Wind and Turbulence, Part 1

A. Processes

A. Wind and Turbulence

1. Concepts

a. Definition of Turbulence

b. Reynolds' Number

c. Conservation equation for wind

d. TKE budget, conceptual

Away out here they got a name
For rain and wind and fire
The rain is Tess, the fire Joe,
And they call the wind Mariah

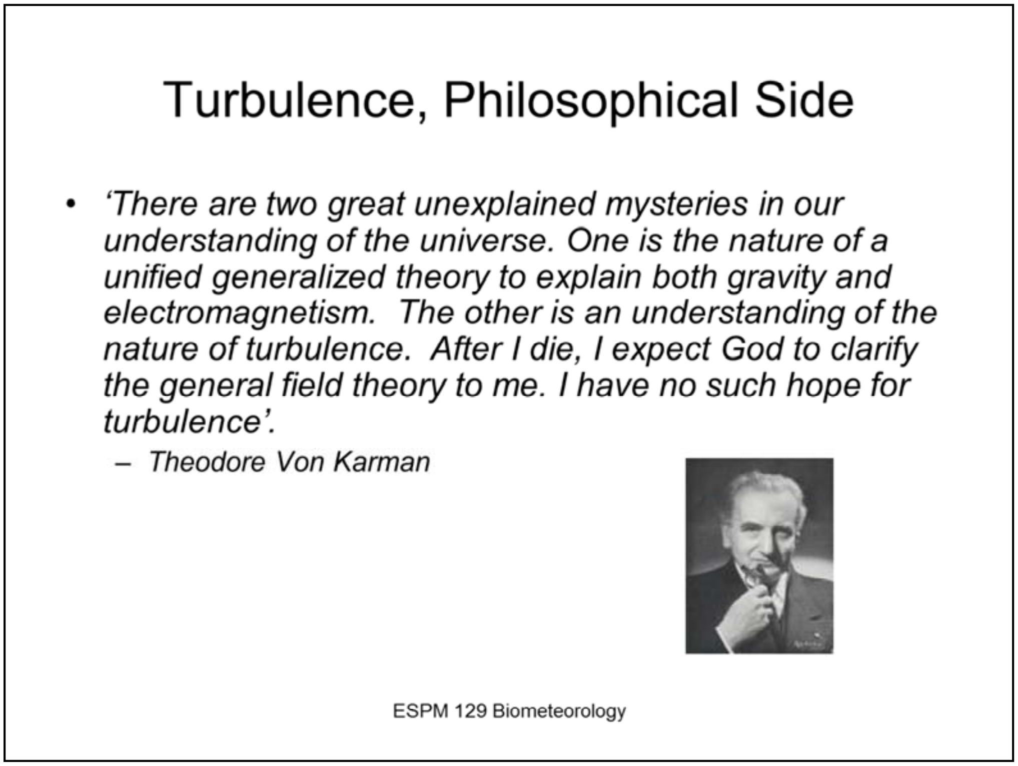
Lerner and Lowe, 'Paint your Wagon'

When I can, gotta find a poem that relates to some aspect of biometeorology

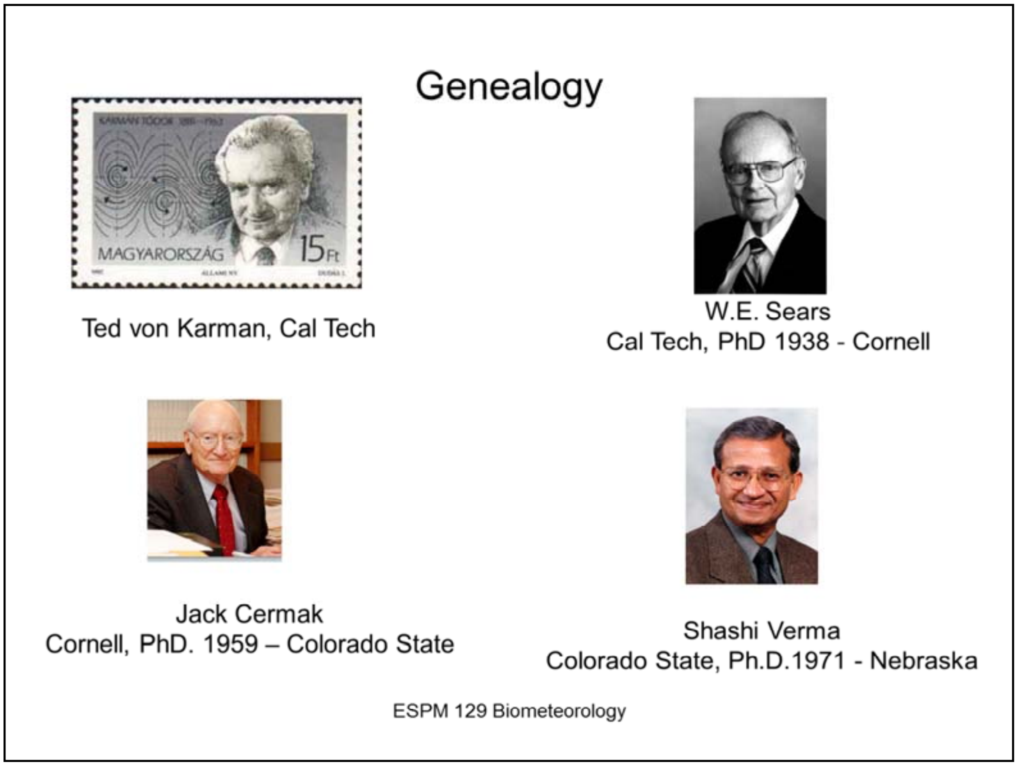
Roles of Turbulence in Biometeorology

- Transfers heat, momentum and mass (water vapor, carbon dioxide, biogenic gases, pollutants) between the biosphere and the atmosphere and diffuses pollutants in the atmosphere.
- Imposes drag forces on plants, causing them to wave, bend and break
- Mixes the air and diffusing air parcels with different properties, thereby forming spatial gradients
- Gusts place loads on the surface, which can erode soils and eject dust, spores, pollen, seeds, bacteria/viruses and insect eggs into the atmosphere.

Turbulence plays many roles in biometeorology



Turbulence is one of the great unsolved problems of physics. Why? It is non-linear, complex, multi-scaled, sensitive to initial conditions, yet forms coherent and organized structures.



In science, we sometimes like to follow our genealogy. Years ago I was told I, we, are linked to von Karman..(this guy has a constant named after him and was on a stamp?)

Back in 1993 or 94 I was sitting in the Toronto airport waiting for a flight home from Boreas experiment field work up in Canada. Prof. Wilford Brutsaert walked to me and said ‘you are Shashi Verma’s student? , yes…you are 3 generatations removed from von Karman’..and walked away. Prof Brutsaert was author of the leading book on Evaporation, so I was a bit taken away by this encounter. So in 2011 I was at a meeting in Lausanne with Prof. Brutsaert and during a field trip I asked him about this genealogy. Here is his connections.

General Attributes of Turbulence

- Complex
  - Spectrum of scales
  - Highly Organized
  - Chaotic
- Mechanical turbulence
  - Produced by Shear
- Convective Turbulence
  - Produced by Buoyancy

Reasons turbulence remains an unsolved problem.

Specific Properties of Turbulence

- Turbulence is Non-Linear, Chaotic, Complex
  - du/dt ~ u du/dx
    - acceleration is forced by advection
- Turbulence is non-Gaussian
  - It is skewed and kurtotic
- Turbulence is three-dimensional
  - Motions are rotational and anisotropic (eg vortices)
- Turbulence is diffusive
- Turbulence is dissipative
  - Energy of motion is degraded into heat
- Turbulence consists of multiple length scales
  - Large scales of energy input break down into smaller and smaller scales

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There are important concepts to appreciate as to what is complexity and why turbulence is a complex topic.

Complex, Chaotic, but Self-Organization

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Coherent structures and self organization can form. Some of the amazing attributes of turbulence.

Poetic Side of Turbulence

Great whirls have little whirls
That feed on their velocity
And little whirls have lesser whirls
And so on to viscosity

Lewis Fry Richardson

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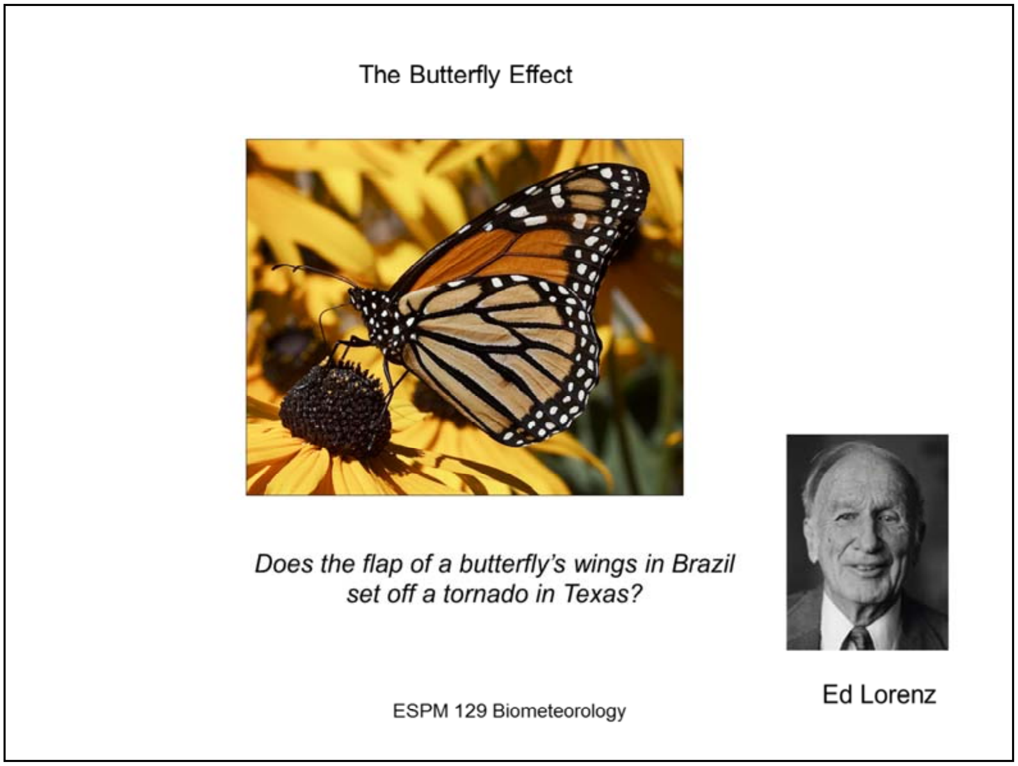
Today we get a second poem as this one illustrates the scales of turbulence. LF Richardson is one of the great scientists of turbulence. He is associated with the Richardson number, which tells us about the type of turbulence (mechanical vs buoyant), he is the father of numerical weather prediction and on fractals. He was a pacifist during WWI and developed his theories while working on an ambulance between battles.



Another aspect of turbulence is its association with chaos theory. Ed Lorenz was famous for finding limits to weather prediction due to the sensitivity to initial conditions of complex systems. But is this story about the butterfly effect true or not? Remember turbulence is dissipative, too.

Wind

- Speed
- direction

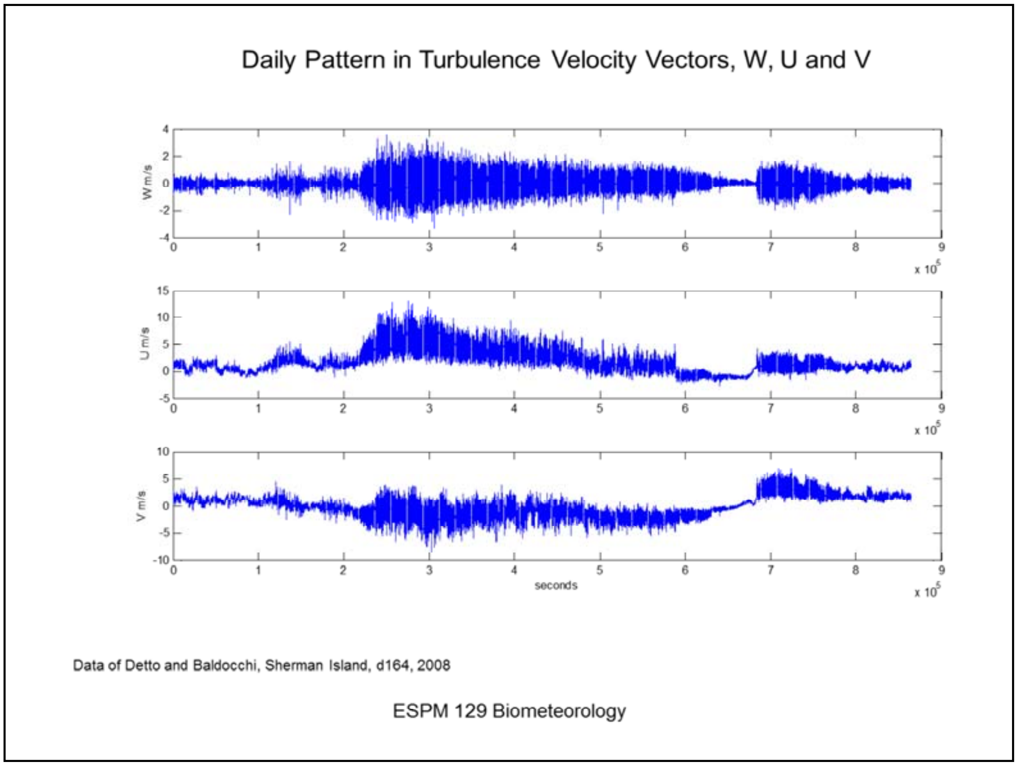
$$U = (u^2 + v^2 + w^2)^{1/2}$$

u, longitudinal velocity, dx/dt

v, lateral velocity, dy/dt

w, vertical velocity, dz/dt

To study turbulence we study the wind vectors. Vectors have direction and magnitude. Vertical velocity is w (in z direction), longitudinal velocity is u (in x direction) and lateral is v (in y direction)



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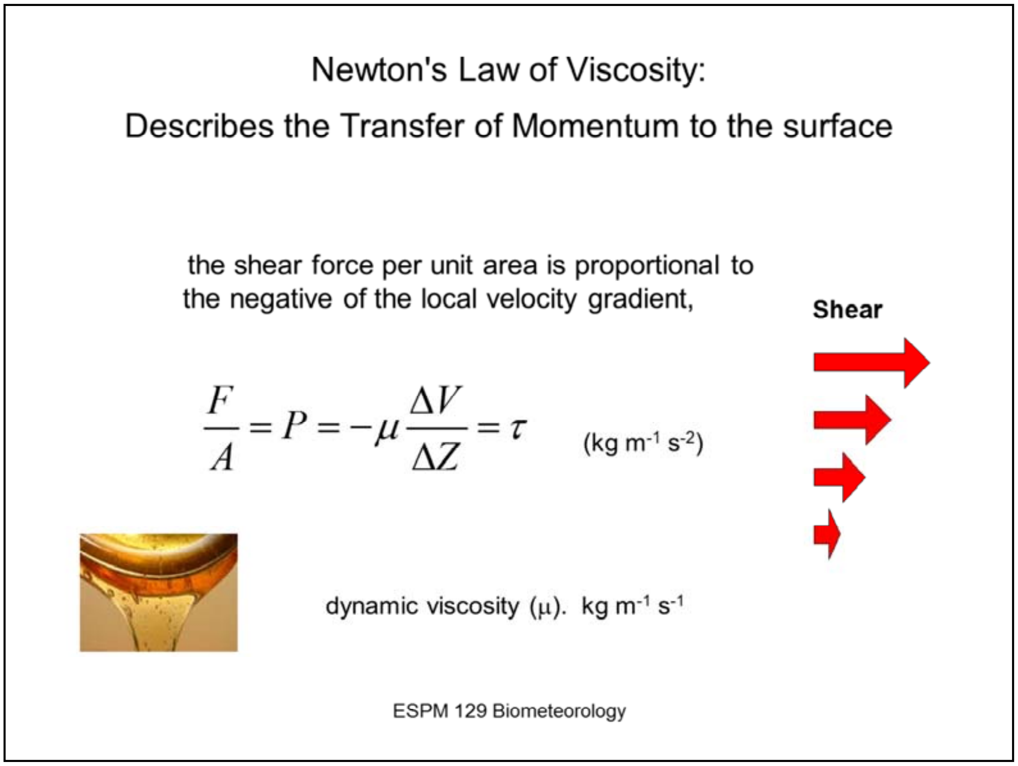
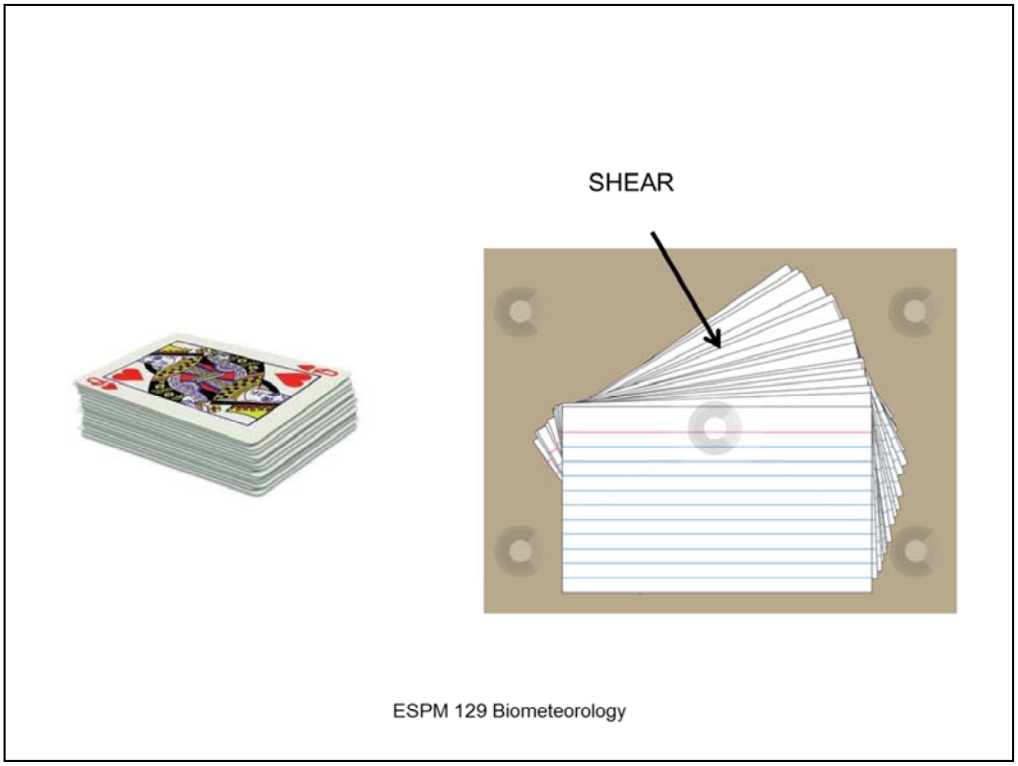
Here you see small w fluctuations at night and the generation of turbulence and lots of vertical motion during day. Same can be said for lateral flows



Shear is an important concept to study. It is associated with a force per unit area due to the transfer of momentum to the surface. More momentum aloft, than below, causes a gradient in momentum. With the application of Fickian Diffusion theory we can infer a flux density of momentum to the surface that is related to shear. This process will be the basis of understanding wind profiles in the surface layer.



Ways to think of shear. Push the top card of a deck of cards and watch what you see. Those closest to the table do not move. The others above move so more proportionally to the movement of the top card.

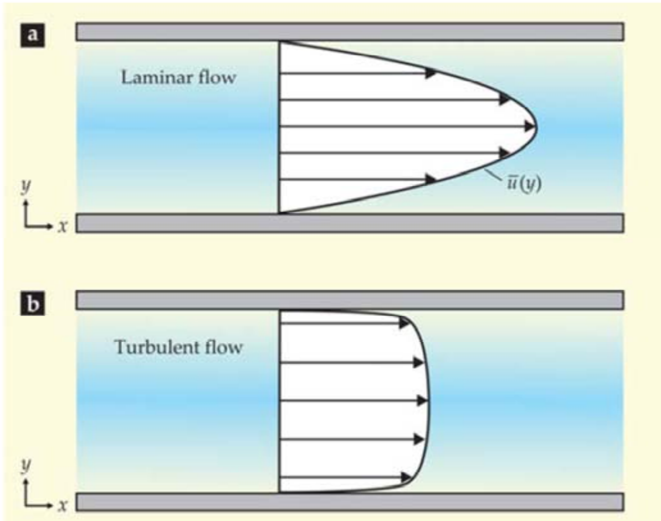
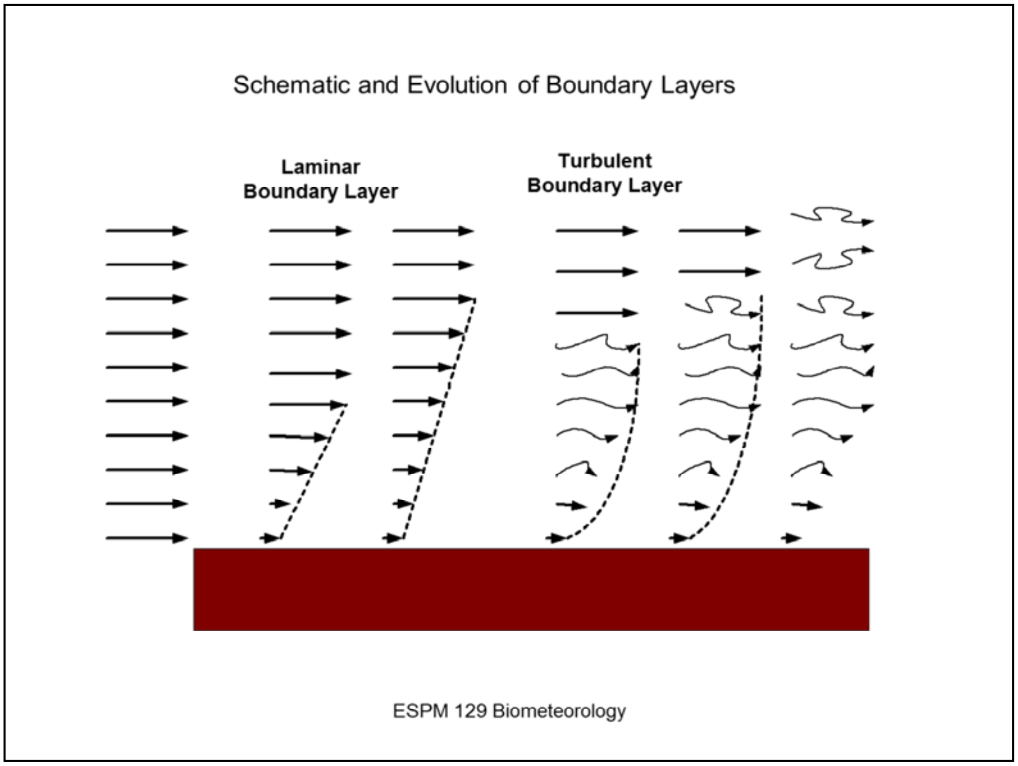


Figure 2. Pipe-flow velocity profiles. (a) In a laminar flow, velocity varies smoothly—as a parabola—across the pipe cross section. (Here, $\overline{u}$ is the time-averaged streamwise velocity at a given distance *y* from the wall.) **(b)** In a turbulent flow, mixing driven by turbulent eddies results in a time-averaged velocity profile that's nearly uniform in the bulk flow but falls sharply in the thin region near the wall.

One of the first ideas to stress is that of boundary layers, as biometeorology is replete with boundary layers. We have two kinds. There are laminar and turbulent boundary layers. They occur under different conditions and have different characteristics.

Smits and Marusic 2013 Physics Today



We also see transitions between shear and turbulence due to interactions with a surface and the distance from the edge. In time, and distance, even a laminar flow can become turbulent, as you see in this picture. The shear that occurs at the surface becomes unstable and trips the laminar flow into turbulence. We also see regions with laminar flow overlaying turbulent flow.

Transition between Laminar and Turbulent Flows

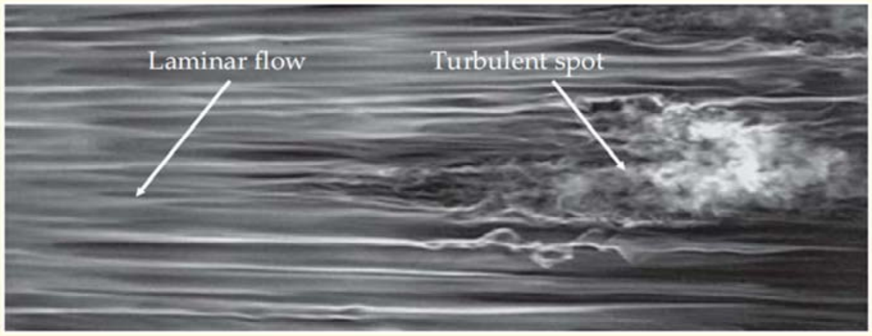


Figure 1. A turbulent spot develops in an initially laminar air flow across a flat plate—a geometry known as a boundary-layer flow. The view is from above, and the flow, from left to right, is visualized using streaks of smoke. Downstream, the spot grows to encompass the full domain of flow. (Adapted from ref. 15.)

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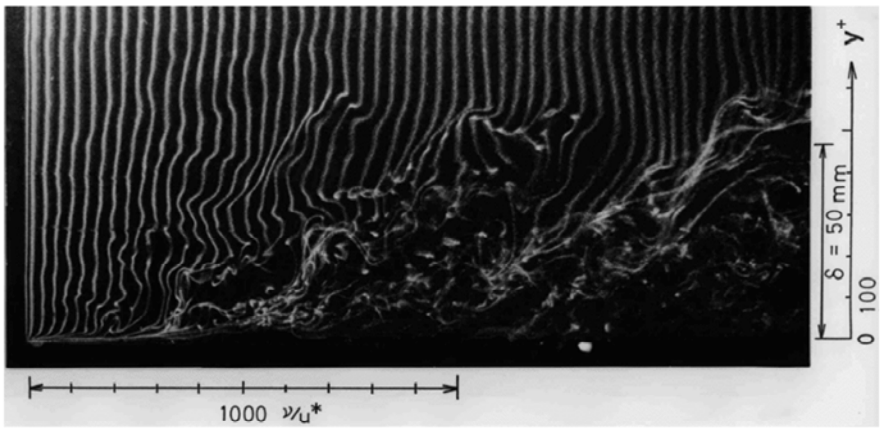
Smits and Marusic 2013 Physics Today..Transition between laminar and turbulent flow

Transition from Laminar to Turbulent Flows



http://www.thtlab.t.u-tokyo.ac.jp/index.html

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Great photo demonstrating this laminar to turbulent transition. Here we see smoke

Frictional Shear Stress

(kg m^{-1} s^{-1})

$$\tau = -\rho \nu \frac{\partial u}{\partial z} = -\mu \frac{\partial u}{\partial z}$$

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Frictional shear stress is a function of the longitudinal velocity gradient, or shear, and properties of the fluid, denoted by its density and its dynamic viscosity.

Kinematic viscosity

Defined as dynamic viscosity (kg m^{-1} s^{-1}=Pa s) normalized by the density of the fluid:

$$\upsilon = \frac{\mu}{\rho} \qquad (m^2\ s^{-1})$$

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Dynamic and kinematic viscosity are interchangeable. The later is normalized by density

Defining whether the flow is turbulent or laminar.

Reynolds Number

$$\mathrm{Re} = \frac{d \cdot u}{\nu}$$

Osborne Reynolds

Re is the ratio between inertial and viscous forces

d, physical dimension
u, fluid velocity
ν, kinematic viscosity

Re < 2000, laminar

Reynolds number is critical for knowing if the flow is turbulence of laminar or turbulent

Decoupled Flow: Breakup of Stable Boundary Layer

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Stability is also important. One can see in nature stable boundary layers at night which decoupled more energetic flows aloft. Look at still water under the fog layer, the ripples in the forefront and the turbulence starting to form above the fog.

Is Atmospheric Flow Laminar or Turbulent?

$$\mathrm{Re} = \frac{d \cdot u}{\nu}$$

$$\mathrm{Re} = \frac{0.1 \cdot 1}{10^{-5}} = 1000$$

Leaf

$$\mathrm{Re} = \frac{3.0 \cdot 3}{10^{-5}} = 90000$$

Corn field

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Do we live in a laminar or turbulent world? Here are some simple computations of Reynolds number. What do you think.

Smallest Scales of Turbulence: Kolmogorov Microscale, ~ 10^{-3} m

Andrei Kolmogorov

$$\eta = (\frac{\upsilon^3}{\varepsilon})^{1/4}$$

kinematic viscosity (ν)
rate of dissipation (ε),
scales with velocity cubed
Over a length scale, u^3/l.

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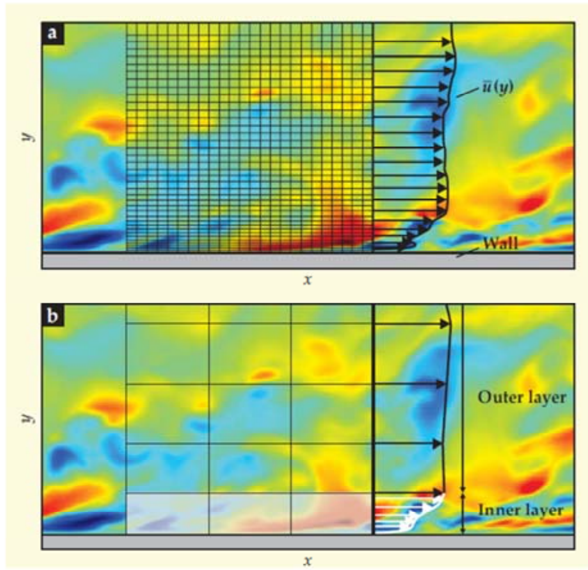
The smallest scales of turbulence are at the millimeter scale and are defined by Kolmogorov's microscale. Why is this important? If one is to perform direct numerical simulations of turbulence in the real world we would need to resolve motions as small as this scale. Think of the numerical cost. Halving the grid causes an 8 fold increase in computer nodes. Large eddy simulation (LES) models tend to work at the meter scale and parameterize smaller scale turbulence.

Computations Fluid Dynamics



10 Zetaflops are needed to Accomplish DNS

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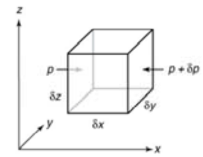
From Smit and Marusic 2013 Physics Today. 10 zetaflops, 10^21 floating point computations per second are needed to perform direct numerical computations for Re ~ 10^5, fastest computers are at 33 Petaflops

Equation of Motion



Equation of Motion defines how wind velocity accelerates or decelerates

$$\rho \frac{\Delta u}{\Delta t} = \frac{\Delta (F / A)}{\Delta x} = \frac{\Delta P}{\Delta x} + \frac{\Delta F_{friction} / A}{\Delta x}$$

Changes in wind velocity with time (acceleration) are promoted by lateral difference In forces per unit area (pressure) and are retarded by differences in frictional forces

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Now we start with an important equation, the equation of motion. In many ways we are lucky, we have an equation that describes fundamental fluid flow. Other aspects of biometeorology, eg those associated with biology tend not to. Here we want to describe the forces that accelerate or decelerate wind velocity. It is due to gradients in pressure and in frictional forces.

Navier-Stokes Equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \approx -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\mu}{\rho}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right]$$

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The full derivation leads to the famous Navier Stokes equation. The first tern on the left is the time rate of change in horizontal velocity. The next 3 terms are advection terms, associated with gradients in u along x, y and z. These are balanced by a pressure gradient term and frictional forces

Understanding non-linear terms

Total Derivative

$$\frac{du(t,x,y,z)}{dt} = \frac{\partial u}{\partial t} + \frac{dx}{dt}\frac{\partial u}{\partial x} + \frac{dy}{dt}\frac{\partial u}{\partial y} + \frac{dz}{dt}\frac{\partial u}{\partial z}$$

Time Rate of Change

Advection

$$\frac{du(t,x,y,z)}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}$$

u=dx/dt *v=dy/dt* *w=dz/dt*

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Remember at the beginning we spoke of non linear forcings. What are they and why? Here we start with the total time derivative in u with t and see how it shakes out. From it comes the advection terms and a local time derivative, like that which we measure at a meteorological tower. This has what is called an Eulerian framework.

Explaining 2nd derivative terms

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\approx-\frac{1}{\rho}\frac{\partial p}{\partial x}-\frac{\mu}{\rho}[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}]$$

Divergence of Frictional Shear Stress, τ

$$\frac{\partial \tau}{\partial z}=\nu\frac{\partial(\partial u/\partial z)}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2}$$

$$\frac{\mu}{\rho}[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}]$$

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Why are there 2nd derivative terms. This comes from expanding the flux divergences in frictional shear.

Reynolds' Averaging

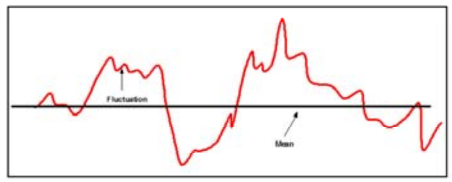
- Instantaneous Velocity (u) is the sum of
  - The mean velocity (<u>)
  - Fluctuation from the Mean (u')

$$u = \overline{u} + u'$$

$$v = \overline{v} + v'$$

$$w = \overline{w} + w'$$

- Substitute instantaneous values in the Navier-Stokes Equation with Reynolds terms, multiple and average



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Next big idea is Reynolds averaging to separate the mean from the fluctuating components.

Mean velocity budget equation

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + \overline{w}\frac{\partial \overline{u}}{\partial z} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x} - \frac{\partial \overline{w'u'}}{\partial z}$$

Note: addition of new and unknown term, the Reynolds Shear Stress. It's introduction is the essence behind the Turbulence Closure Problem $\overline{w'u'}$

Note: Frictional Shear terms are small and disappear $\frac{\partial \tau}{\partial z} = \nu \frac{\partial (\partial u / \partial z)}{\partial z} = \nu \frac{\partial^2 u}{\partial z^2}$

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Applying Reylolds rules of averaging to the Navier Stokes equation gives us a new equation for mean fluid flow but it introduces a new term the flux diverence in the covariance between w and u. This is turbulent shear. It is much greater than the frictional terms for laminar flow. It also adds another aspect of complexity, more unknowns than equations, the classical closure problem of turbulence.

$$Kinetic\ Energy = \frac{1}{2}mv^2$$

turbulent kinetic energy or velocity variance

$$\frac{1}{2}\overline{q'^2} = \frac{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}{2}$$

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Couple of other concepts to be aware of. Here we define kinetic energy and turbulent kinetic energy.

production of turbulent kinetic energy, v1.0

$$\frac{\partial \frac{1}{2}\overline{q'^2}}{\partial t} = 0$$

$$-\overline{w'u'}\frac{\partial \overline{u}}{\partial z} = \varepsilon$$

Kinetic energy, produced by shear, is in balance
with dissipation of energy **into heat by viscous processes**

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If there is no production there is a balance between shear produced turbulence and viscous dissipation.

production of turbulent kinetic energy, v2.0

shear and buoyant production of tke
must equal the rate at which energy
is **dissipated into heat by viscous processes**

$$-\overline{w'u'}\frac{\partial \bar{u}}{\partial z}+\frac{g}{\theta_v}\overline{w'\theta_v}=\varepsilon$$

g, acceleration of gravity
θ_v, virtual potential temperature

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In natural flows turbulence is generated by shear and buoyance and these are destroyed by dissipation.

Summary

- Turbulence
  - transfers heat, momentum and mass
  - imposes drag forces on plants, causing them to wave, bend and break
  - mixes the air and diffusing air parcels
  - Complex Process with many scales
  - Shear Production of Turbulence eventually dissipates into heat