

# Dynamics of osmotic fluid flow

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## Introduction

The classical thermodynamic treatment of osmotic pressure is quite sufficient to describe equilibrium situations and compute most quantities of interest without reference to any molecular model (see, for example, Finkelstein, 1987). However, one's intuition is always aided by molecular models, and a microscopic picture can be quite useful when thinking about osmotic flow in unfamiliar situations. The equilibrium situation is well described in the classical literature (see especially Hill, 1960). However, osmotic flows are handled generally by employing equilibrium quantities to describe nonequilibrium situations. Therefore, we offer here a surprisingly simple and intuitive molecular model for describing solvent flow driven by osmotically generated pressure gradients. We will show that osmotic pressure arises from the external force on solute particles which is transmitted to the solvent by viscous drag. This simple picture enables us to propose a simulation method that can be used to compute the osmotic flows associated with quite complex situations arising in polymer and gel swelling.

## A microscopic model for osmotic pressure

We begin with the simplest situation of an ideal solute in water. The molecular picture we shall adopt is the following. A population of  $N$  noninteracting particles of mass  $m$  immersed in water is driven by Brownian motion. The motions of the particles and the fluid are coupled by simple hydrodynamic drag. Let  $\mathbf{X}_k$  be the location of the  $k$ th particle. The equation of motion for a particle in a fluid flowing with velocity  $\mathbf{u}$  is:

$$m \frac{d^2 \mathbf{X}_k}{dt^2} = - \underbrace{\nabla \phi(\mathbf{X}_k)}_{\text{External force on particle } k} + \underbrace{\beta \left[ \bar{\mathbf{u}}(\mathbf{X}_k(t), t) - \frac{d\mathbf{X}_k}{dt} \right]}_{\text{Frictional force between particle } k \text{ and fluid with velocity } \mathbf{u}} + \underbrace{\mathbf{R}_k(t)}_{\text{Random force exerted on particle } k \text{ by the fluid}} \quad [1]$$

Here  $\phi$  is a potential function that defines all of the forces acting on the particle; in the simplest case,  $-\nabla\phi$  is the external force acting on the particle.  $\beta$  is the coefficient of the frictional drag between the fluid and the particle, which depends on their relative velocity and the shape of the particle (e.g. for a sphere,  $\beta$  is given by Stokes law,  $\beta = 6\pi\mu a$ , where  $a$  is the particle radius and  $\mu$  the fluid viscosity).  $\bar{\mathbf{u}}$  is a weighted average fluid velocity in the neighborhood of the particle:

$$\bar{\mathbf{u}}(\mathbf{X}_k, t) = \int \mathbf{u}(\mathbf{x}, t) \delta_d(\mathbf{x} - \mathbf{X}_k) d\mathbf{x} \quad [2]$$

where  $\mathbf{x}$  locates a point in the fluid, and the function  $\delta_d$  defines the average fluid velocity in the neighborhood of the particle. The reason for performing such an average instead of just evaluating the fluid velocity at the particle will appear below. The random force,  $\mathbf{R}(t)$  has the following statistics:

$$\langle \mathbf{R}_k(t) \rangle = 0, \quad \langle \mathbf{R}_{jp}(t) \mathbf{R}_{kq}(t') \rangle = 2k_B T \beta \delta_{jk} \delta_{pq} \delta(t - t') \quad [3]$$

where  $k_B$  is Boltzmann's constant,  $T$  the absolute temperature, and  $\delta_{jk}$  and  $\delta_{pq}(t-t')$  are the Kronecker and Dirac delta functions, respectively. Equation [3] is the usual definition of "white" noise employed in Langevin dynamical equations such as [1] (Chandrasekhar, 1943).

The particle motions are coupled to the local fluid flow via the incompressible Navier-Stokes equations:

$$\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) + \nabla p = \mu \nabla^2 \mathbf{u} - \underbrace{\sum_{k=1}^N \delta_d(\mathbf{x} - \mathbf{X}_k(t)) \left[ \beta \left( \bar{\mathbf{u}}(\mathbf{X}_k(t), t) - \frac{d\mathbf{X}_k}{dt} \right) + \mathbf{R}_k(t) \right]}_{\text{Frictional and random forces exerted on the fluid by the particles}} \quad [4]$$

$$\nabla \cdot \mathbf{u} = 0 \quad \text{incompressibility} \quad [5]$$

where the last term in [4] is the equal and opposite force applied to the fluid by the particles in the neighborhood defined by the weighting function  $\delta_d$ . That is, the viscous drag force exerted by the fluid on each particle is felt equally, but in the opposite direction, by the fluid. Also, the random force acting on the particles arises from the statistical fluctuations in the fluid near the particle, and so the same force must act reciprocally on the fluid volume surrounding the particle. If this force were applied at a point, it would

produce an infinite velocity there. Instead, we spread the force out over a fluid volume of diameter  $d$  by making use of the weighting function  $\delta_d(\mathbf{x} - \mathbf{X}_k)$ . Once we have decided to spread the force in this way, conservation of energy demands that we also average the fluid velocity using the same weighting function. This is the reason for the use of the spatially averaged velocity given by Equation [2] as the convection velocity in Equation [1].

Equations [1] - [5] constitute our "microscopic" model. It is not really microscopic, since we describe the solvent flow by continuum equations; however, molecular dynamics simulations have shown that the Navier- Stokes equations are approximately valid even for flow of  $1\text{\AA}$  particles flowing through a  $3.2\text{\AA}$  channel (Levitt, 1973).

For the situations of biological interest we can neglect the particles' inertia, so that the equation for the particles is:

$$\nabla\phi(\mathbf{X}_k) = \beta\left(\bar{\mathbf{u}}(\mathbf{X}_k(t), t) - \frac{d\mathbf{X}_k}{dt}\right) + \mathbf{R}_k(t) \quad [6a]$$

or

$$\beta\frac{d\mathbf{X}_k}{dt} = -\nabla\phi(\mathbf{X}_k) + \beta\bar{\mathbf{u}}(\mathbf{X}_k(t), t) + \mathbf{R}_k(t) \quad [6b]$$

Now [6a] can be used to eliminate  $\mathbf{R}_k(t)$  from the fluid equations:

$$\rho\left(\frac{\partial\mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla\mathbf{u}\right) + \nabla p = \mu\nabla^2\mathbf{u} - \sum_{k=1}^N \delta_d(\mathbf{x} - \mathbf{X}_k(t))[\nabla\phi(\mathbf{X}_k)] \quad [7]$$

Notice that the only randomness in the fluid equations is the location of the particles, and that the forces,  $-\nabla\phi$ , felt by the particles are now transferred to the fluid as an additional force term.

To see the consequences this, consider the case where the fluid inertia is also neglected, so that the fluid equations become simply

$$\nabla^2\mathbf{u} = \frac{1}{\mu}\left[\nabla p + \sum_{k=1}^N \delta_d(\mathbf{x} - \mathbf{X}_k(t))[\nabla\phi(\mathbf{X}_k)]\right] \quad [8]$$

It may appear strange that the frictional coupling coefficient,  $\beta$ , doesn't appear in Equation [8]. Note, however, that Equation [8] must be solved simultaneously with Equation [6] for the particle locations,  $\mathbf{X}_k(t)$ , and the fluid velocity field,  $\mathbf{u}(\mathbf{x},t)$ . The limiting situation where  $\beta \rightarrow 0$  is singular: with no drag, the particles move at infinite velocity, and so neglect of the particle mass is not possible. For any finite particle drag, all of the force,  $-\nabla\phi$ , on the particle is transmitted to the fluid, and so the fluid equations do not contain  $\beta$ .

Imagine, for example, a collection of particles released in the center of a vesicle (e.g. a liposome) bounded by a semipermeable membrane. The only force acting on the particles is the repulsion of the membrane, which is not felt until the particles are very close to the vesicle walls. Thus nothing interesting happens until the particles diffuse to the membrane. Once the particles feel the repulsive force of the membrane, they transfer this force to the fluid by viscous drag. This drag force is in the same direction as the membrane repulsion, and it raises the pressure inside the vesicle with respect to the outside. To analyze this situation further, we adopt a macroscopic view of the membrane and consider it as uniformly porous, i.e., we ignore the discreteness of the pores. Then the whole situation is spherically symmetric, and it is easy to see that the (incompressible) fluid cannot move. Nevertheless, the semipermeable membrane will be driven outward through the static fluid by the high internal pressure until it stretches sufficiently to balance the internal pressure. (Of course, from the standpoint of the membrane, this looks exactly like fluid flowing through it.) Thus the vesicle will swell until it reaches an equilibrium state, or bursts.

### Equilibrium: van't Hoff's law

At equilibrium, the fluid velocity vanishes,  $\mathbf{u} = 0$ , and Equation [8] reduces to

$$\nabla p = - \sum_{k=1}^N \delta_d(\mathbf{x} - \mathbf{X}_k) \nabla \phi(\mathbf{X}_k) \quad [9]$$

The right hand side of [9] is a force field which is generated by applying the external force acting on each particle to the fluid which surrounds that particle. The pressure,  $p$ , generated by this force field is just the osmotic pressure. This can be seen as follows.

Consider the situation shown in Figure 1: two fluid compartments separated by a semipermeable membrane. We introduce into the compartment on the left  $N$  solute molecules, and wait until equilibrium is reached. To analyze the equilibrium situation, we take the ensemble average of both sides of Equation [9]. Recall that the ensemble average of any function,  $f(X)$ , is defined by

$$\langle f(\mathbf{X}) \rangle \equiv \int f(\mathbf{X})P(\mathbf{X})d\mathbf{X},$$

where  $P(\mathbf{X})$  is the probability density function of the random variable  $\mathbf{X}$ . In our case, the random variables are the locations of the particles,  $\mathbf{X}_k(t)$ . At equilibrium these all have the same probability density function  $P(\mathbf{X}) \equiv c(\mathbf{X}) / N$ , where  $N$  is the total number of particles and  $c(\mathbf{X})$  is their equilibrium concentration (number per unit volume). At equilibrium,  $c(\mathbf{x})$  obeys the Boltzmann distribution

$$c(\mathbf{x}) = c_0 e^{-\frac{\phi(\mathbf{x})}{k_B T}} \quad [10]$$

where  $c_0$  is the concentration at a location where  $\phi = 0$ . Taking the gradient of both sides of this equation and rearranging the result, we get

$$k_B T \nabla c = -c \nabla \phi \quad [11]$$

Taking the ensemble average of Equation [9] in the manner described above results in

$$\langle \nabla p(\mathbf{x}) \rangle = \sum_{k=1}^N \int \delta_d(\mathbf{x} - \mathbf{X}) (-\nabla \phi(\mathbf{X})) \frac{c(\mathbf{x})}{N} d\mathbf{X} \quad [12]$$

Since the quantity being summed is independent of  $N$ , the sum of  $N$  terms just cancels the factor  $1/N$ , and we are left with

$$\langle \nabla p(\mathbf{x}) \rangle = \int \delta_d(\mathbf{x} - \mathbf{X}) (-\nabla \phi(\mathbf{X})) c(\mathbf{x}) d\mathbf{X} \quad [13]$$

At this point, we make two approximations. First, since  $p(\mathbf{x})$  is a macroscopic variable, we assume it is close to its ensemble average. Thus, we replace  $\langle p(\mathbf{x}) \rangle$  by  $p(\mathbf{x})$ . Second, since  $\phi$  and  $c$  are also macroscopic, and hence slowly varying in comparison to  $d$ , we may replace  $\delta_d$  by the Dirac  $\delta$ -function itself. This makes it possible to evaluate the integral in Equation [13]. The result is

$$\nabla p(\mathbf{x}) = -\nabla \phi(\mathbf{x}) c(\mathbf{x}) \quad [14]$$

Finally Equations [11] and [14] may be combined to yield

$$\nabla p(\mathbf{x}) = k_B T \nabla c(\mathbf{x}) \quad [15]$$

If the membrane is completely impermeable to solute and if there is no solute to the right of the membrane in Fig. 1, we integrate Equation [15] across the membrane to obtain

$$\Delta p = k_B T c = k_B T \frac{N}{V} \quad [16a]$$

where  $V$  is the volume in which the solute is distributed. Multiplying numerator and denominator on the right-hand side of Equation [16a] by Avogadro's number,  $N_A = R/k_B$ , we obtain van't Hoff's law for the osmotic pressure of a dilute solution (Finkelstein, 1987):

$$\Pi = RT \cdot c \quad [16b]$$

where  $\Pi$  is the osmotic pressure and  $C = N/N_A V$  is the molar concentration of solute. The reasons why we get an osmotic pressure law appropriate to a dilute solution are: (1) our model particles do not interact, and (2) the particles are treated as points, so that there is no excluded volume effect (despite the presence of the parameter  $d$ ) in the simple form of the model that we have presented so far.

**Figure 1.** (a) The classical osmotic flow experiment. A semipermeable membrane separates two compartments of volume  $V$ ;  $N$  particles of solute are introduced into the left compartment. The resulting osmotic flow induces a hydrostatic pressure head,  $\Delta p$ , just equal to the osmotic pressure,  $\Pi$ .

(b) A microscopic view of the trajectory of particles near the membrane. The random diffusion of particles is biased by the membrane repulsion which induces a drift component directed away from the membrane. Therefore, the particles drag more fluid to the left than to the right until mechanical equilibrium is achieved.

The picture of osmotic pressure that emerges from this model is shown in Figure 1b. According to Equation [9] particles diffusing close to the membrane have their random motions biased away from the membrane by the force  $-\nabla\phi$ , and this force exerted on the particles by the membrane is transferred to the fluid. The membrane pushes on the particles—or, equivalently, the particles push on the membrane—and the particles drag fluid away from the membrane. Until equilibrium is reached the particles drag more fluid away from the membrane than towards it. At equilibrium this drag force on the fluid near the membrane is still present, but it is balanced by a pressure gradient that develops near the membrane. Seen from this viewpoint, osmotic pressure is generated by biasing the random motions of the solute particles, which induces a viscous drag of the fluid by the particles.

### Nonequilibrium and nonideal osmotic flows

The analysis of the microscopic model that we have just done for the equilibrium case generalizes readily to the nonequilibrium situation. The key result of the foregoing analysis was that the external forces exerted on the solute particles (e.g., by a membrane) are transferred to the fluid (see Equations [8] and

[9]). This same result holds in non-equilibrium situations. Thus, to obtain macroscopic equations corresponding to our microscopic model, we need only add an external force density to the fluid equations of the form  $c(\mathbf{x},t)(-\nabla\phi(\mathbf{x},t))$ , since  $c(\mathbf{x},t)$  is the local density of solute particles and  $(-\nabla\phi(\mathbf{x},t))$  is the external force per particle. In the case where fluid inertia may be neglected, the macroscopic equations are:

$$\nabla p = \mu \nabla^2 \mathbf{u} - c \nabla \phi \quad (\text{Stokes equations}) \quad [17]$$

$$\nabla \cdot \mathbf{u} = 0 \quad (\text{Incompressibility}) \quad [18]$$

Here the local body force in the fluid equations is just the force per particle times the local particle concentration. These equations must be supplemented by the solute particle conservation equation, which takes the form:

$$\frac{\partial c}{\partial t} = -\nabla \cdot \left[ \underbrace{-\frac{kT}{\beta} \nabla c}_{\text{Diffusion}} + \underbrace{\mathbf{u}c}_{\text{Convection}} + \underbrace{\frac{-\nabla \phi}{\beta} c}_{\text{Drift}} \right] \quad \text{Conservation of particles} \quad [19]$$

Note that the velocity multiplying  $c$  in Equation [19] is a sum of two terms: the macroscopic convection velocity,  $\mathbf{u}$ , and the drift velocity relative to the fluid generated by the external force per particle:  $\mathbf{v}_{\text{drift}} = -\nabla\phi/\beta$ . At equilibrium, both the fluid velocity and the solute flux (the quantity in square brackets in Equation [19]) must be zero. Thus,

$$c \nabla \phi = -kT \nabla c \quad [20]$$

which, when substituted into the fluid equations yields:

$$\nabla p = kT \nabla c \quad [21]$$

as before.

It is not easy to generalize the macroscopic equations [17] - [19] to cover the case of interacting particles (although it is easy to include chemical reactions by simply adding a reaction term to Equation [19] and a separate force term to Equation [17] for each species). The advantage of the microscopic equations, however, is that they can describe not only ideal solutions, where the particles do not interact, but also polymer solutions where the particles are coupled. For example, instead of placing the particles inside a vesicle, let them be polymerized into a chain. The equations for polymer osmotic pressure are obtained by adding the inter-particle forces to the potential in Equation [1]. Neglecting inertial forces, the equations for a flexible polymer inside a vesicle take the following form (Figure 2a).

$$\underbrace{\beta_k \frac{d\mathbf{X}_k}{dt}}_{\text{Viscous drag}} = \underbrace{-\frac{\partial \Phi}{\partial \mathbf{X}_k}(\mathbf{X}_1, \dots, \mathbf{X}_N)}_{\text{Forces on } k\text{th monomer}} + \underbrace{\mathbf{R}_k(t)}_{\text{Random force}} \quad [22]$$

Where the potential,  $\Phi$ , is given, for example, by:

$$\Phi(\mathbf{X}_1, \dots, \mathbf{X}_N) = \underbrace{\sum_{i=1}^n \phi(\mathbf{X}_i)}_{\text{Membrane potential}} + \underbrace{\sum_{k=1}^{N-1} \frac{k_0}{2} (|\mathbf{X}_{k+1} - \mathbf{X}_k| - D_0)^2}_{\text{Extensional rigidity of the polymer}} + \underbrace{\sum_{k=2}^{N-1} S_k \frac{k_1}{2} |\mathbf{X}_{k+1} + \mathbf{X}_{k-1} - 2\mathbf{X}_k|^2}_{\text{Bending rigidity of the polymer}} \quad [23]$$

The forces acting on the particles now include those holding the chain together. These forces are directed (on the average) towards the center of mass of the polymer so as to oppose the diffusive dispersal of the monomers that would result from the random force if the monomers were not linked together into a polymer chain. Just as in the simpler case of independent solute particles (see Equation [8]), these forces are transferred to the fluid in the same direction. Since the fluid is incompressible, there can be no net flow into the center of the polymer coil, but there will be an increase in hydrostatic pressure at the center as the polymer hydrates. If the particles are contained inside a vesicle, then some of the force they formerly imparted to the membrane (and thus to the fluid in the opposite direction) is now transmitted through the interparticle bonds. Then the net force on the fluid at the membrane is less than when the monomer particles were uncoupled, and so the osmotic pressure of the polymer is less than that of its independent monomer particles. How much does the length and flexibility of the polymer chain affect the osmotic effect (i.e. the solvent drag) of each monomer? This is an interesting question that we hope to answer through computer simulation.

If several charged polymers are crosslinked into a gel, the contribution of the counterions to the swelling pressure can be computed as a Donnan equilibrium (Figure 2b). This result can be obtained from Equation [9] as well by letting the particles be the mobile counterions and setting the force acting on them,  $-\nabla\phi$ , equal to the Donnan potential. That is, the Donnan potential acts as an electrostatic membrane around the gel which pushes on the particles, and so imparts forces to the solvent. There are many interesting dynamic situations that one can address with the osmotic model. For example, it will be intriguing to see if our osmotic equations produce the same swelling dynamics as predicted by Tanaka's well accepted diffusion equation (Tanaka and Fillmore, 1979). The Brownian drag model predicts that the counterions would generate an increased hydrostatic pressure inside the gel as they attempt to pump incompressible fluid into the gel interior. The Tanaka model, however, does not predict this effect, although it fits the swelling kinetics quite well. In principle, this increase in internal hydrostatic pressure could be measured as a test of the model's validity.

**Figure 2.** (a) Solute molecules joined into an elastic polymer. Since we neglect inertial forces, the velocity vector of the  $k$ th monomer,  $dX/dt$ , is proportional to the resultant of the random force,  $R(t)$ , the external force,  $-\nabla\phi$ , and the internal torsional and extensional elastic forces. This will not be the same as the velocity of the monomer in isolation, and so the solvent dragged by the  $k$ th monomer will be smaller, on the average, when it is a member of a polymer than when it acts alone.

(b) Because of the constraint of electroneutrality, a polyelectrolyte gel generates a Donnan potential at its surface which acts as an "electrostatic membrane" confining the mobile counterions. This increases the hydrostatic pressure inside the gel, contributing to its swelling pressure.

## Discussion

The physical origins of osmotic flow have been the subject of considerable controversy (e.g. Hammel, 1979; Kiil, 1989). It is not our purpose to enter into that debate here. Rather we offer a simple dynamical model for computing the osmotically driven fluid flow in a variety of complex, nonequilibrium situations. Intuitively, the model is appealing, for one can interpret osmotic flows and pressures entirely in terms of the viscous drag of fluid by Brownian particles. The key idea is that the solute particles are excited by Brownian forces generated by the fluid in their immediate neighborhood and that they are simultaneously retarded by viscous drag of the same fluid. It follows that the fluid sees these same forces equally and in the opposite direction. Consequently, since we neglect the mass of the solute particles, any external force applied to the particles—for example, repulsion from a semi-permeable membrane—is transferred to the fluid by viscous drag. That is, the membrane acts on the fluid using the particles as its agent.

Parenthetically, we note that the same model applies to another colligative property: vapor pressure lowering. Here the "membrane" is the liquid-vapor interface, which is permeable to solvent (by evaporation), but impermeable to a nonvolatile solute. The thin region near the surface where the solvent is being dragged away from the interface is at a lower pressure than the bulk of the solution, and since this interface governs the escape probability of solvent molecules, it is clear that solutes must lower the vapor pressure.

In its simplest form, this microscopic model of osmotic pressure involves four parameters: (i) the fluid viscosity,  $\mu$ ; (ii) the drag coefficient,  $\beta$ , between particles and fluid, which depends on the size and shape of the particles; (iii) the absolute temperature,  $T$ , which influences the intensity of the random force (see Equation [3]), and (iv) the diameter,  $d$ , of the local region of fluid that that interacts with each solute particle (one might also vary the shape of the function  $\delta_d$ , but the diameter of its support is presumably its most important characteristic). Of course  $\beta, \mu$ , and  $T$  are physical parameters which can be measured or calculated for each situation. The meaning of the parameter  $d$  is less clear, however. It is tempting to equate it with the diameter of the solute particle, but it remains to be seen whether the physical

consequences of changes in the effective particle diameter can be simulated by varying  $d$ . For polymer molecules there are additional parameters such as bending rigidity that must be given to completely specify the model.

The model equations reduce to the usual equations at equilibrium. They generalize easily to quite complex situations where particles interact with one another, for example if they are polymerized and/or crosslinked into a gel. Therefore, one can compute the effect of polymer mechanical properties such as bending or extensional stiffness on osmotic pressure and flow. In principle, this permits one to compute the effects of coupling mechanical and osmotic stresses, as measured by Ito and his colleagues (Ito et al., 1989; Ito et al., 1987; Suzuki et al., 1989). In order to carry out such calculations, however, one must solve a rather large set of molecular dynamical equations<sup>1</sup>. Fortunately, computers of sufficient speed are now available for this task. Moreover, recent progress in molecular dynamics algorithms has made it possible to take large enough time steps to make this task a reasonable undertaking (Peskin & Schlick, 1989; Schlick & Peskin, 1989; Peskin, 1990), and the computational apparatus for coupling immersed elastic particles to a viscous incompressible fluid has already been developed in a different context (Fogelson & Peskin, 1988). We shall report on such calculations in subsequent publications. For now, however, we present this picture of osmotic flow in the hopes that it will provide both a new intuitive view of the phenomena as well as a practical method for computing osmotic flows and pressures in complex situations.

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<sup>1</sup> Note, however, that molecular dynamics is applied only to the solute particles; the solvent is modeled macroscopically.

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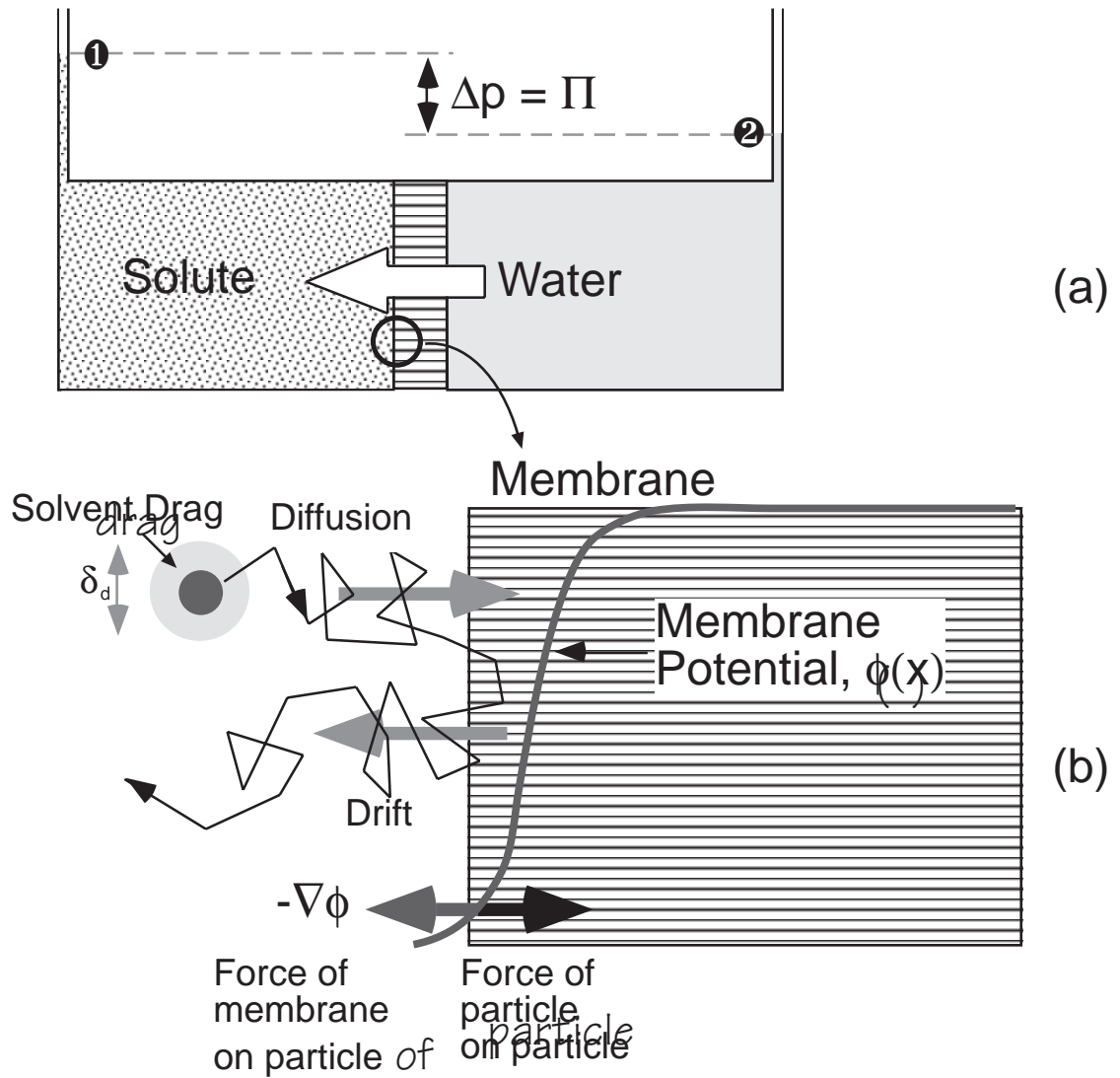


FIG. 1

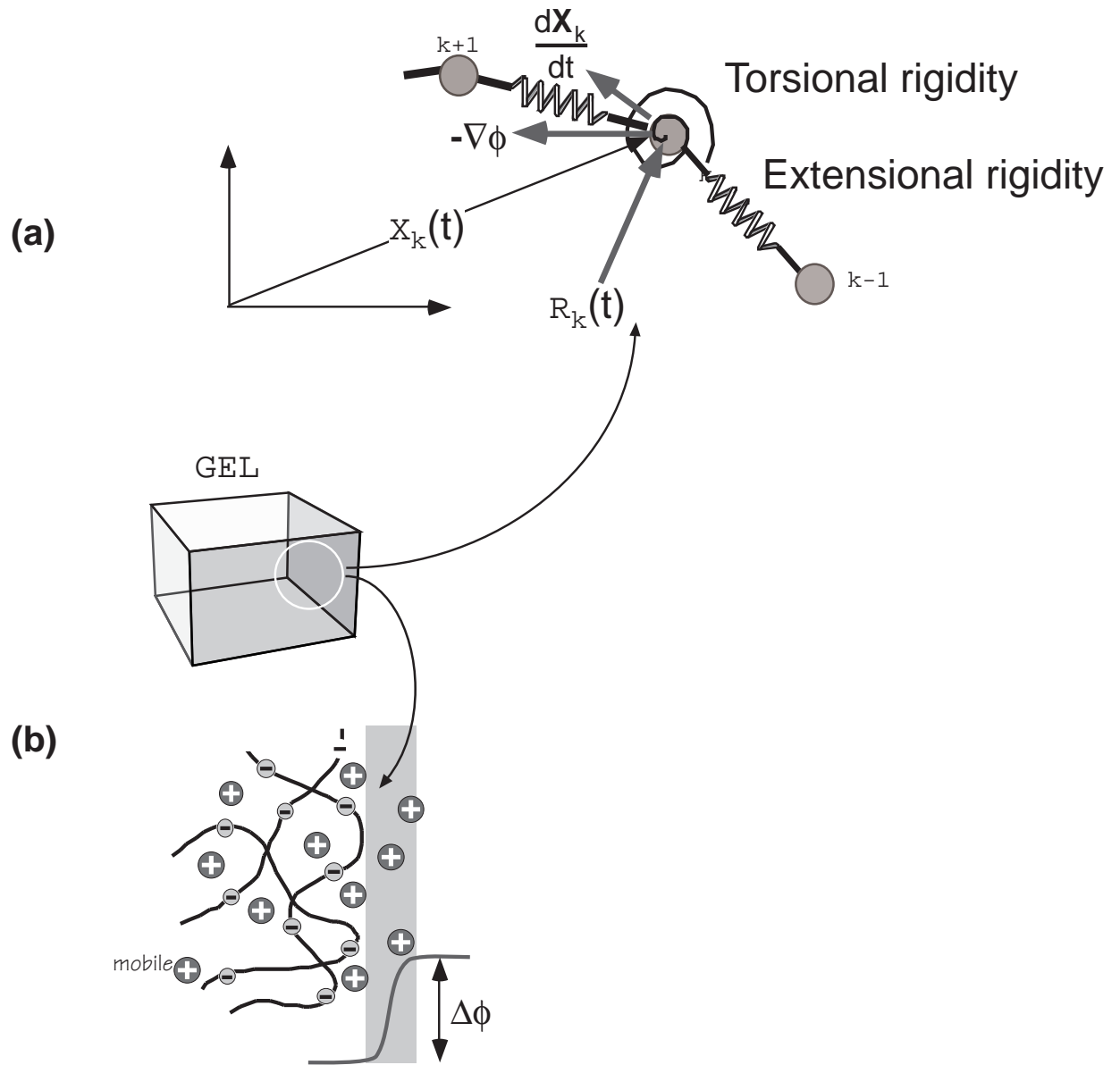


FIG. 2