Evaluating a new method for reconstructing forest conditions from General Land Office survey records

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Abstract

Historical forest conditions are often used to inform contemporary management goals because historical forests are considered to be resilient to ecological disturbances. The General Land Office (GLO) surveys of the late nineteenth and early twentieth centuries provide regionally quasi-contiguous datasets of historical forests across much of the Western United States. Multiple methods exist for estimating tree density from point-based sampling such as the GLO surveys, including distance-based and area-based approaches. Area-based approaches have been applied in California mixed-conifer forests but their estimates have not been validated. To assess the accuracy and precision of plotless density estimators with potential for application to GLO data in this region, we imposed a GLO sampling scheme on six mapped forest stands of known densities (159-784 trees ha\(^{-1}\)) in the Sierra Nevada in California, US, and Baja California Norte, Mexico. We compared three distance-based plotless density estimators (Cottam, Pollard, and Morisita) as well as two Voronoi area (VA) estimators – the Delincé and mean harmonic Voronoi density (MHVD) – to the true densities. We simulated sampling schemes of increasing intensity to assess sampling error. The relative error (RE) of density estimates for the GLO sampling scheme ranged from 0.36 to 4.78. The least biased estimate of tree density in every stand was obtained with the Morisita estimator and the most biased was obtained with the
MHVD estimator. The MHVD estimates of tree density were 1.2-to-3.8 times larger than the true densities and performed best in stands subject to fire exclusion for 100 years. The Delincé approach obtained accurate estimates of density, implying that the Voronoi approach is theoretically sound but that its application in the MHVD was flawed. The misapplication was attributed to two causes: 1) the use of a crown scaling factor that does not correct for the number of trees sampled; and 2) the persistent underestimate of the true VA due to a weak relationship between tree size and VA. The magnitude of differences between true densities and MHVD estimates suggest caution in using results based on the MHVD to inform management and restoration practices in the conifer forests of the American West.

Key words: Forest, California, density, plotless density estimator, Voronoi area

Introduction

Forests in the western United States are threatened by a suite of stressors that include altered fire regimes, legacy effects from timber harvesting, a warming climate, chronic air pollution, and uncharacteristically severe attacks by insects and pathogens (Bytnerowicz et al. 2013, Perry et al. 2011, Hessburg et al. 2016). In response, management seeks to restore diverse landscapes that maintain native species and characteristic processes (North et al. 2009, Hessburg et al. 2016, Stephens et al. 2016). These management goals are informed by the structure and composition of the forest prior to European settlement (Churchill et al. 2013), a time when western forests are thought to have sustained form and function despite fire, drought, and insect attack (Mast et al. 1999, Stephens et al. 2015). Given the current emphasis on forest restoration
and resilience in public lands (USFS 2011, USFS 2013) accurate characterizations of historical forests are particularly important.

Estimates of pre-settlement forest conditions are derived from data with inherent limitations. Historical inventories are a primary source of information (Stephens 2000, Hagmann et al. 2013) but even the most geographically extensive records (e.g., > 10,000 ha, Stephens et al. 2015) may not necessarily represent regional forest characteristics. Forest reconstructions using fire scars, stumps, and tree-rings also provide useful references (Taylor 2004, North et al. 2007) but physical evidence of the pre-settlement forest degrades with time and the effort involved constrains the spatial extent of the reconstruction. In contrast, the public land survey system conducted by the General Land Office (GLO) is a systematic, historical sample of trees over a broad domain from Ohio to the West Coast of the US (Schulte and Mladenoff 2001).

The purpose of the GLO survey was to delineate boundaries of federal lands for sale. The survey consisted of 9.7 x 9.7 km (6 x 6 mile) townships containing thirty-six 1.6 x 1.6 km (1 x 1 mile) sections. Section corners marked with permanent monuments demarcated the end of 1.6-km section lines and quarter corners were located equidistant to two section corners. In order to provide reference points to the corners, nearby bearing trees (also called witness trees) were selected. Section corners were referenced with four bearing trees; quarter corners were referenced with two bearing trees. For each bearing tree, the distance and direction from the point along with species identification and stem diameter were recorded. In effect, these bearing tree records represent a systematic sample of forest conditions (Schulte and Mladenoff 2001).

While the GLO bearing tree data overcome the restricted geographic extent shared by historical inventories and forest reconstructions, it is a sparse sample (i.e., 1 point per 0.8 km) beset by questions regarding data quality and analysis (Bourdo 1956, Bouldin 2008, Hanberry et
Cottam (1949) was one of the first to use the pre-settlement survey to estimate past forest characteristics. As he noted, the key challenge is an accurate estimate of tree density from the information contained in the bearing trees. Although surveyor bias and error in the selection of bearing trees are concerns (Bourdo 1956, Grimm 1984, Bouldin 2008, Williams and Baker 2011, Liu et al. 2011), this paper focuses on the fundamental task of calculating tree density from distance data (e.g., Cottam and Curtis 1956, Morisita 1957, Pollard 1971, Williams and Baker 2011). Specifically, we evaluated the performance of a new plotless density estimator (PDE) developed to maximize the utility of GLO data to reconstruct pre-settlement forests in the arid western US by using area-based metrics (*sensu* Williams and Baker 2011, Baker 2014) as opposed to traditional distance-based estimators.

**Background**

Plotless density estimators (PDEs) are a frequent alternative to plot-based sampling for forest inventory (Kronenfeld 2009). They rely on point-to-tree and/or tree-to-tree distances to efficiently sample heterogeneous landscapes. Of the many possible PDEs (e.g., Engeman et al. 1994), we tested the bias and precision of three PDEs that have been applied by previous studies to GLO data. These PDEs are appropriate to GLO data because they rely only on the distance data recorded by surveyors when documenting bearing trees. We refer to these PDEs by the name of their most often-cited source: Cottam, Pollard, and Morisita (Table 1). The equations share a similar format to estimate tree density from a sample of points where the distance to the nearest tree is measured in each sector (four 90° sectors for corner points; two 180° sectors for quarter corners). This common format for tree density (λ) includes: a unit scalar (K), an
expression of design parameters (\(\Omega\)), and an estimate of either the squared mean or the mean squared point-to-tree distance (\(ptd\)):

\[
\lambda \propto K \ast \Omega \ast \frac{1}{ptd^2 \text{ or } (ptd)^2}
\]

Equation 1

Applications of the Cottam estimator to GLO survey data include Rhemtulla et al. (2009) and Maxwell et al. (2014); the Pollard — Kronenfeld and Wang (2007); the Morisita — Hanberry et al. (2012).

The theoretical basis for these three PDEs rests on the assumption that the distribution of the point-to-tree distances indicates the average surface area occupied by the average tree (Cottam and Curtis 1956, Morisita 1957). Cottam and Curtis (1956) consider this sample space as the exclusive surface area occupied by the nearest trees. Thus the reciprocal of the average area occupied (area per tree) is the density (trees per area). However point-to-tree distances are unbiased variables only when the spatial distribution of trees follows a random pattern and the density of trees remains constant (Kleinn and Vilčko 2006). The PDEs vary in their performance, but in general accuracy and precision decrease as the trees diverge from random spacing and/or tree density varies across the landscape (Cottam and Curtis 1956, Engeman et al. 1994, Kronenfeld 2009). The challenge posed by a non-random distribution of trees is particularly important in dry forests of western North America, where clumped tree spacing is typical (Larson and Churchill 2012).

The new PDE recommended by Williams and Baker (2011) uses the predicted Voronoi area (VA) of individual trees to overcome the limitations of distance-based estimators. VAs are applied in a variety of disciplines to describe spatial positions of points on a plane (Okabe et al. 2000). Mark and Esler (1970) were among the first to suggest a design-based, point-to-tree PDE that relied on the exclusion zone of individual trees to avoid potential biases introduced by the
point pattern of the trees. This zone delineates a polygon where the tree defining the polygon will be the closest to any sample point in the polygon. Delincé (1986) subsequently provided the theoretical justification for this approach and the application to calculate tree density from point samples (Table 1). In Delincé’s (1986) method, the exclusion zone is the VA with the vertices of the VA defined by half the distance to each neighboring tree. The obvious constraint with the Delincé PDE with regard to the GLO data is that only the distance to the nearest tree in each sector is recorded at the corner points. There is no information on the location of the neighboring trees in GLO data.

The innovation in Williams and Baker (2011) was the adaptation of Delincé’s (1986) approach to the GLO data. Since there was no way to measure the VA of bearing trees in historical data sets such as the GLO data, they developed region-specific regression equations from contemporary data that predict VA as a function of tree diameter and local tree density, which can then be applied to historical data (Table 1). The accuracy of the VA prediction is crucial. The expectation that tree size is a good indicator of VA assumes that competition is a major factor driving the spatial arrangement of trees. In short, bigger trees are expected to have larger exclusion zones. A host of distance-dependent tree competition indices incorporate tree size to account for this size-dependency (Biging and Dobbertin 1992, Aakala et al. 2013) but competition is not only distance-dependent (Lorimer 1983). For mixed-conifer forests in the Sierra Nevada, Biging and Dobbertin (1995) reported that distance-independent competition indices performed slightly better than the best distance-dependent measures. The assumption inherent in Williams and Baker’s (2011) methodology that the size of a tree is the most important determinant of its spatial configuration relative to neighboring trees must be investigated in order to determine whether the density estimates derived from this approach are accurate.
Research objectives

The application of this new area-based PDE to GLO data from dry conifer forests across the western US has often produced drastically higher estimates of pre-settlement tree density than expected (Williams and Baker 2012, Baker 2012, Baker 2014). For example, in the Californian mixed-conifer forests on the western slopes of the Sierra Nevada, pre-settlement tree density estimates based on Baker’s (2014) analysis of GLO data exceeded estimates based on historic forest inventories by 200 to 500% (Stephens et al. 2015). Hagmann et al. (2013, 2014) found similar discrepancies between the historical forest inventories they analyzed in mixed-conifer forests of central and northern Oregon and density estimates for the same areas reported by Baker (2012) using the new area-based PDE. These results challenge our understanding of the natural dynamics of the Sierra Nevada and eastern Oregon mixed-conifer forests where frequent low- and moderate-severity fires were thought to maintain relatively open forests (Hessburg et al. 2015, Safford and Stevens, in press). Moreover, Baker’s (2014) reconstructions raised important questions regarding the appropriate strategy to restore contemporary Sierra Nevada forests (Odion et al. 2014, Hanson and Odion 2016).

This new perspective on the historical western forest has engendered criticism (e.g., Fulé et al. 2014, Hagmann et al. 2013, Williams and Baker 2014, Baker 2014) but the performance of the novel density estimator that underpins these controversial findings has not been independently tested. Therefore, we simulated a GLO sampling scheme in six 4 ha mapped stands that span much of the latitudinal range of mixed-conifer forest in the California Floristic Province, some of which had experienced fire exclusion whereas others had relatively intact fire regimes. Mapped stands, where the location of all trees is known, are required to verify the true
VA of trees. We compared the accuracy of traditional PDEs to the method used by Baker (2014) and assessed whether VA regression equations are able to accurately predict true VA. Given the potential of the GLO bearing tree data to characterize pre-settlement forest conditions of the United States, it is essential to review the theoretical basis of any new PDE and to test its application.

**Methods**

**Study sites**

We identified six old-growth mixed-conifer and pine-dominated stands across the latitudinal gradient of the Sierra Nevada and Sierra de San Pedro Mártir of California, United States and Baja California, Mexico (Table 2). Sites included three stands on the western slope of the Sierra Nevada: one stand in the Plumas National Forest (PLUM), one stand in the Teakettle Experimental Forest (TEAK), and one stand in Yosemite National Park (YOSE). A fourth site (BRID) was located on the eastern slope of the Sierra Nevada in the Humboldt-Toiyabe National Forest. All of the California stands had not been harvested but experienced approximately 100 years of fire exclusion. The remaining two stands were in the Sierra de San Pedro Mártir National Park on the western slope of the Sierra de San Pedro Mártir, where fire suppression did not begin until the 1970s (Stephens et al. 2010). The two sites in Baja differ in their underlying geology, with one site on soil derived from metamorphic parent material (META) and the other site on soil derived from granitic parent material (GRAN; Stephens and Gill 2005, Fry et al. 2014). In addition to covering a latitudinal gradient of the mixed-conifer and pine forests, these stands also encompass a density gradient ranging from 159-784 trees ha$^{-1}$ (Table 2). Each stand included in this study was 4 ha in area and included the mapped locations of all stems ≥ 9.5cm.
diameter at breast height (DBH; 1.37 m; Figure 1). Five of the stands were 200 × 200 m in configuration and the PLUM stand was 100 × 400 m. To illustrate the range of results, we included figures from two sites: YOSE, a site that is representative of contemporary density in the Sierra Nevada (FIA 2015), and GRAN, a site that is representative of pre-settlement density (Stephens et al. 2015). Figures for the other four sites can be found in Appendix S1.

Site-specific spatial patterns

We applied an inhomogeneous Ripley’s $L$ function ($L$) to determine the spatial patterning of the trees at each of the six stands. The inhomogeneous function avoids assuming a null model characterized by a homogeneous Poisson process, which may result in the misinterpretation of the point structure when point density varies within a mapped study area (Wiegand and Moloney 2004). We used a radius of 25 m at the PLUM stand (the maximum allowable radius due to the stand configuration) and a radius of 30 m at the other five stands. Thirty meters was chosen as the cutoff because this radius was larger than the maximum distance required to locate a nearest neighbor to a random sampling point in the least dense stand (GRAN, 28 m maximum distance to nearest neighbor). Thus, a 30-m radius captures all potential tree-to-tree competitive interactions relevant to PDE calculations. For each stand, we simulated 1,000 runs of a random distribution of trees to determine the 95% confidence interval (CI). The empirical $L$ value from the mapped trees was compared to the simulated 95% CI to determine at what scales the distribution of trees significantly differed from a random distribution (Cressie 2015). The criterion for significance was the non-overlap of observed $L$ with the 95% CI of the random simulations.
GLO density estimators

We compared density estimates from the six mapped stands based on the three traditional PDEs (i.e., the Cottam, Pollard, and Morisita) as well as the mean-based harmonic Voronoi density (MHVD, Table 1). Williams and Baker (2011) found the MHVD to be one of the most accurate Voronoi-based estimators they tested, and Baker (2014) applied the MHVD to sites in the Sierra Nevada of California. The calculation of the traditional PDEs relies on the information contained in the GLO survey, namely the number of corners (n), the number of sectors (m), and the point-to-tree distances (r) in each sector (Table 1). In contrast, the MHVD requires predicting the mean Voronoi area (MVA) via a three-step process: 1) predict a tree’s crown radius (CR) from the allometric relationship between crown size and stem diameter; 2) calculate the mean neighborhood density (MND) correction factor; and 3) estimate a tree’s MVA from its CR scaled by the MND (Williams and Baker 2011). The CR for tree i was predicted from the diameter at breast height (DBH, breast height = 1.37 m) of tree i according to the equation:

$$\ln(CR) = a + b \times \ln(DBH)$$  \hspace{1cm} \text{Equation 2}

where a and b are species-specific regression parameters for CR (Table D1 in Baker 2014). At each corner, the MND was calculated as:

$$MND_j = \frac{1}{(\sum_{i=1}^{m} r_{ij})^2}$$  \hspace{1cm} \text{Equation 3}

where r is the distance from the sampling point j to the nearest neighbor tree i in m sectors (Williams and Baker 2011). The MVA of tree i at sampling point j was then estimated as:

$$MVA(tree_i) = \exp \left( a + b \times \left( \frac{\ln(CR(tree_i))}{MND_j} \right) \right)$$  \hspace{1cm} \text{Equation 4}

where a and b are species-specific regression parameters for MVA (Table D1 in Baker 2014), MND j from Equation 3, and CR(tree i) from Equation 2. Note: we used DBH to estimate CR in
accord with the equation in Baker (2014). However, an unpublished erratum (W. Baker, personal communication) corrected the CR predictor to diameter at stump height (DSH; 0.30 m). Thus we also calculated MHVD with DSH. See Appendix S2 for details.

**Bias and precision assessment**

We applied simulations to quantify the performance of the PDEs. Each test included 1,000 realizations of randomly placed sampling corners in each of the six mapped stands. Sample corner intensity spanned the range of GLO point pools considered by Williams and Baker (2011) — three, six, nine, and 21 — as well as the recommended minimum intensity of 50 points (Kronenfeld 2009, Hanberry et al. 2011) and a saturation intensity of 1,000 points. To minimize edge effects, the location of the random points was excluded from a buffer zone along the stand boundaries with the buffer width ranging from 10-20 m depending on tree density. Less dense stands required a larger buffer to ensure the presence of a bearing tree in each sector for every simulated point.

The GLO sampling regime includes section corners with four bearing trees (one tree in each 90° quadrant) and quarter corners with two bearing trees (one tree in each 180° semicircle). Bearing trees are referred to as nearest neighbors (nn) for the purposes of density calculation. When calculating the PDEs, we treated all corners as either sections corners with four bearing trees (4nn) or quarter corners with two bearing trees (2nn). We did not apply the correction factors developed in Williams and Baker (2011) to allow the mixing of results from section corners and quarter corners. While the correction factors increase the number of trees available from the GLO data because both 4nn and 2nn corners can be included, the use of the weights has not to our knowledge only been vetted for the Cottam (Cottam and Curtis 1956). Moreover, it is...
clear from both theory and practice that the number of neighbors measured greatly affects the estimate (Morisita 1957, Engeman et al. 1994, Kronenfeld 2009). For example, Kronenfeld (2009) demonstrated for the Pollard that the bias in the estimated density diminishes as nn increases, ultimately approaching an asymptote near the true density as nn approaches 50. By excluding correction factors, our comparisons avoided a potential confounding factor.

Although not appropriate for GLO data because it requires measuring the VA, we included the Delincé PDE in the simulation because it constitutes the theoretical underpinning of the MHVD approach. Its inclusion can help to deduce the source of any bias observed. To compute the Delincé (Table 1), the nearest bearing tree to each point (1nn) was selected and its true VA was calculated from the mapped trees.

Results from the 1,000 realizations were summarized by the median as the measure of central tendency and the 95% CI as the measure of variation. Bias was defined by the relative error (RE):

$$RE = \frac{\lambda_{\text{sim}}}{\lambda_{\text{true}}}$$  \hspace{1cm} Equation 5

where $\lambda_{\text{sim}}$ is the median density from the 1,000 realizations and $\lambda_{\text{true}}$ is the true density of the mapped stand. $RE < 1$ implies underestimates; $RE > 1$ implies overestimates. Precision was defined by the relative root mean square error (rRMSE):

$$rRMSE = \frac{\text{RMSE}_{\text{sim}}}{\lambda_{\text{true}}}$$  \hspace{1cm} Equation 6

where $\text{RMSE}_{\text{sim}}$ is the root mean square error of the 1,000 realizations in each simulation and $\lambda_{\text{true}}$ is the true density of the mapped stand. Larger values of rRMSE imply less precision.
**MHVD deconstruction**

We evaluated the three steps needed to calculate the MHVD. For crown radius (CR, Equation 2), we compared the regression fits described in Baker (2014) to equations estimated from Forest Health Monitoring (FHM) data collected as part of the Forest Inventory and Analysis Program. This database included measured crown radii from monitoring plots in the mixed-conifer forest of the Sierra Nevada in California (http://www.fia.fs.fed.us/tools-data/other_data/index.php). Predictions of CR were fit from the FHM data for all species present in the six stands (ten species total; \( n \) per species ranging from 59-746).

Williams and Baker (2011) used the mean neighborhood density (MND, Equation 3) to adjust the MVA prediction by the local density, with the assumption that a tree of a given diameter will have a larger MVA in a less dense stand and a smaller MVA in a more crowded stand. To isolate the effect of the MND correction, we simulated a 6-point section corner (4nn) and a 6-point quarter corner (2nn) sampling scheme in each of the six mapped stands and reported the median MND and 95% CI (from 1,000 realizations) for the 2nn and 4nn sampling schemes. This 6-point scheme is recommended by Williams and Baker (2011) as being sufficient for estimating density with a 22% relative mean absolute error (RMSE). We used the 1,000 simulated MND values to estimate the median and 95% CI of the MVA for a representative tree (Equation 4). For each stand, the representative tree was defined by the dominant species and the median DBH; MVA was fit using the equation parameters in Baker (2014).

We tested the strength of the relationship between CR/MND and true VA by least-squares regression. We calculated the MVA regression parameters in Equation 4 for the common species (>5% dominance) in each stand and compared them to the species-specific fits in Baker (2014). Only section corners (4nn) were considered for the MND values in this analysis to avoid...
potential confounding errors from calculating MND with two or four bearing trees. CR was estimated using parameters derived from the FHM data set (as above); MND was calculated for 21 regularly spaced section corners (4nn). True VAs were calculated from mapped tree locations. Thus in this analysis we had large, well-distributed samples for both the CR and MVA regressions.

We assessed the accuracy of the MVA regression equation by comparing the predicted VA of individual trees to the true VA. We used the tree maps to calculate the true VA of each tree. For each stand, a buffer ranging from 5-18 m from the stand boundaries was applied with the width set to ensure that trees included in the analysis had a neighbor on all sides. MND was derived from 21 regularly spaced section corners (4nn, as above). Only section corners were considered in this analysis to avoid potential confounding errors from calculating MND with two or four bearing trees. The MVA for trees was computed using the CR and MVA coefficients in Baker (2014); the recorded tree species and DBH; and the MND for the point nearest to each tree (Equation 4). We also estimated the mean difference (\( \overline{VA_{difference}} \)) between MVA and true VA, calculated as:

\[
\overline{VA_{difference}} = \sum_{i=1}^{n} \frac{VA_{i, true} - VA_{i, MVA}}{n}
\]

where \( i \) refers to the individual tree and \( n \) is the total number of trees. We calculated the comparison for each stand with three different size classes of trees: trees \( \geq 9.5 \) cm DBH, trees \( > 20 \) cm DBH, and trees \( > 60 \) cm DBH. All data processing and analyses were conducted in R ver. 3.2.4 (R Development Core Team 2014); spatial metrics relied on functions in the spatstat package (Baddeley et al. 2015).

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Results

Tree spatial distribution in each stand

Despite differences in geography, density, and composition, the overall spatial pattern was consistent among all six stands. Trees were significantly clumped at spatial scales relevant to the PDE calculation and showed inhibition at larger scales due to the presence of gaps throughout the stand (Appendix S1: Figure S1). The inhomogeneous Ripley’s L values exceeded the 95% CI for randomly spaced trees from 0-1 m at the minimum and 13-29 m at the maximum — an indication of an aggregated distribution (Appendix S1: Figure S1).

Bias and precision assessment

Considering only the GLO appropriate PDEs for a 50-point sampling intensity, the least biased estimate of tree density in every stand was obtained with the Morisita and the most biased was obtained with the MHVD (Table 3). The direction of the bias in the Morisita varied with some underestimates and some overestimates. In contrast, the Cottam and Pollard consistently underestimated true density and the MHVD always overestimated true density. In general, the 4nn sampling had less bias than the 2nn sampling. This difference was particularly pronounced in the MHVD. For the 2nn MHVD, the median REs ranged from 2.04 to 3.58 whereas for the 4nn MHVD, it ranged from 1.16 to 1.59 (Table 3). The performance of the Delincé PDE was exceptional (Table 3). The biases were small and non-directional with median REs ranging from 0.92 to 1.14. The performance in regard to the bias of individual estimators documented for the 50-point sampling intensity was consistent at other sampling intensities (Appendix S1: Tables S1-S9).
The precision of the tree density estimates systematically increased with sampling intensity for all PDEs. Not only did the width of confidence interval shrink with increasing point density (Figure 2, Appendix S1: Figure S2 for the Delincé) but the rRMSE also declined systematically with increasing point density in every case (Appendix S1: Tables S1-S9). Moreover, the 4nn sampling scheme consistently produced more precise estimates (i.e., lower rRMSE) than the 2nn.

Among the PDEs, the MHVD was the least precise for a given sampling scheme and intensity (Figure 2, Table 3). The Morisita tended to produce less precise density estimates at sample intensities <50. The improvement in precision with increasing sample intensity was steepest for the MHVD and Delincé. For both PDEs, the rRMSE was an order of magnitude lower for the 1,000-point sample compared to the 3-point sample (Appendix S1: Tables S1-S9).

In the 50-point sampling simulations, the Delincé obtained not only the least biased estimates of density but also the most precise (Table 3). The Morisita 4nn estimator was the next best in terms of minimizing bias and maximizing precision.

**MHVD deconstruction**

Tree diameter proved to be a robust predictor of crown radius. Based on the coefficient of determination ($r^2$), the fits of ln(CR) to ln(DBH) for conifer trees in the FHM data ranged from 0.43 for *Pseudotsuga menziesii* (PSME, n=196) to 0.83 for *Pinus monticola* (PIMO, n=59). The $r^2$ values for the hardwoods were lower, ranging from 0.22 for *Populus tremuloides* (POTR, n=65) to 0.44 for *Quercus kelloggii* (QUKE, n=254). All fits showed a significantly increasing relationship of ln(CR) to ln(DBH) ($p<0.05$; Appendix S1: Figure S3). The slope and intercepts of
the relationship of ln(CR) to ln(DBH) were generally similar to those listed in Baker (2014) for the same species.

The 4nn sampling scheme resulted in a significantly higher mean distance to nearest trees compared to the 2nn sampling (Figure 3A, C). This difference translated into a lower MND. When we applied the MND to the estimation of the MVA of a representative tree, the 4nn MND correction resulted in significantly higher estimations of MVA for the same tree relative to the 2nn MND (Figure 3B, D). The results were replicated at every site — the 2nn sampling resulted in a higher MVA estimate (Appendix S1: Figure S4).

For the mapped trees in the six stands we tested, CR was a very weak predictor of VA (Figure 4). Although the slope of the regression line was positive and often significant (i.e., p < 0.05), the fits were very poor. The coefficient of determination ($r^2$) ranged from a minimum of 0.003 for *Abies concolor* (ABCO) at the GRAN site (n=73) to a maximum 0.29 for *Pinus jeffreyi* (PIJE) at the META site (n=773; Appendix S1: Table S10).

The MVA calculated with the parameters provided in Baker (2014) consistently underestimated the true VA of the trees at all sites (Figure 5 and Appendix S1: Figure S5). The difference increased with increasing tree size (Table 4). For example, the median VA$_{\text{difference}}$ for all mapped trees (DBH ≥ 9.5 cm) in the six stands was -21.2 m$^2$; for large trees (DBH > 60 cm) the median difference was -36.7 m$^2$. The underestimate of MVA relative to the true VA increased as tree density decreased (Table 4).

Calculating MVA using DSH as opposed to DBH reduced the bias of the MHVD 4nn estimator. However, both the MHVD 2nn and 4nn estimators overestimated stand density in all cases. Additionally, MVA consistently underestimated true VA, regardless of whether MVA was
calculated with DSH or DBH. Results for relevant analyses using DSH in place of DBH are presented in Appendix S2.

**Discussion**

The mean harmonic Voronoi area (MHVD) as specified in Baker (2014) consistently provided the least accurate estimate of tree density among the plotless density estimators (PDE) tested. In every scenario, the MHVD was biased toward overestimating tree density with lower precision than alternative metrics. Two factors contributed to this bias: 1) the difference in mean neighborhood distance (MND) between 2nn and 4nn sampling; and 2) the persistent underestimate of the true Voronoi area (VA).

Williams and Baker (2011) recognized that local tree density influences the allometric relationship between tree diameter and crown radius, specifically that for a given species and DBH, the crown width decreases with increasing tree density (Bragg 2001). They evaluated three different crown scaling factors and found that the MND was ultimately the best. Baker (2014) subsequently applied this MND to scale CR in the Sierra Nevada based on the recommendation of Williams and Baker (2011) and supported by local validation (Appendix D in Baker 2014). The well-known influence of the number of sectors in the CR scaling factor (e.g., Cottam and Curtis 1956) was clearly a concern — one of the alternative neighborhood density equations Williams and Baker (2011) tested explicitly included a correction for mean distances obtained with 2nn vs 4nn sampling (the correction factor neighborhood density, CFND, Table 2 in Williams and Baker 2011). However, the PDE that included the CFND (the correction factor geometric Voronoi estimator, CFGVD) was generally not as accurate as the MHVD in their evaluation (Table 3 in Williams and Baker 2011). In contrast, in our simulations the lack of a
correction factor for the number of bearing trees included in the calculation of MHVD almost doubled (1.85x), on average, the estimate of tree density in the 2nn sampling compared to the 4nn (Table A7, A8). Why this flaw was not noted earlier may be due to two facts. Baker (2014) developed local MVA regressions as functions of CR and MND (Baker 2014, Appendix D) but only used points with four bearing trees. In addition, the performance of the MHVD in the Sierra Nevada was not compared to plot estimates of tree density as done in previous applications (e.g., Williams and Baker 2012, Baker 2012).

Independent of the MND, the MVA equations in Baker (2014) systematically underestimated true VA (Figure 5, Table 4). In particular, the MVA (calculated using a MND based on 4nn) reached a maximum area much lower than the true VA. For example, at YOSE, the median true VA was 18 m² while the median MVA was only 8 m², and 18% of the mapped trees had a true VA that exceeded the maximum MVA (Figure 4). Similar results were obtained at the five other stands and the effect appeared to increase as density decreased (Appendix S1: Figure S5). One explanation for the mismatch could be that samples included in Baker’s (2014) MVA regressions did not capture the full range of variation.

We found a weak predictive ability of CR in our stands, with the r² of true VA~CR/MND for conifers ranging from 0.003-0.29 (Table A10). In contrast, Baker (2014) found strong relationships between MVA and CR with r² values ranging from 0.52-0.95 for these same conifer species. The weak predictive ability of CR in these mixed-conifer forests may be due to the fact that tree spacing can sometimes be more dependent on edaphic factors (Meyer et al. 2007) or disturbance legacies than on competitive interactions. On drier microsites or nutrient-poor soils or in old canopy gaps, some trees grow in relative isolation due to factors unrelated to competition or tree size. These circumstances occur across the semi-arid Western US forests in
general (Larson and Churchill 2012) and specifically in the stands included in our analysis (e.g., Figure 1, North et al. 2002, Fry et al. 2014). Only at our most dense site (PLUM, Table 2) where canopy approaches closure (canopy cover = 90%, Kayler et al. 2005) from the effects of 100 years of fire suppression might spacing be influenced by crown area. Interestingly the least bias in the MHVD density estimate was observed at PLUM (RE = 1.16 for MHVD, 4nn, 50 points, Table 3). Thus, it seems likely that the MHVD approach will overestimate tree density in the Sierra Nevada and in other forests of the arid Western US with similar stand structure.

Our critique of the MHVD does not extend to its underlying theory. Using the Voronoi area to measure the inclusion probability of the nearest tree in a PDE (Delincé 1986) proved to be the best predictor of tree density in all simulations (Table 3, Table S9). However, at the lower sampling intensities (i.e., < 50 sampling points), the precision of the Delincé PDE was low resulting in large confidence intervals. This imprecision is not surprising given that the Delincé PDE only considers the nearest tree to each point (1nn) in its calculation (Table 1). The Delincé PDE also had a tendency to underestimate the true tree density at the lower sampling intervals (Appendix S1: Figure S2). Klein and Vilčko (2006) extend the VA-based PDE to include more than the nearest tree to each point in an effort to improve precision. Advances in tree mapping technology (e.g., laser hypsometers) make measuring the VA of trees in the field more efficient. Thus the VA-based PDEs are viable plotless methods for forest inventory. However, for GLO applications, the challenge of predicting VA with the available data severely limits its utility.

Our assessment of the MHVD as a theoretically sound but technically flawed means to reconstruct historical forest density from GLO data was constrained by the availability of data and the specificity of our question. The spatial scale of our analysis was on the order of hectares whereas the GLO data span km$^2$. As often noted (e.g., Engeman et al. 1994, Kronenfeld and
Wang 2007, Bouldin 2008, Hanberry et al. 2011), the non-random dispersion of trees at the local level and differences in tree density at the regional level affects PDE performance. Thus the stand-level tests presented here probably do not capture the full range of variation in forest structure present in the landscape sampled by GLO survey points, although we attempted to account for this by testing six stands of widely varying densities across >8° of latitude. We also restricted our focus to PDE performance under ideal conditions without complications from sampling inconsistencies or surveyor biases (sensu Bourdo 1956). Ongoing research in the Sierra Nevada and Sierra de San Pedro Mártir (unpublished data, C. Restaino and H. Safford) seeks to address these constraints by testing PDEs at the landscape scale while also quantifying the influence of survey data quality.

The Morisita was consistently the least biased estimator of tree density from GLO data across a gradient of forest conditions (Tables A1-A8). This result supports previous research that found the Morisita to be the best choice when bearing trees have an aggregated distribution. The Cottam (Bouldin 2008) and Pollard (Hanberry et al. 2011) tended to underestimate true density in these situations while the Morisita provided more accurate estimates. Note that we relied on the median as the measure of central tendency instead of the more commonly reported mean (e.g., Engemen et al. 1994, Hanberry et al. 2011). Given the right skew in the distribution of the simulations due to the fact that the minimum density was constrained at 0 while the maximum was unconstrained (i.e., zero-truncation), the median was less than or equal to the mean. Thus our conclusions regarding overestimates are based on a more conservative measure of central tendency. However, we also report the mean density from the simulations to allow comparison with previous efforts (Appendix S1: Tables S1-S9).

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In our simulations, both the Morisita 4nn and Morisita 2nn predictors were unbiased in estimating density at a range of sampling intensities. Although the Morisita 4nn was less biased and more precise than the 2nn (e.g., Table 3), the Morisita 2nn has the advantage of a larger potential sample intensity because both section and quarter corners can be included in the density estimate. Moreover, the Morisita 4nn appears to be sensitive to local non-random dispersion (C. Cogbill, personal observation). The well-vetted performance of the Morisita 2nn suggests it should be considered the current standard for GLO applications.

Pre-settlement forest conditions derived from the GLO records for the Sierra Nevada (Baker 2014) differ substantially from results based on inventories conducted before EuroAmerican influence and on reconstructions from live and dead plant material (Taylor 2004, North et al. 2007, Scholl and Taylor 2010, Collins et al. 2011, Barth et al. 2015, Collins et al. 2015, Stephens et al. 2015, Safford and Stevens, in press). Specifically, Baker (2014) rejected the prevailing hypothesis that the pre-settlement Sierra Nevada mixed-conifer forests were mostly open, park-like landscapes with low tree densities maintained by low to moderate severity fires. For example, in the vicinity of Yosemite National Park in the central Sierra Nevada, Collins et al. (2011) reported an average tree density of 52 trees ha$^{-1}$ for trees ≥15.2 cm DBH in a 1911 timber inventory. Working in the same area, Scholl and Taylor’s (2010) reconstruction of the 1899 forest put tree density at 86 trees ha$^{-1}$ for trees ≥10 cm DBH. In the Yosemite Forest Dynamics Plot, a more mesic forest than Scholl and Taylor (2010), Barth et al. (2015) reconstructed forest density in 1900 to be between 62 – 122 trees ha$^{-1}$ for trees ≥10 cm DBH. In contrast, the GLO estimate for the area including all of these direct density estimates (Table G1 in Baker 2014) was 212 trees ha$^{-1}$. The much greater tree densities obtained from the MHVD analysis of GLO data (Baker 2012, Baker 2014) have been noted for forests in Oregon.
(Hagmann et al. 2013, Hagmann et al. 2014) and California (Stephens et al. 2015, Collins et al. 2015). Notably the magnitude of the differences — namely GLO densities 2 to 5 times larger than ones based on inventories or reconstructions — match the methodological bias detected in our analysis.

The management implications of these contrasting perspectives of the pre-settlement forest are significant. Baker (2014) used GLO-derived tree densities to infer that extensive stand-replacing fire was a major component in the natural disturbance regime of the Sierra Nevada mixed-conifer forests. This conclusion implies that ongoing efforts by forest managers to mitigate wildfire behavior (e.g., North et al. 2009, USFS 2011, USFS 2013) are misguided. Subsequent papers have attempted to add support for this alternate interpretation about historical forest and fire interactions (Odion et al. 2014, Baker 2015), which collectively promote management practices that foster denser forests susceptible to high-severity fire. Although these authors posit other lines of evidence supporting this alternative perspective, the much higher estimates of tree density are the quantitative linchpin of the thesis (Baker 2014). The propensity of the MHVD to significantly overestimate tree density challenges the validity of the argument.

In this era of global change, historical conditions play an increasingly important role as we seek to inform the future by understanding the past (Safford et al. 2012). As forest conditions and disturbance impacts deviate from the range of natural variation (Safford and Stevens, in press), effective intervention depends on our knowledge of forest dynamics and our ability to explain the processes involved (Stephens et al. 2010). The GLO survey represents a spatially extensive window into the pre-settlement forest of the American West that complements the detailed site-specific information obtained from historical inventories and forest reconstructions. Given the paucity of data, there is a premium on methods that extract as much insight as
possible. Innovation must be encouraged. At the same time, new methods must be independently validated, especially when they directly impact management and policy. It is in this spirit of sound scientific practice that we present our findings.

Acknowledgements

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FIA. 2015. USFS Forest Inventory and Analysis National Program. Database version FIADB_1.6_0.0.02. https://www.fia.fs.fed.us/tools-data/

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Supporting Information

Additional supporting information may be found in the online version of this article at


Data Availability

Data associated with this paper have been deposited in figshare

https://dx.doi.org/10.6084/m9.figshare.4749892

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Figures and Tables

Table 1. Equations used to estimate tree density (trees ha\(^{-1}\)) in this study. Term definitions: \(m\) = number of sectors; \(n\) = number of points, \(r_{ij}\) = is the distance from point \(j\) to tree \(i\); \(K\) is the scaling coefficient to return density in trees ha\(^{-1}\); \(\lambda\) is tree density; VA is the measured Voronoi area; and \(MVA\) is the mean Voronoi area (estimated). In this paper, \(r_{ij}\) is measured in meters, thus \(K = 10,000\). Also note in all cases only the nearest tree in each sector is measured. Thus the total number of trees is \(m*n\) (\(mn\)).

<table>
<thead>
<tr>
<th>Name</th>
<th>Equation</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cottam (C)</td>
<td>[\lambda_C = K \times \frac{m}{4} \times \frac{1}{\left(\sum_{j=1}^{n} \sum_{i=1}^{m} \frac{r_{ij}}{nm}\right)^2}]</td>
<td>1</td>
</tr>
<tr>
<td>Pollard (P)</td>
<td>[\lambda_P = K \times \frac{(mn-1)m}{\pi} \times \frac{1}{\sum_{j=1}^{n} \sum_{i=1}^{m} r_{ij}^2}]</td>
<td>2</td>
</tr>
<tr>
<td>Morisita (M)</td>
<td>[\lambda_M = K \times \frac{(m-1)m}{\pi n} \times \sum_{j=1}^{n} \frac{1}{\sum_{i=1}^{m} r_{ij}^2}]</td>
<td>3</td>
</tr>
<tr>
<td>Delincé (D)</td>
<td>[\lambda_D = K \times \frac{1}{mn/\sum_{i=1}^{mn} \left(\frac{1}{VA(tree_i)}\right)}]</td>
<td>4</td>
</tr>
<tr>
<td>Mean Harmonic Voronoi Density (MHVD)</td>
<td>[\lambda_{MHVD} = K \times \frac{1}{mn/\sum_{i=1}^{mn} \left(\frac{1}{MVA(tree_i)}\right)}]</td>
<td>5</td>
</tr>
</tbody>
</table>

References: 1 from Cottam and Curtis (1956); 2 from equation #16 in Pollard (1971); 3 from equation #31 in Morisita (1957); 4 from equation #7 in Delincé (1986); 5 from Table 2 in Williams and Baker (2011).
Table 2. Site description for the six stands included in this study. Density and basal area are for trees ≥9.5 cm DBH.

<table>
<thead>
<tr>
<th>Site</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Elevation (m)</th>
<th>Density (trees ha(^{-1}))</th>
<th>Basal area (m(^2) ha(^{-1}))</th>
<th>% Fir</th>
<th>% Pine</th>
<th>Ref</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLUM</td>
<td>W 121º 02’</td>
<td>N 39º 55’</td>
<td>1158-1219</td>
<td>784</td>
<td>87</td>
<td>87%</td>
<td>3%</td>
<td>1</td>
</tr>
<tr>
<td>YOSE</td>
<td>W 119º 49’</td>
<td>N 37º 46’</td>
<td>1774-1911</td>
<td>562</td>
<td>56</td>
<td>79%</td>
<td>13%</td>
<td>2</td>
</tr>
<tr>
<td>TEAK</td>
<td>W 119º 02’</td>
<td>N 36º 58’</td>
<td>1880-2485</td>
<td>313</td>
<td>56</td>
<td>64%</td>
<td>27%</td>
<td>3</td>
</tr>
<tr>
<td>META</td>
<td>W 115º 59’</td>
<td>N 31º 37’</td>
<td>2400-2500</td>
<td>254</td>
<td>23</td>
<td>0%</td>
<td>100%</td>
<td>4</td>
</tr>
<tr>
<td>BRID</td>
<td>W 119º 28’</td>
<td>N 38º 24’</td>
<td>2500-2600</td>
<td>236</td>
<td>36</td>
<td>38%</td>
<td>50%</td>
<td>4</td>
</tr>
<tr>
<td>GRAN</td>
<td>W 115º 59’</td>
<td>N 31º 37’</td>
<td>2400-2500</td>
<td>159</td>
<td>25</td>
<td>13%</td>
<td>87%</td>
<td>4</td>
</tr>
</tbody>
</table>

Site code: PLUM refers to Plumas National Forest; YOSE refers to Yosemite National Park; TEAK refers to Teakettle Experimental Forest; META refers the site in the Sierra de San Pedro Mártir with soil derived from metamorphic parent material; BRID refers to the site in the Humboldt-Toiyabe National Forest near Bridgeport, CA; GRAN refers to the site in the Sierra de San Pedro Mártir with soil derived from granitic parent material. % Fir and % Pine refer to the percentage of the number of trees in either category in each 4 ha stand. References: 1 from Levine et al. (2016), 2 from Lutz et al. (2012), 3 from North et al. (2007), 4 from Fry et al. (2014).
Table 3. Relative performance of the density estimates for the 50-point simulations. Results are reported as relative values with the results from the 1,000 realizations divided by the true density. rRMSE refers to the relative root mean square error. Bold text indicates site estimates where the 95% CI of the simulation overlaps the true density.

<table>
<thead>
<tr>
<th>Measure (nn trees)</th>
<th>Site</th>
<th>True density (trees ha(^{-1}))</th>
<th>PLUM</th>
<th>YOSE</th>
<th>TEAK</th>
<th>META</th>
<th>BRID</th>
<th>GRAN</th>
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<tr>
<td>Cottam (4 trees)</td>
<td>2.5% CI</td>
<td>0.79</td>
<td>0.66</td>
<td>0.49</td>
<td>0.52</td>
<td>0.63</td>
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<tr>
<td></td>
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<td>0.79</td>
<td>0.62</td>
<td>0.66</td>
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<td></td>
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<td>0.74</td>
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<td>0.47</td>
<td>0.49</td>
<td>0.57</td>
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<td>1.59</td>
<td>1.21</td>
<td>1.41</td>
<td></td>
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<tr>
<td></td>
<td>rRMSE</td>
<td>0.10</td>
<td>0.12</td>
<td>0.34</td>
<td>0.24</td>
<td>0.14</td>
<td>0.18</td>
<td></td>
</tr>
</tbody>
</table>
Table 4. The mean difference between the true VA and MVA (m²; Equation 7) for all trees ≥9.5 cm DBH, trees >20 cm DBH, and trees > 60 cm DBH at each site. Negative values indicate the extent to which the MVA underestimated the true VA. Sites are listed in order of decreasing stand density with PLUM being the most dense and GRAN the least dense.

<table>
<thead>
<tr>
<th>Site</th>
<th>Trees ≥ 9.5 cm DBH</th>
<th>Trees &gt;20 cm DBH</th>
<th>Trees &gt; 60 cm DBH</th>
</tr>
</thead>
<tbody>
<tr>
<td>PLUM</td>
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<td>-6.02</td>
<td>-6.87</td>
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<td>YOSE</td>
<td>-10.1</td>
<td>-9.61</td>
<td>-14.4</td>
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<td>TEAK</td>
<td>-15.6</td>
<td>-16.4</td>
<td>-28.3</td>
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<td>META</td>
<td>-26.7</td>
<td>-39.2</td>
<td>-83.7</td>
</tr>
<tr>
<td>BRID</td>
<td>-33.8</td>
<td>-33.7</td>
<td>-45.1</td>
</tr>
<tr>
<td>GRAN</td>
<td>-43.6</td>
<td>-45.7</td>
<td>-71.2</td>
</tr>
</tbody>
</table>

Figure 1. An example (YOSE) of the mapped tree plots used in the analysis. (A) The spatial distribution of trees in the mapped plot; (B) section corner sampling points (blue) and neighboring trees for a six-point sampling scheme; (C) and true Voronoi areas.

Figure 2. Results of simulations testing the effect of sampling intensity (number of sampled points) on the Cottam, Pollard, Morisita, and MHVD estimators at the YOSE and GRAN sites. Bars show the median value of 1,000 realizations and error bars show the 95% confidence intervals. Simulation summaries can be found in tables A1-A9.

Figure 3. Simulation results (1000 runs) of the mean neighborhood distance (A, C) when two nearest neighbors (2nn) or 4 nearest neighbors (4nn) are used for a 6-point sampling scheme for
the YOSE and GRAN sites, respectively. The discrepancy in mean neighborhood distance is reflected in the estimate of the Voronoi area of a single tree of the dominant species (B, D) of the median DBH in that plot using the method and equation parameters in Baker (2014). Results for other sites are shown in Figure A4. ABCO refers to Abies concolor (white fir); PIJE refers to Pinus jeffreyi (Jeffrey pine).

**Figure 4.** The relationship of true Voronoi area (VA) to crown radius (CR) for the dominant tree species at each of the six study sites. The crown radius was estimated using parameters for species-specific fits derived from Forest Health Monitoring measurements in Sierra Nevada mixed-conifer plots (PIJE, n=164; ABCO, n=746). MND is the mean neighborhood distance (see Equation 3). ABCO refers to Abies concolor (white fir); PIJE refers to Pinus jeffreyi (Jeffrey pine).

**Figure 5.** The relationship between the true VA and the estimated VA (i.e., MVA) in the YOSE and GRAN sites. Estimated VA was calculated using the species-specific equation parameters provided in Baker (2014) and a MND estimated from a grid of 21 sampling points with four bearing trees at each point. Results for the other stands are shown in Appendix S1, Figure S5.
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