underline{mv}$, the total quantity of motion put into the universe at the time of creation could not vary although the parts of matter contained more or less of it at various times. Conservation for Leibniz was so important that it obscured his thinking on certain physical problems. Further although he knew that _vis viva_ was not conserved in inelastic impact, he still argued for conservation on the basis that it simply went into the small parts of matter and was retained for release at some other time. Conservation was also of significance in Bernoulli's position which closely followed that of Leibniz in that he depended on the law of continuity and hence on conservation of mv^2 in rejecting the existence of "hard" bodies and the loss of force encountered in their collision.

The laws which hard bodies would obey in impact constituted a problem for scientists in this period and created confusion in the momentum-*vis viva* controversy. Descartes' hard bodies behaved as inelastic bodies in some impact problems and as elastic bodies in others. Hard bodies for Wallis were equivalent to inelastic bodies in behavior, whereas for Huygens they behaved elastically. Later because the question of whether matter consisted of hard bodies, elastic atoms or no atoms at all had become an issue the laws of impact for hard atoms were discussed. MacLaurin who accepted *mv* as the measure of force showed that in the case of hard atoms colliding from opposite directions there was a loss of momentum. The 1724 contest concerning hard atoms, however, did not ask the question whether hard bodies existed in nature, but only the theoretical laws they would obey. Leibniz and Bernoulli denied their existence on the basis of the law of continuity.

Mairan held that hard bodies differ from elastic only in that the communication of force is instantaneous instead of successive; hard bodies continue to move uniformly with lesser velocity after collision. Other participants in the controversy argued that there were no atoms, but a plenum instead. Thus Mazière followed Descartes in the supposition that the universe was full of subtle matter swirling in vortices according to the law of centifugal force.

Another relevant philosophical issue behind the positions chosen was whether the original source of motion or force was external or internal to matter. For Descartes motion was a state of the body moved, not an inner force. Motion was originally put into the world by God, an external source and thereafter conserved by him. For Leibniz the primitive force of striving forming the essence of the monad was an internal source of motion. The communication of motion between bodies was for him merely a phenomenon. In the real world of monads there was no real communication of motion but only the pre-established unfolding of the lives of monads. Crousaz accepted Descartes' view, saying that by nature a body is indifferent to motion or rest, motion being a state of the body moved. Voltaire scoffed at the possibility of force existing primatively in simple beings or monads, for these can no more produce moving force than zeros can form a number.

Finally there was a predisposition in the antagonists to take a certain position depending upon what scientific training was popular in their own country or depending upon the receptivity of their own mind to a more theoretical or more empirical approach to natural philosophy. Thus there is some significance to the predisposition of British scientists to an empirical and experimental viewpoint, and this, coupled with the growing background of Newtonianism and its teaching in British colleges, caused these scientists to argue for mv as a measure of force. In France Cartesianism was still popular well into the eighteenth century and the fact that Descartes had discussed conservation of mv influenced French scholars such as Mazière, Louville, Mairan, and Deidier, as well as the Swiss, Crousaz, in following his mathematical formula. In Voltaire we see the Cartesian and Newtonian influences combined. The Leibnizian synthesis of force as mv^2 representing conservation of force as the inner dynamic principle of matter causing all motion appealed to Bernoulli. This also fired the mind of Madame du Châtelet who had received training in French and Newtonian science. 'S Gravesande represents a certain independent inquiring mind set mainly on expositing the truth as it appeared to him, but basing this on experimental demonstrations.

The controversy over living force was called "a dispute over words" by Leibniz, 's Gravesande, Boscovich and d'Alembert. This of course is true. Thus the relationship between momentum *mv*, and dead force, *mdv*, was confused by many participants as Leibniz had pointed out. D'Alembert sharply distinguished between these two concepts in his *Treatise* in 1758. The crucial confusion was that between momentum and *vis viva* which were both called force by all participants. As Leibniz said in his reply to Papin anyone is free to define force as he chooses in order to exclude all verbal misunderstanding. In 1758 d'Alembert likewise clarified the definitions of *mv* and mv^2, as had Boscovich in 1745. Maclaurin and 's Gravesande both discussed the force lost in an inelastic collision, but 's Gravesande meant the living forces lost in an inelastic collision while Maclaurin calculated the momentum lost in a hard body inelastic collision. Louville gave a different mathematical meaning to both dead and living force than had Leibniz and Bernoulli, claiming that their expressions were incorrect and his correct. However Louville's was based on *mv* and Bernoulli's on mv^2.

The controversy was more than verbal disagreement however. Many arguments rested on logical confusion or outright mathematical errors. Thus Leibniz tacitly assumed that since mv^2 could be shown to be the measure of force that the same proof showed that it was conserved. Maclaurin made an arithmetical blunder in comparing the velocities of relative motions which seemed to indicate inconsistencies in the measure mv^2. In many collision examples (Descartes, Malebranche, Hermann, Mairan, Maclaurin, Voltaire) the sign of the velocity in mv was not taken into consideration. Mairan compared the mv^2's before collision with the mv's after collision obtaining results which seemed to show mv^2 to be in error.

The main reason however that the controversy was more than a verbal disagreement was that the concepts dead force, living force and momentum were not clearly developed and clarified until the mid-eighteenth century. The controversy itself was the instrument for sharpening those definitions.

The general conclusion which this study has shown therefore is that the complex meaning of the concepts kinetic energy and momentum, both taken as "force" (which is really a third concept) was the cause of the confusion surrounding the controversy over living force. The area and limits of application of each concept, as well as the realization that each formed the basis of an independent approach to mechanics had to be penetrated and understood before the arguments died out. Philosophical considerations although important and influencial in initiating the controversy were of secondary significance in causing it to continue through the eighteenth century.