Leaf Boundary Layer Resistances and Mass and Momentum Exchange , part II:

- Dimensionless Numbers
  - Sherwood Number, Sh
  - Schmidt Number, Sc
  - Grasshof Number, Gr
  - Nusselt Number, Nu
  - Prandtl Number, Pr

10/31/2016 ESPM 129 Biometeorology 1

To be able to compute boundary layer resistances for mass, momentum and heat transfer we will need to rely on theories that apply to flat plats and the dimensionless quantities that help us scale.

Reynolds number	Re	$\mathrm{Re} = \frac{ul}{\upsilon}$	Inertial to visous forces
Schmidt	Sc	$Sc = \frac{\upsilon}{D_c} = \frac{\mu}{\rho D_c}$	Kinematic viscosity to molecular diffusivity
Prandtl	Pr	$\mathrm{Pr} = \frac{\upsilon}{D_t}$	Kinematic viscosity to thermal diffusivity
Sherwood	Sh	$Sh = \frac{g_c l}{D_c}$	Dimensionless mass transfer conductance (conductance divided by the ratio of the molecular diffusivity and a length scale, l)
Grasshof	Gr	$Gr = \frac{l^3 \rho^2 g \beta \Delta T}{\mu^2}$	Buoyant force times an inertial force to the square of the viscous force
Nusselt	Nu	$Nu = \frac{g_h l}{D_t}$	Dimensionless heat transfer conductance

Here is the laundry list of applicable dimensionless numbers. In this lecture we will consider their application.

Conductance is f(dimensionless Number, Diffusivity and Length scale)

$$g_h = \frac{D_h Nu}{d} \qquad g_c = \frac{D_c Sh}{d} \qquad \text{(m/s)}$$

- g_h: heat conductance
- g_c: scalar conductance
- d: length scale
- D_h: thermal diffusivity
- D_c: scalar diffusivity
- Nu: Nusselt Number
- Sh: Sherwood Number

In essence the boundary layer conductance (m/s) for heat and mass are a product of the appropriate diffusivity (thermal or scalar) times the right dimensionless number divided by a length scale.

Nusselt Number, Nu

Nusselt number, is the ratio of the turbulence conductance, *gh*, and a length scale, *l*, to the thermal diffusivity, *Dt*,

$$Nu = \frac{g_h l}{D_t} = \frac{Hl}{\rho C_p D_h (T_s - T_a)}$$

$$Nu = a\,Pr^b\,Re^c$$

$$g_h = \frac{D_h Nu}{d}$$

Conductance for Heat is a f(Nu)

ESPM 129 Biometeorology 4

We'll start with Nu. It self is computed as a function of the Reynolds number and Prandtl number. So we need a cookbook for the recipies

Prandtl Number, Pr

Ratio between kinematic viscosity, ν and thermal diffusivity, D_t

$$\Pr = \frac{\upsilon}{D_t} = \frac{\mu}{\rho_a D_t}$$

Defining whether the flow is turbulent or laminar.

Reynolds Number

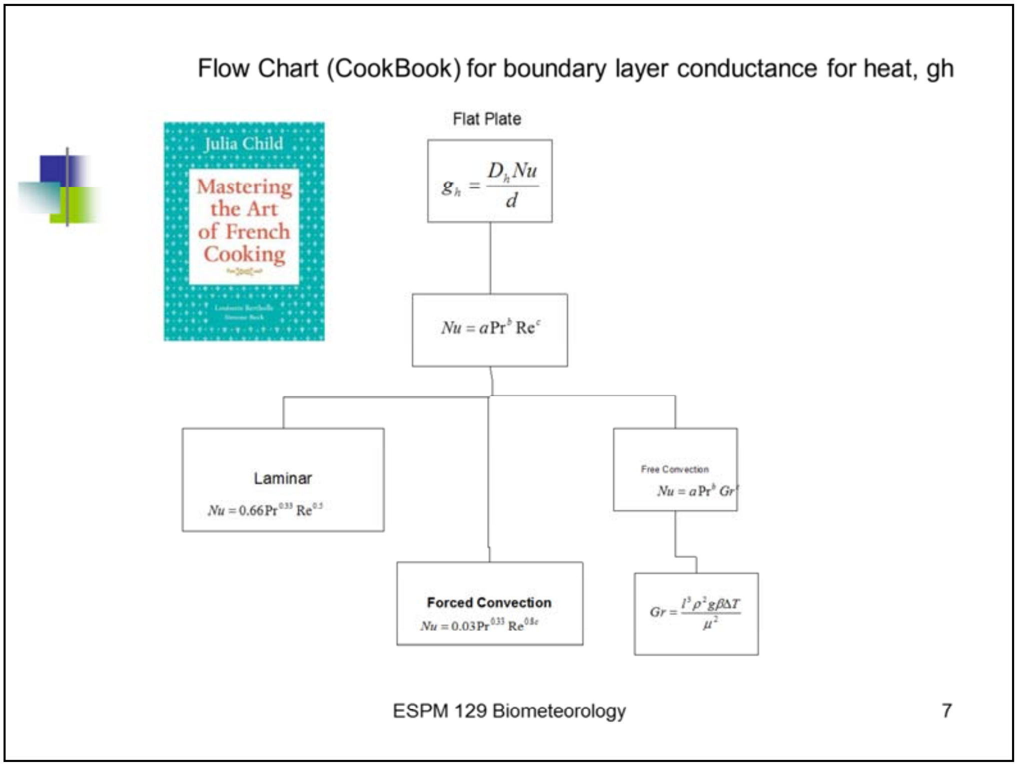
$$\mathrm{Re} = \frac{d \cdot u}{\nu}$$

Osborne Reynolds

Re is the ratio between inertial and viscous forces

d, physical dimension
u, fluid velocity
ν, kinematic viscosity

Re < 2000, laminar



To compute gh we have to first compute Nu. To do that we need to compute Re and decide if the flow is laminar or turbulent. If it is turbulent is it free or forced convection. If free convection we also need to compute the Grasshof number, Gr..then compute the appropriate Nu.

Laminar Flow

$$Nu = 0.66\,\mathrm{Pr}^{0.33}\ \mathrm{Re}^{0.5}$$

Forced Convection

$$Nu = 0.03\ Pr^{0.33}\ Re^{0.8}$$

Free Convection Gr >> Re^2

$$Nu = a\,Pr^{b}\ Gr^{c}$$

ESPM 129 Biometeorology 8

Typical power law equations for Nu

Grashof Number, Gr

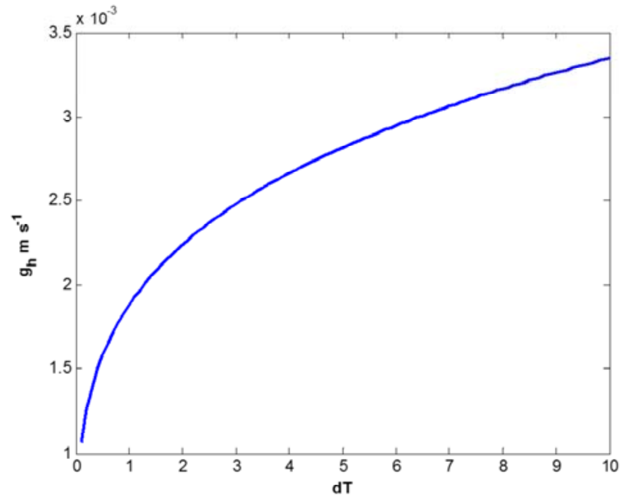
Buoyant force times an inertial force
to the square of the viscous force

$$Gr = \frac{l^3 \rho^2 g \beta \Delta T}{\mu^2}$$

l, length scale
g,acceleration due to gravity
ρ, air density
T, air temperature
μ, viscosity

Grashof number

Plot gh with Delta T



ESPM 129 Biometeorology 10

Visual on how conductance changes with temperature gradients and convection

Sherwood Number, Sh

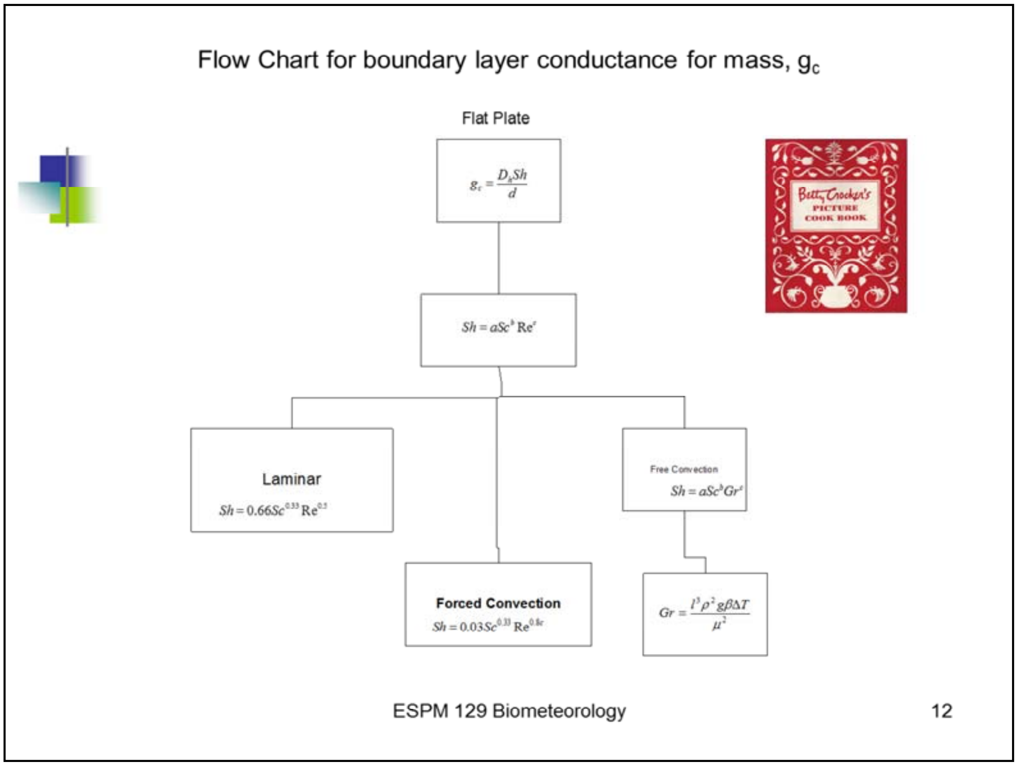
Sherwood number, is the ratio of the turbulence conductance, g_c, and a length scale, l, to the molecular diffusivity, D_c,

$$Sh = \frac{g_c l}{D_c} = \frac{Fl}{D(\chi_s - \chi_a)}$$

$$g_c = \frac{D_c Sh}{d}$$

Conductance =f(Sh)

For mass transfer we substitute the Nu with Sh.



To compute gc we have to first compute Sh. To do that we need to compute Re and decide if the flow is laminar or turbulent. If it is turbulent is it free or forced convection. If free convection we also need to compute the Grasshof number, Gr..then compute the appropriate Sh.

Sherwood Number, Sh

$$Sh = aSc^b \operatorname{Re}^c$$

Laminar Flow, Flat Plate

$$Sh = 0.66Sc^{0.33} \operatorname{Re}^{0.5}$$

Forced Convection, Flat Plate

$$Sh = 0.037Sc^{0.33} \operatorname{Re}^{0.8}$$

Re, Reynolds Number
Sc, Schmidt Number

Schmidt Number, Sc

Ratio of kinematic viscosity to molecular diffusivity

$$Sc = \frac{\upsilon}{D_c} = \frac{\mu}{\rho D_c}$$

Why is CO2 resistance related to water vapor resistance in terms of the ratio of Diffusivities Raised to the 2/3 power?

$$r_{ac} = r_{av} (\frac{D_v}{D_c})^{2/3}$$

$$\frac{r_{ac}}{r_{av}} = \frac{D_v \, Re^{1/2} \, Sc^{1/3}}{D_c \, Re^{1/2} \, Sc^{1/3}}$$

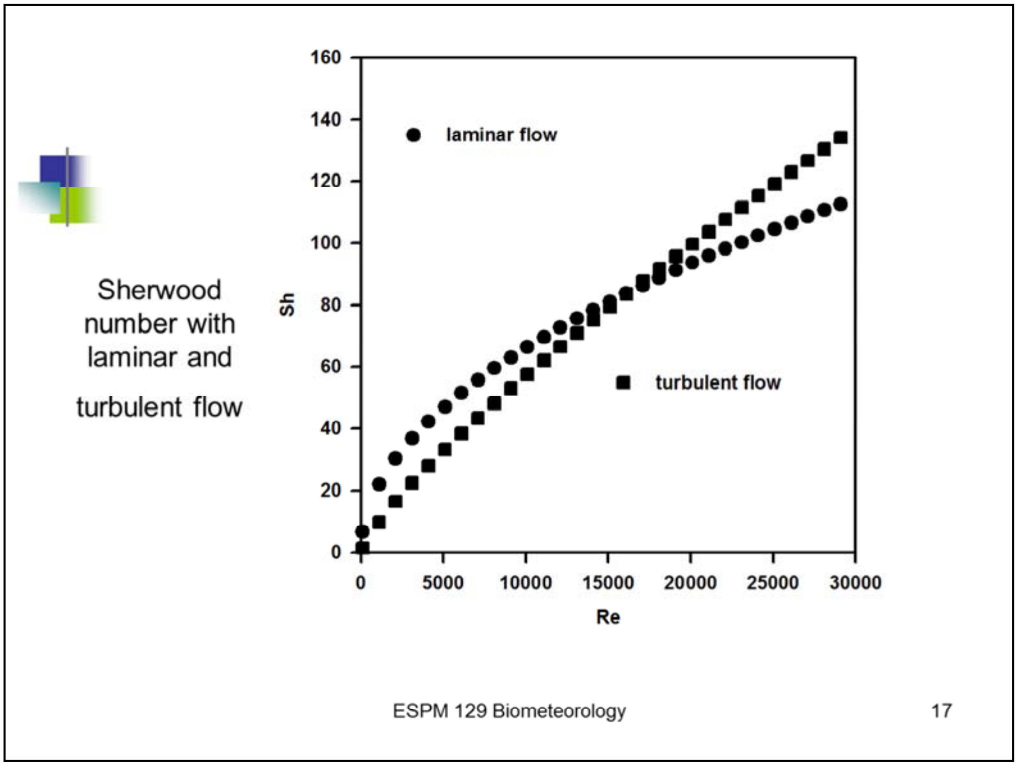
$$\frac{r_{ac}}{r_{av}} = \frac{D_v \, \mathrm{Re}^{1/2} \, D_c^{1/3}}{D_c \, \mathrm{Re}^{1/2} \, D_v^{1/3}} = (\frac{D_v}{D_c})^{2/3} = 1.39$$

We often see that the boundary layer resistance for CO2 is related to that for water vapor times the ratios of the diffusivities raised to the 2/3 power. Why? We can answer this by examining the ratios of the Sh. Re cancels as do part of the Sc.

Sherwood Number, Free Convection

$$Sh = aSc^b Gr^c$$

Gr >> Re^2

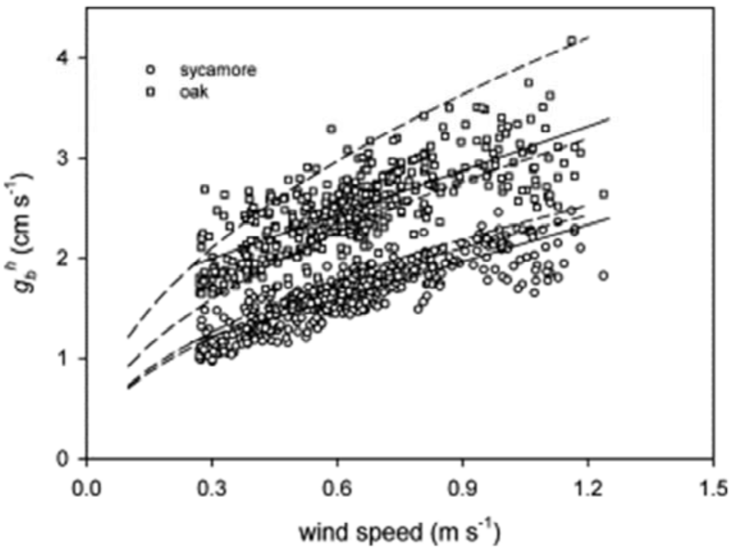


Switch points in Sh between laminar and turbulent flows

Prandtl and Schmidt numbers for a variety of temperatures. (D: mm s^{-1})

T	k	n	D_v	D_c	Pr	Sc h_2o	Sc co_2
-5	18.3	12.9	20.5	12.4	0.705	0.629	1.040
0	18.9	13.3	21.2	12.9	0.704	0.627	1.031
5	19.5	13.7	22	13.3	0.703	0.623	1.030
10	20.2	14.2	22.7	13.8	0.703	0.626	1.029
15	20.8	14.6	23.4	14.2	0.702	0.624	1.028
20	22.2	15.5	24.9	15.1	0.698	0.622	1.026
25	22.5	15.75	25.3	15.3	0.700	0.623	1.029
30	22.8	16	25.7	15.6	0.702	0.623	1.026
35	23.5	16.4	26.4	16	0.698	0.621	1.025

Flat Plate Theory vs Observations



Stokes et al 2006 AgForMet

Forced Convection

$$g_{aH} = \frac{1}{r_{aH}} = 6.62(\frac{u}{d})^{0.5}$$ Flat Plate

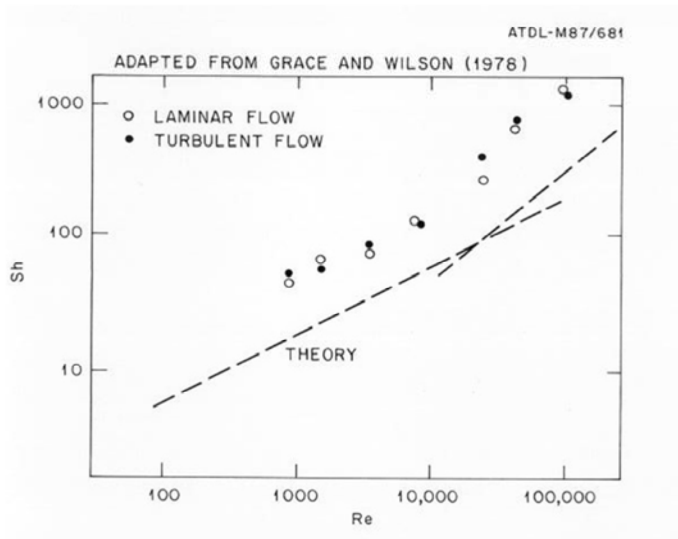
$$g_{aH} = \frac{1}{r_{aH}} = 4.03(\frac{u^{0.6}}{d^{0.4}})$$ Cylinder

$$g_{aH} = \frac{1}{r_{aH}} = 5.71(\frac{u^{0.6}}{d^{0.4}})$$ Sphere

ESPM 129 Biometeorology 20

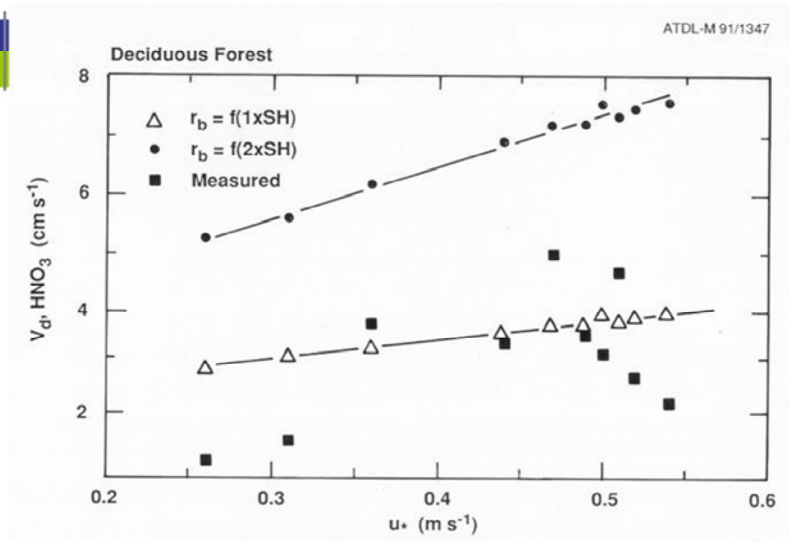
Not all leaves are flat plates. Other equations can be used for cases of cylinders and spheres.

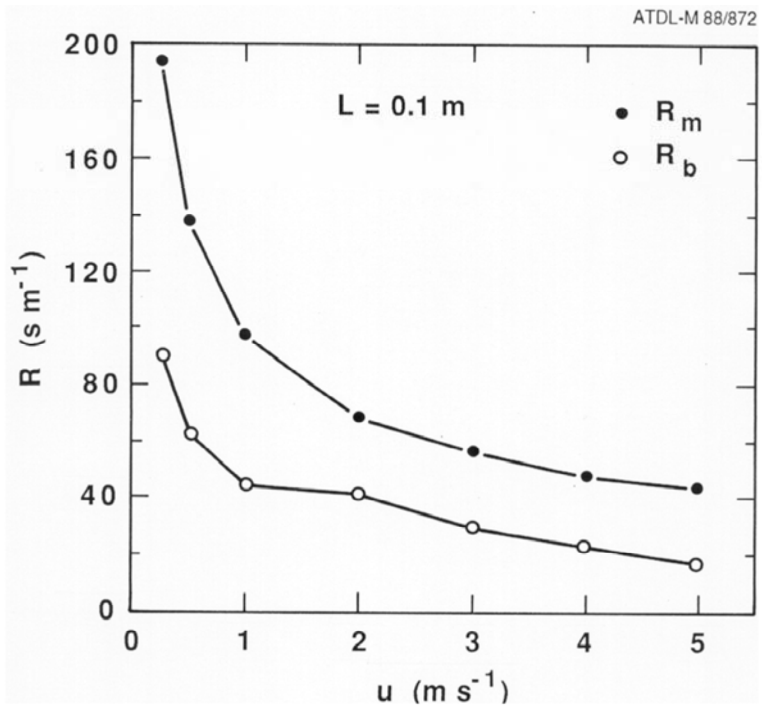
Computations of Sherwood number for real and theoretical leaves



Real leaves are clumped and can affect the theory. Grace and Wilson suggest that the Sh should be doubled in the field.

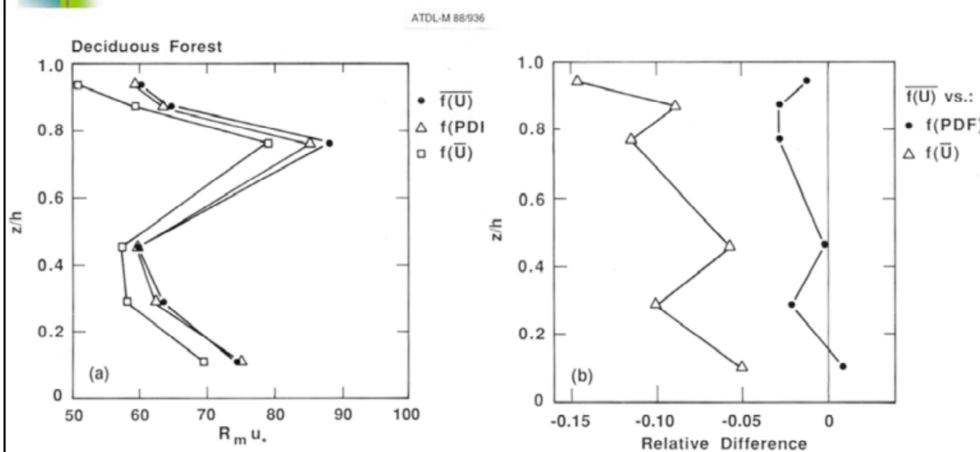
Application of leaf resistance theory to compute nitric acid vapor flux





23

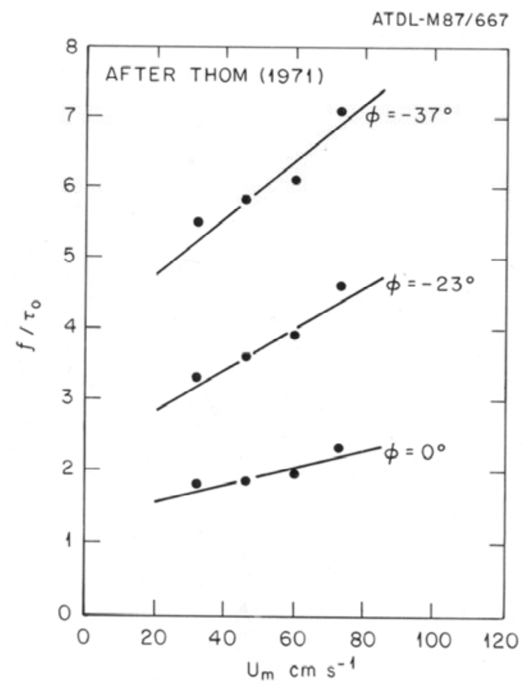
Impact of non-linearity and non-Gaussian turbulence on the evaluation of leaf resistance models



ESPM 129 Biometeorology 24

What happens when you try to compare the mean of the function vs the function of the mean for heterogeneous turbulence in the canopy. After Baldocchi and Meyers, 1988

Leaf angles and momentum transfer



Leaves are not flat and there is drag with those angels. After Thom 1971

Summary

- A laminar sublayer always exists close to the surface of leaves, even when experiencing turbulent flow
- The Reynolds' number quantifies whether a leaf is experiencing turbulent or laminar flow and increases with characteristic leaf size.
- The conductance for mass transfer is proportional to the molecular diffusivity and the Sherwood number and is inversely proportional to the characteristic leaf size.
- A constant flux layer exists for heat and mass transfer through the laminar and turbulent boundary layers. The products of molecular (or turbulent) diffusivities and concentration gradients interact to preserve this constancy.

Summary

- Wind flow over a leaf can be laminar or turbulent.
- The Reynolds' number is used to discriminate between turbulent and laminar flow. It is the ratio between inertial and viscous forces
- The orientation of leaves, its irregular shape, leaf hairs, clumping trip laminar flow at lower Re numbers than for isolated plates in wind tunnel.
- The dimension-less functions depend on the characteristic length scale and shape of the object (flat plate, sphere, cone
- The Nu number is used to evaluate the transfer conductance to heat transfer.
- The Sherwood number is used to evaluate mass transfer.
- Both are functions of the Reynolds number

$$g_h = \frac{D_h Nu}{d}$$

$$g_c = \frac{D_c Sh}{d}$$