INTRODUCTION

Gottfried Wilhelm Leibniz, in the year 1686, made the following accusation, a statement which was to spark one of the longest, most bitter, and yet most interesting quarrels in the history of science:

> Our new philosophers commonly make use of the famous rule that God always conserves the same quantity of motion in the world. This rule is indeed most plausible and I have in the past regarded it as beyond doubt. But more recently I have discovered wherein it is in error. This is that Descartes and many other able mathematicians believed that the quantity of motion, that is the velocity multiplied by the magnitude of the moving body coincides exactly with the moving force; or to speak geometrically that the forces are proportional to the product of the velocities and masses. [Yet] there is a great difference between quantity of motion and force.... force must be estimated by the quantity of the effect it can produce, for example by the height to which a heavy body of a certain size and kind can be lifted; and this is quite different from the velocity which can be imparted to it.[1]

As a consequence of this and other statements of the same year arose the controversy over which quantity was to be taken as the measure of "force", mv, the quantity of motion,

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[1] Gottfried Wilhelm Leibniz, "Discours de metaphysique," Philosophische Schriften, ed. C. I. Gerhardt, 7 vols., Berlin, 1875-1890, 4, 442,443.

or mv^2, living force (vis viva).[2]

The first recognition that both quantities were valid physical concepts was made in 1743 by Jean d'Alembert's statement that the controversy was a dispute over words, a futile metaphysical discussion:

> ...neanmoins comme nous n'avons d'idée précise et distincte du mot de force, qu'en restreignant ce terme à exprimer un effet, je crois qu' on doit laisser chacun le maître de se decider comme il voudra làdessus, et toute la question ne peut plus consister, que dans une discussion métaphysique très futile, ou dans une dispute de mots plus indigne encore d'occuper des Philosophes.[3]

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[2] The main historical outlines of the course of the controversy are brief:

René Dugas, La Mecanique au XVII siecle, des antecedents scholastiques a la pensee classique, Paris, Neuchatel, 1954. This describes the early part of the controversy in which Leibniz played a direct part.

______ A History of Mechanics, trans., J. R. Maddox, Neuchatel, 1955, 235-238. This is on the period, 1724-1743.

J. F. Montucla, Histoire des matematiques, 3 vols., Paris, 1799-1802, (first edition, 2 vols., 1758) 3, 629-643.

William Whewell, History of the Inductive Sciences, 3rd. ed., 2 vols., New York, 1872, 1, 361.

[3] Jean d'Alembert, Traité de dynamique, 1st ed., Paris, 1743, xxi. This evaluation was further refined by d'Alembert in the second edition (1758) of the Treatise on Dynamics; xxx-xxxi:

> ...dans le second [cas] l'effet est l'espace parcouru uniformement dans un temps donné, et cet effet est l'espace proportionnel à la vitesse; dans le troisième l'effet est l'espace parcouru jusqu' à l'extinction totale du mouvement, et cet effet est comme le carré de la vitesse. Or, ces différents effets sont évidemment produits par une même cause; donc ceux qui ont dit que la force était tantôt comme la vitesse tantôt comme son carré, n'ont pu entendre parler que de l'effet, quand ils se sont exprimés de la sorte.... Enfin ceux mêmes qui ne seraient pas en état de remonter jusqu'aux Principes metaphysiques de la question des forces vives, verront aisément qu'elle n'est qu'une dispute de mots s'ils considèrent que les deux partis sont d'ailleurs entièrement d'accord sur les principes fondamentaux de l'equilibre et du mouvement...

The contents of several articles concerning the question of

The controversy however was far more than a dispute over words. It involved the definition and refinement of the concepts of the quantity of motion, *mv*, and *vis viva*, mv^2 as they evolved into the concepts of momentum, *mv*, and kinetic energy, $\frac{1}{2}mv^2$. It further concerned the meaning and limits of validity of both measures of force, these being at that time, particularly difficult to define with precision. In retrospect the main issue was: which of these two concepts (or both) should be used to describe which physical situations and solve which problems? A concept is defined by its use and by the limitations on its generality. The controversy may then be viewed as the unfolding and unraveling of the definitions of *mv* and mv^2.

This evolution was by no means completed in the period from 1686-1743. However during these years the concepts were discussed in the context of three general types of physical problems: (1) Impact problems: soft-body inelastic, hard-body inelastic, and elastic, (2) the problems of the *vis viva* acquired by freely falling bodies, and (3) the action of bodies accelerated by expanding springs.

Secondary reasons for the longevity of the controversy are provided by certain philosophical issues: (1) Is

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living forces written between 1743 and 1758 are briefly outlined in ch. IX and the appendix. Some indicate that the controversy was not completely resolved by 1743. Other articles on *vis viva* are concerned with the generalization of the principle itself or with its use as a device in calculations.

"force" conserved in the universe or is the universe running down so that its force must be periodically renewed? (2) If mv or mv^2 is the measure of force does this mean that they are also conserved in the universe? (3) Does the source of motion of a moving body lie in an external cause or is it internal to matter? (4) Is matter itself composed of hard atoms, elastic atoms or no atoms at all? (5) To what extent were the participants in the controversy predisposed to their position by the "natural philosophy" accepted in their native countries? Philosophical considerations were of prime importance in initiating the controversy but of secondary significance in its continuation into the eighteenth century.

A few highlights of the chronological events of the dispute will set the stage for the development of the above problems. First a brief examination of Descartes's concept of the quantity of motion m|v| is important for an understanding of the main issue. It was out of Leibniz's 1686 discussion of Descartes' treatment of the quantity of motion that this controversy grew. The work of Wallis, Wren, and Huygens on impact forms a necessary part of the historical background for the discussions and confusions over collision problems developing after vis viva was proposed as a measure of force. A discussion of Leibniz's dynamics will show that the origins of the controversy stem from an identification of the terms measure of force and conservation of force, confusion by the Cartesians of mv with mdv, used in statics, incomplete physical analysis in the problems chosen by Leibniz to illustrate his arguments, and a desire on the part of Leibniz to establish *vis viva* as a foundation for his philosophical system.

In response to Leibniz's treatment of Descartes' quantity of motion arose the long dispute between Leibniz and his followers and the defenders of Descartes. Within a few months after Leibniz had published his first paper (1686) stating that Descartes' measure of "force" was in error, he was involved in a series of discussions (1687-1691) with Cartesians who sought to defend Descartes' hypothesis. The Leibniz-Clarke correspondance of 1716 marked the entrance into the controversy of Newtonians who argued for the correctness of *mv*.

Despite an early work (1719) which treated impact by the use of *mv* considerations, 's Gravesande after performing experiments on falling bodies (1722), became convinced that mv^2 was the only correct measure of force. Experimental work done in England by such men as Pemberton, Desaguliers, and Eames (1722-1728) was designed to verify *mv* as a measure of force and to reinterpret the meaning of 's Gravesande's experiments. Simultaneously in France in response to competitions on hard and on elastic body impact sponsored by the French Academy, arguments were given in support of *mv* (Crousaz, Maclaurin, Mazière). The sole essay in these contests which supported the mv^2 view, written by Jean Bernoulli, merited only honorable mention and gave rise to a series of papers on whether a body accelerated by an expanding spring acquired momentum or vis viva (eg. Camus, Louville). A paper by Mairan (1728), secretary of the French Academy, supporting mv was hailed by the Academy as settling the issue.

In the 1740's in France the issue of vis viva was re-examined in response to a criticism of Mairan by Madame du Châtelet. Their discussions sparked still other papers by such men as Voltaire and Abbé Deidier (1741). The publication of d'Alembert's 1743 Treatise on Dynamics, has traditionally been cited[4] as resolving the argument by calling it a dispute over words. 'S Gravesande however

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[4] Ernst Mach in his Science of Mechanics (6th ed., La Salle, 1960, 365) says: "The dispute raised by Leibniz rested, therefore, on various misunderstandings. It lasted fifty-seven years 1686-1743, till the appearance of d'Alembert's Traité de dynamique in 1743."

William Whewell in his History of the Inductive Sciences (3rd. ed., 2 vols., New York, 1872, 1, 361) writes: "Finally D'Alembert in 1743 declared it to be, as it truly was, a mere question of words; and by the turn which dynamics then took it ceased to be of any possible interest or importance to Mathematicians."

J. Bernard Stallo, The Concepts and Theories of Modern Science, New York, 1884, 72, implies the date 1743 rather than that of the second edition, 1758, as ending the dispute by reference to a contribution by Immanual Kant of 1746, /"Gedanken von der wahren Schätzung der lebendigen Kräfte" Königsberg, 1746, in Immanual Kant's Werke, Berlin, 1922, 1, 1-187/: "Here was the origin of the famous controversy between the Leibnizians and the Cartesians respecting the true measure of the forces of the universe which was participated in by so many mathematicians and philosophers and to which is well known a late and inapposite contribution was made by Kant."

J. F. Montucla in his Histoire des Mathématiques, op.cit., in 1729 had also called the controversy a dispute over words, but neither he nor d'Alembert in 1743 really clarified in what way this was true. D'Alembert's insight that mv was the measure of a force acting through time and mv^2 that of a force acting through a distance appeared in the 1758 edition of the Treatise. Although through lack of textual comparison on the part of early chroniclers of the dispute, d'Alembert is usually given credit for its resolution, he was anticipated in this fundamental clarification by Roger Boscovich's De Viribus Vivis of 1745. The date 1743, prevalent in the literature then has relatively little significance as a date for the resolution of the controversy. Although the issues of the early 1740's remained alive in some of the papers written on living force through the end of the decade, contributions to the controversy gradually died out. While no date can be set for its terminus the clarification provided by Boscovitch (1745) and d'Alembert (1758) represent a resolution of the disputes initiated by Leibniz's attack on Descartes' quantity of motion. Later articles on vis viva dealing with the

641, does not distinguish between the 1743 and 1758 editions of d'Alembert's Traité. He implies that the controversy ended in 1743 by mentioning the contribution of Voltaire in 1741 and then closing his discussion with a summary of the Traité de dynamique: "D'Alembert, dans son traité de Dynamique, publié peu de temps après, ne pouvoit manquer de dire son avis sur une question agitée avec tant de chaleur; il n'hésite point a déclarer qu'il la regarde comme une pure question de mots."

generalization of the principle and its incorporation into eighteenth century mechanics will not be treated in this study /Eg. d'Alembert (2nd. ed of the Treatise,1758), La Grange,(1788), LaPlace (1799)7.

The simultaneous use of both measures of force in solving physical problems indicating their ultimate acceptance in the general body of physical knowledge did not occur until the nineteenth century, as briefly indicated by the appendix.

In providing a perspective from which to view the controversy, it is useful to reiterate Ernst Mach's discussion of the two spheres of mechanics, the Newtonian approach which takes force as fundamental, and the Huygensian approach to which energy conservation is basic.

Historically the former was developed earlier and more completely through the work of Galileo and Newton than the latter. After first attempting to relate the simple final velocity, v, of a body to the space traversed, s, Galileo discovered the true relationship of the velocity to the time, t, i.e., $v = gt$, where g represents the acceleration due to gravity. From this the free fall relationships, $s = \frac{1}{2}gt^2$ and $2gs = v^2$ can be obtained with the help of the equation $s = vt/2$, representing the distance s through which a uniformly accelerated body moves in time t. Multiplying these equations by the mass, m, and letting the force of the body, f, equal mg, the following equations obtain:

$$(1) \quad \underline{mv} = \underline{ft}$$

$$(2) \quad \underline{ms} = \tfrac{1}{2}\underline{ft}^2$$

$$(3) \quad \underline{fs} = \tfrac{1}{2}\underline{mv}^2$$

The first is the impulse-momentum equation or the result of a force acting through a time; the third is the work-energy equation or the result of a force acting through a distance. The second has no particular name.

In equation (1), the velocity is calculated by reference to the time during which a body moves. In equation (3) the velocity squared is calculated from knowledge of the distance over which it moves, irrespective of the time. In equation (3) the kinetic energy depends only on the force and the end points of the distance, s, and not on the path or the time between them. In the controversy over living force this difference between the dependence of the first relationship upon time and the third upon distance formed the basis for many of the arguments.

Due to the confusion of mass for weight, made by most of the participants in the controversy, where $\underline{f} = \underline{w} = \underline{mg}$, the third relationship appears as an equivalence of $\underline{ms}$ to $\underline{mv}^2$, the same terms being used to indicate mass or weight on both side of the equation, or equivalence. The importance of the factor ½ in $\tfrac{1}{2}\underline{mv}^2$ was contributed by Coriolus in 1829.[5]

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[5] Gustave Gaspard Coriolus, "Calcul de l'effet de machines, ou considerations sur l'emploi des moteurs et sur leur évaluation," Paris, 1829.

In the papers discussed in this dissertation, the formula mv^2 was referred to as "living force", *vis viva*, and *force vive*.

If the time of descent is taken as the factor determinative of velocity, then force is the original concept and work a derived concept. If the distance of descent is determinative of velocity, then work becomes the original concept. Galileo and Newton developed the first approach; Huygens and Leibniz the second. The subsequent "mingling of the two spheres of thought, the independence of which were not always noticed, led to various blunders and confusions,...in the dispute between the Cartesians and the Leibnizians."[6] Exactly what these blunders and confusions were, and some of the reasons for their occurance will be discussed in the course of this dissertation on and exposition of the subject matter of the controversy.

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[6] Mach., *op. cit.*, 312.