Lecture 2:
Fluxes, Part 1: K-Theory

Dennis Baldocchi
ESPM 228

This is the first of several lectures on flux measurements. We will start with the simplest and earliest method, flux-gradient or K theory techniques

What are Fluxes?

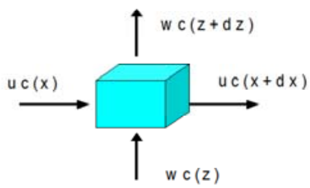
Why Measure Fluxes?

Fluxes, or technically flux densities, are the number of moles of a chemical scalar or Joules of energy passing a unit area per unit time. Typical units are moles m-2 s-1 or J m-2 s-1. Why are they important? Well the state of the atmosphere and how it changes with time is a function of the flux divergence across a unit volume.

How density of some gas, ρ_c, of a volume changes with time

$$\Delta x \Delta y \Delta z \frac{\partial \rho_c}{\partial t}$$

Balance of mass fluxes in and out of horizontal and vertical walls



$$\Delta y \Delta x [\rho_c w |_z - \rho_c w |_{z+\Delta z}]$$

$$\Delta y \Delta z [\rho_c u |_x - \rho_c u |_{x+\Delta x}]$$

This schematic shows the budget of a volume of air and the fluxes into and out of that volume and how they change the molar density with time.

Attributes of Micrometeorological Flux Methods

- *1) in situ,* so they are non-intrusive;
- 2) they can be applied on a quasi-continuous time basis;
- 3) measurements made a point represent an areally-averaged ensemble of mass and energy exchange,with a length scale of 100 m to 2 km..

This is a list of the positive attributes of micrometeorological flux techniques.

Ergodicity

space-based ensemble averages can be substituted with **temporal averaging**.

$$E[f(x)] = \int_{-\infty}^{\infty} f(x)\, p\left(x\right) dx \approx \frac{1}{2UT} \int_{-T}^{T} (u \cdot t) f(ut) dt$$

$$E[f(x)] = \int_{-\infty}^{\infty} f(x)\, p(x) dx = \frac{1}{2T} \int_{-T}^{T} t\, f(t) dt$$

Double check for errors.

Challenges in Measuring Greenhouse Gas Fluxes

- Measuring/Interpreting greenhouse gas flux in a quasi-continuous manner for Days, Years and Decades
- Measuring/Interpreting fluxes over Patchy, Microbially-mediated Sources (e.g. CH_4, N_2O)
- Measuring/Interpreting fluxes of Temporally Intermittent Sources (CH_4, N_2O, O_3, C_5H_8)
- Measuring/Interpreting fluxes over Complex Terrain and or Calm Winds
- Developing New Sensors for Routine Application of Eddy Covariance, or Micrometeorological Theory, for trace gas Flux measurements and their isotopes (CH_4, N_2O, $^{13}CO_2$, $C^{18}O_2$)
- Measuring fluxes of greenhouse gases in Remote Areas without ac line power

In the era of 2010s, many of the simple and basic issues regarding flux measurements, techniques and instrumentation have been addressed. New science involves measuring fluxes under non ideal conditions, for short and long periods, with novel trace gases and in remote areas.

Typical Micrometeorological Measurement Station

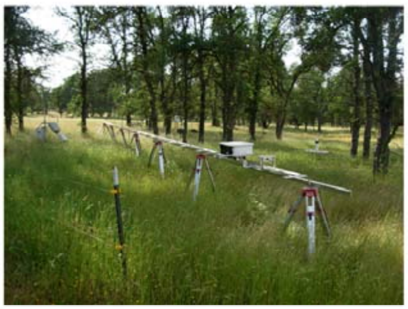
- Wind and 3-D Turbulence Vectors
- Fast-Response CO_2 and H_2O
- Aspirated/Shielded Temperature/ Humidity
- Tipping Bucket Rain Gauge
- Solar and Terrestrial Radiation, in/out
- Net Radiation
- Direct and Diffuse radiation
- Pressure
- Soil Temperature and Moisture
- [CO_2]
- Methane
- Digital Camera for phenology
- Data-loggers and personal computers for data storage and archive

Example of a flux measurement station maintained by my lab. It is stressed that one not only measures fluxes but the coincident meteorological, soil and plant conditions with a suite of sensors.

Understand Net Atmosphere-Surface Exchange by Studying the Understory, too

Understory eddy flux

Tram System to Measure Light Under Forests



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I am a huge fan of measuring fluxes in the understory of forests. The flux measured above a canopy is the net flux between exchanges on going at the soil and from across the vegetation. To derive knowledge on controls of fluxes it is really important to partition the fluxes into the soil and vegetation components. Flux measurements made in the underestory are one of the best and direct methods for doing so. Unfortunately, this approach is highly under appreciated. Too often you'll read more about very indirect methods, using stable isotopes, COS, sap flow and chambers, each with their distinct sampling problems and artifacts.

K Theory

$$F \approx -K \frac{\Delta c}{\Delta z}$$

F: flux density (mol m^{-2} s^{-1} or J m^{-2} s^{-1})
C: scalar or vector velocity
K: eddy exchange coefficient, m^2 s^{-1}

F is positive if the atmosphere is Gaining Material or Energy
F is negative if the atmosphere is Losing Material or Energy

Flux-Gradient Theory

$$\tau = -\overline{\rho_a} K_m \, \partial \overline{u} / \partial z$$

(kg m^{-2} s^{-1})

$$\lambda E = -\overline{\rho_a} \frac{\varepsilon}{P} \lambda K_v \, \partial \overline{e} / \partial z$$

(J m^{-2} s^{-1})

$$H = -\overline{\rho_a} C_p K_h \, \partial \overline{\theta} / \partial z$$

(J m^{-2} s^{-1})

$$F_c = -K_c \, \partial \overline{\rho_c} / \partial z$$

(mole m^{-2} s^{-1})

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These are the equations for the fluxes of momentum (t), sensible heat (H), latent heat (lE) and (Fc) CO2 flux densities. The sign convention is such that fluxes of material into the atmosphere are positive, as they contribute to a build up of scalar. Fluxes out of the atmosphere are negative in sign. Hence during the day with active photosynthesis, CO2 flux density is negative.

The minus signs are introduced to retain this sign convention as the gradient of material is negative when material is transferred into the atmosphere, and vice versa

Flux Methods Appropriate for Slower Sensors, e.g. FTIR

- Relaxed Eddy Accumulation

$$F = \overline{w'c'} = \beta\sigma_w(\bar{c}_{up} - \bar{c}_{dn})$$

- Modified Gradient Approach

$$F_c \sim F_s \frac{\Delta c_z}{\Delta s_z}$$

- Integrated Profile

$$F = \frac{1}{x}\int_0^z \overline{u(\rho_c - \rho_{background})}dz$$

- Disjunct Sampling

It is important to recognize a host of other micrometeorological methods for measuring fluxes that are more suitable for certain circumstances and when slower sensors are needed.

K-Theory Approaches (m^2 s^{-1})

$$K_m = K_H = K_v = K_c$$

- Aerodynamic
- Energy Balance
- Indirect Method

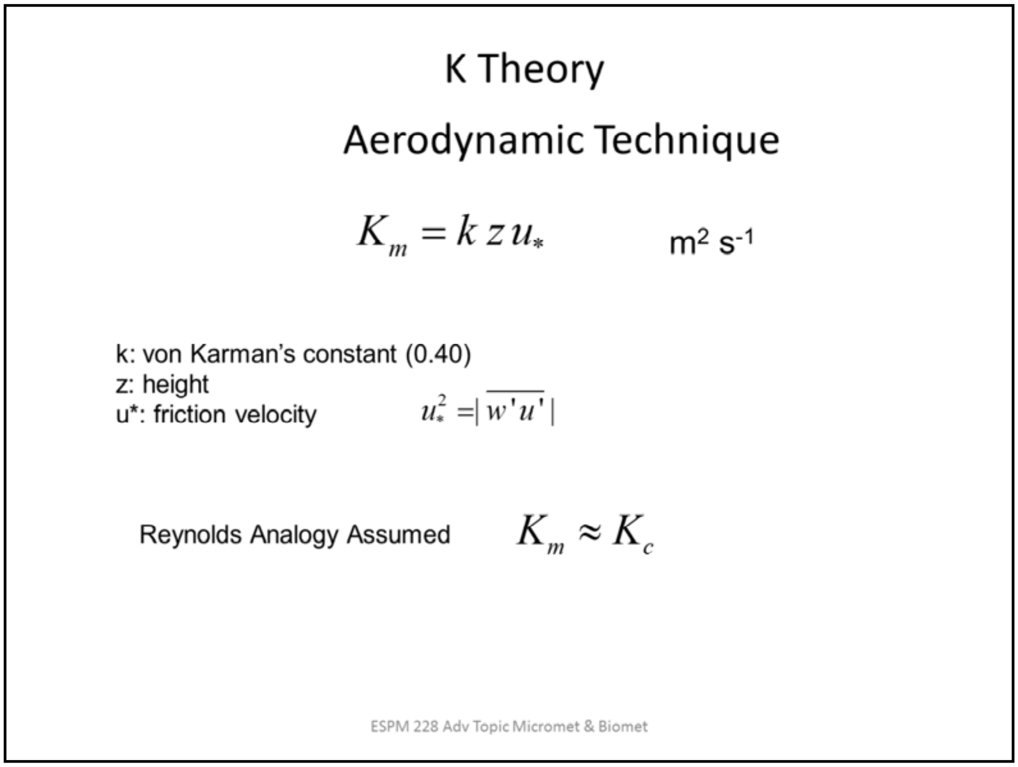
$$F \approx -K \frac{\Delta c}{\Delta z}$$

Here we will discuss 3 key methods for assessing eddy exchange coefficients, K, for fluxes

Restrictions for Application

- K-Theory infers Fluxes; it does not measure them directly.
- Gradients are Measured within the Constant Flux Layer
- Sensors Must be placed Outside the Vegetation Canopy and Roughness Sub-layers
- Extensive, Upwind Fetch must Exist
- Steady-State Conditions

These are many of the important constraints and assumptions in using K theory and many micrometeorological flux methods



We will apply basic momentum transfer theory, discussed in ESPM 129, to derive estimates of K. This is a good rule of thumb method when you need to estimate a ball park value of Km

K Theory

Aerodynamic Technique: Assessed with Wind Velocity Profiles

$$\tau = -\overline{\rho_a}\,\overline{w'u'} = -\overline{\rho_a}\,u_*^2 = -\rho_a C_d u^2 = -\overline{\rho_a} K_m \frac{\partial \bar{u}}{\partial z}$$

http://pages.unibas.ch/geo/mcr/Projects/EBEX/index_profile.en.htm

$$K_m = \frac{u_*^2}{\frac{\partial \bar{u}}{\partial z}} \qquad \frac{\partial \bar{u}}{\partial z} = \frac{u_*}{kz} \qquad K_m = k\,z\,u_* \qquad K_m = k^2 z^2 \frac{\partial \bar{u}}{\partial z}$$

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This derivation assumes near neutral thermal stratification and short vegetation. You can see with simple measurements of the wind velocity gradient one can compute Km

Aerodynamic Technique: Considering Stability

$$K_m = \frac{u_*^2}{\frac{\partial \bar{u}}{\partial z}}$$

$$K_m = k u_* (z-d) / \phi_m(\frac{z-d}{L})$$

$$\frac{\partial \bar{u}}{\partial z} = \frac{u_*}{k(z-d)} \phi_m((z-d)/L)$$

$$K_m = k^2 (z-d)^2 \frac{\partial \bar{u}}{\partial z} \varphi(\frac{z-d}{L})^{-2}$$

A more general form considers non-neutral thermal stratification. Then we have to apply stability functions of Monin-Obukhov similarity theory. We also consider tall vegetation which causes a displacement of the log wind profile, hence the introduction of d, the zero plane displacement.

Non-Dimensional Diabatic Stability Function

$$\varphi_m\left(\frac{(z-d)}{L}\right) = \frac{\partial u}{\partial z}\frac{k(z-d)}{u_*}$$

$$\phi_m(z/L) = (1-\gamma z/L)^{\beta}$$

L: Monin-Obukhov Length Scale

Phi is a non-dimensional form of the log wind profile. It is a function of the ratio between height, z, an the Monin-Obukhov lengths scale, L

Monin-Obukhov Similarity Theory and Non-Dimensional Wind Shear

Figure 2. (a) A.S. Monin (source: Russian Academy of Science: http://hp.iitp.ru/ger/pers.ac.htm). (b) A.M. Obukhov (source: Yaglom, 1990).

(from Foken, 2006, BLM)

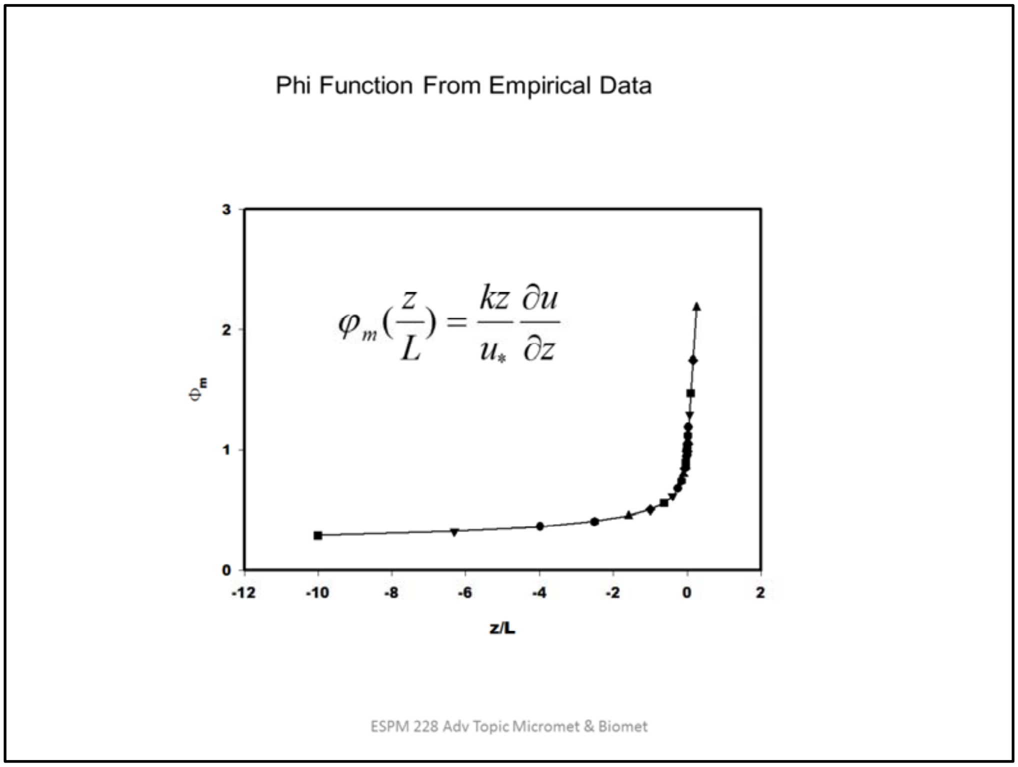
$$\varphi_m\left(\frac{z}{L}\right) = \frac{kz}{u_*}\frac{\partial u}{\partial z}$$

L is the Monin-Obukhov length scale $L = -\frac{\rho_a C_p u_*^3 \theta_v}{k g \overline{w'\theta_v'}}$

Tall Canopies

$$\frac{\partial u}{\partial z} = \frac{u_*}{k(z-d)}\varphi_m\left(\frac{(z-d)}{L}\right)$$

Z over L is defined from Buckhingham Pi theorem for non-dimensional quantities. It is also the ratio between buoyant and shear generated turbulent kinetic energy. Its evaluation incorporates information on sensible heat transfer and shear stress. If one is working exclusively with gradient measurements one can use estimates of phi that are a function of the gradient Richardson number



Classic shape of the phi function. It is one at z/L =0, it is less than one under convective conditions and it is greater than one during stable thermal stratification

$$\phi_m(z/L) = (1 - \gamma z/L)^{\beta}$$

Table 1 Parameters for Phi functions for momentum transfer, unstable thermal stratification

Citation	k	γ	β
[*Businger*, 1971]	0.35	-15	-1/4
[*Hogstrom*, 1996]	0.40	-19	-1/4

Table 2 Parameters for Phi functions for momentum transfer, stable thermal stratification

Citation	k	γ	β
[*Businger*, 1971]	0.35	4.7	1
[*Hogstrom*, 1996]	0.40	5.3	1

Coefficients to compute phi

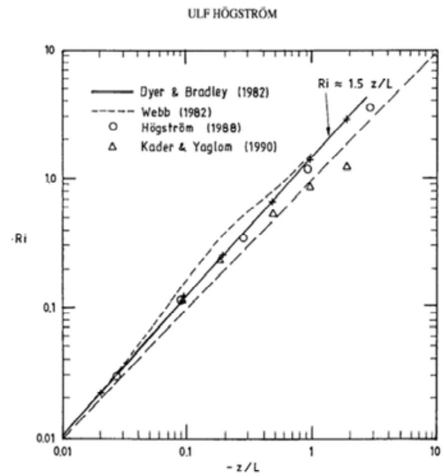
Richardson Number

$$Ri = -\frac{g}{\theta} \frac{\partial \theta / \partial z}{(\partial u / \partial z)^2}$$

g, acceleration due to gravity
u, horizontal wind velocity
θ, potential temperature

As promised above, we can compute stability in terms of the Richardson number, which needs information on wind and temperature gradients. In the surface layer temperature gradients and potential temperature gradients are nearly identical. Though technically this equation is a function of potential temperature.

Scaling Ri with z/L, which is a function of turbulent fluxes



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Energy Balance Method

http://pages.unibas.ch/geo/mcr/Projects/EBEX/img_profile/profile02.jpg

$$R_n = H + \lambda E + G$$

$$K = \frac{R_n - G}{\overline{\rho_a}(C_p \frac{\partial \theta}{\partial z} + \frac{\varepsilon}{P} \lambda \frac{\partial e}{\partial z})}$$

R_n, net radiation flux density, W $^{m-2}$
H, sensible heat flux density
λE, latent heat flux density
G, soil heat flux density
Cp, specific heat of air

Energy balance method is the 2nd way to compute Ks. This is the method I used in my PhD in Nebraska. One solves for K by assigning the flux gradient forms of sensible and latent heat exchange to the surface energy balance. Now we need to measure gradients of temperature, humidity and the net radiation balance and soil heat flux.

Reynolds Analogy Fails

$$K_m \neq K_v = K_h = K_c = K_x$$

$$1.35K_m = K_v = K_h = K_c = K_x$$

$$\frac{K_h}{K_m} \approx \frac{K_w}{K_m} = (1 - Ri)^{0.25}$$

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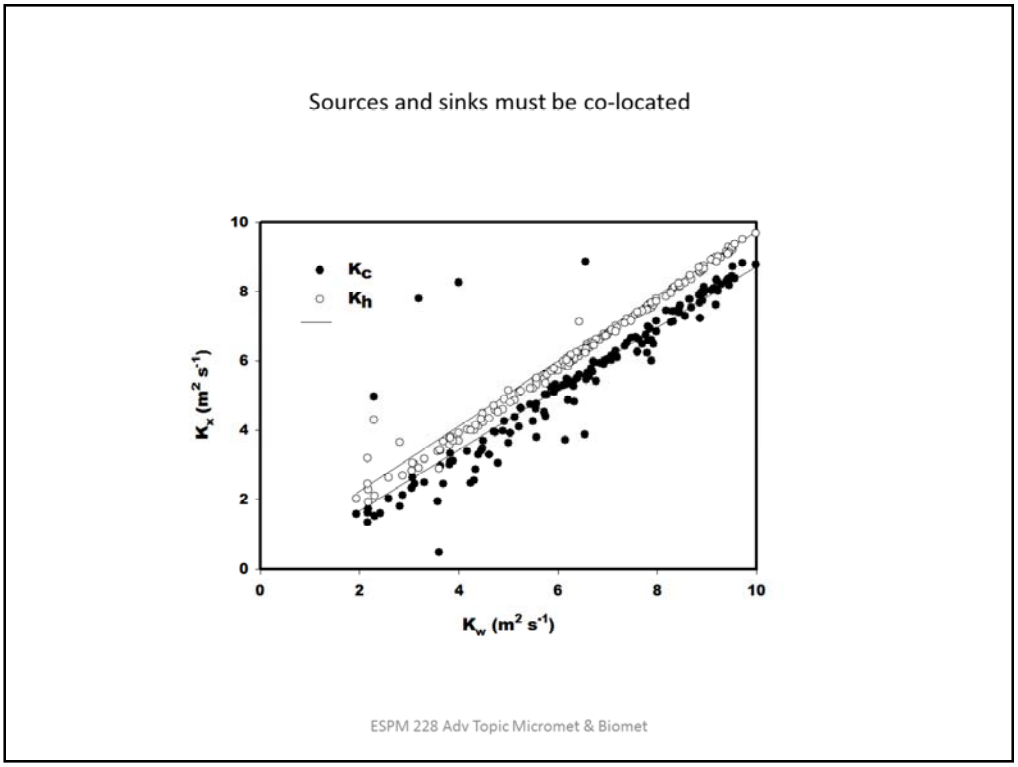
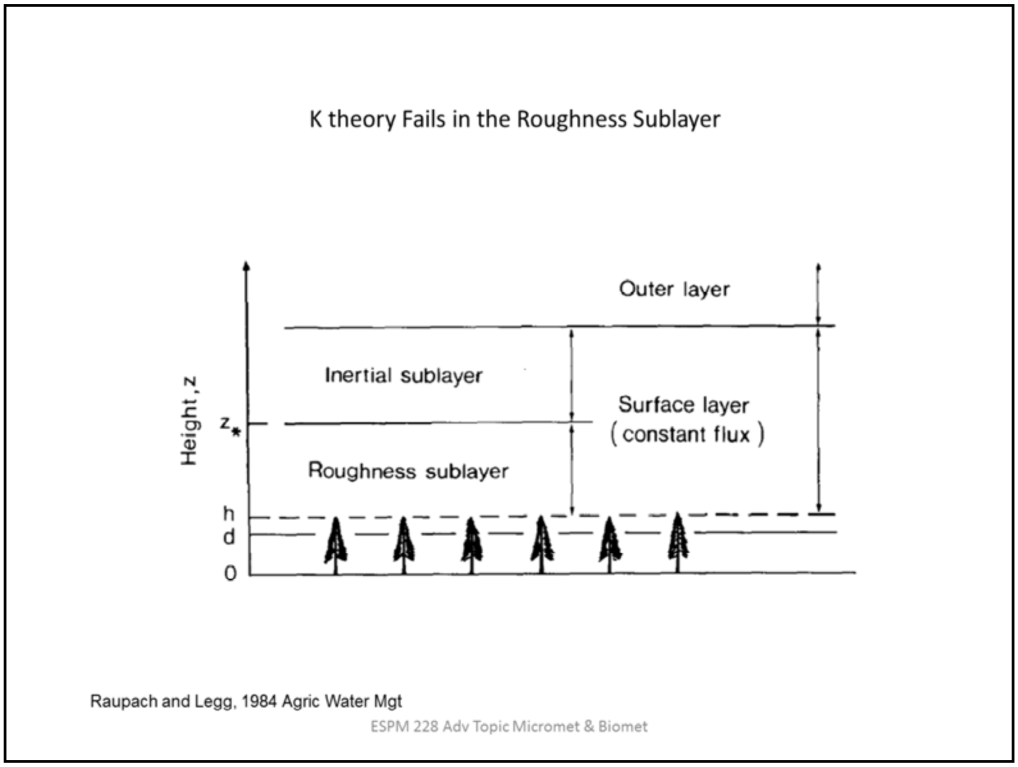
In reality the K for momentum does not equal that for scalars due to different sources and sinks and transfer processes. Pressure fluctuations and drag affect momentum transfer, but not scalars, per se.



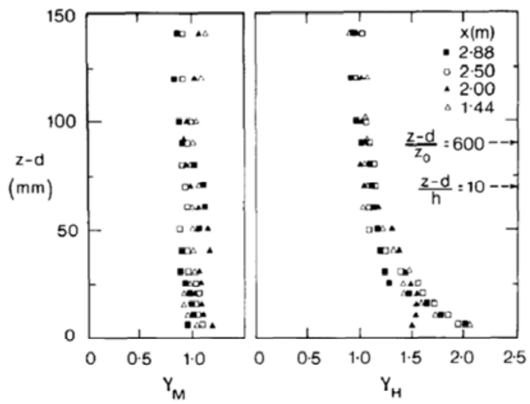
Yet, even the sources and sinks for CO2, water and temperature must be the same. This is often a good assumption for short crops, but can fail across tall forests, as deduced by these computations with my Lagrangian CANOAK model for tranfer over a 25 m tall deciduous forest. Because there is substantial CO2 exchange at the soil surface of a forest, its source-sink level is lower than that for heat and moisture that occur in the upper reaches of the canopy.



While it is acceptable, and necessary, to measure eddy fluxes in the surface layer, or constant flux layer, there are major problems assessing gradients in the roughness sublayer, which is withing 2 times canopy height

Roughness Sublayer

$$\gamma_e = \frac{K_{obs}}{K_{mo}} = \frac{\phi(z/L)_{mo}}{\varphi_{obs}}$$



Raupach and Legg, 1984

This problem in computing K in the roughness sublayer causes a breakdown in Monin Obukhov similarity theory because great shear causes non local transport.

Roughness Sub Layer: Why Monin Obukhov Theory Fails?

$$\frac{\partial \overline{w'c'}}{\partial t} = 0 = -\overline{w'^2}\frac{\partial \bar{c}}{\partial z} - \frac{\partial \overline{w'w'c'}}{\partial z} - \overline{c'\frac{\partial p'}{\partial z}} + g\frac{\overline{\theta' c'}}{\theta}$$

Budget Eq for turbulent Flux

Closure Approximation for Pressure term $\overline{c'\frac{\partial p'}{\partial z}} = \frac{\overline{w'c'}}{\tau}$

$$\overline{w'c'} = -\tau\overline{w'^2}\frac{\partial \bar{c}}{\partial z} - \tau\frac{\partial \overline{w'w'c'}}{\partial z}$$

Flux K $\Delta c/\Delta z$ Non-local transport

Application of the budget equation for scalar fluxes can show why K theory fails in the surface roughness layer when non local transport is present

Question

- What is the implication of Roughness Sublayer on Weather and Climate models, which use M-O Theory as the basis of computing the Flux Boundary Condition over rough surfaces?

In-Direct Method

$$F_c \sim F_s \frac{\Delta c_z}{\Delta s_z}$$

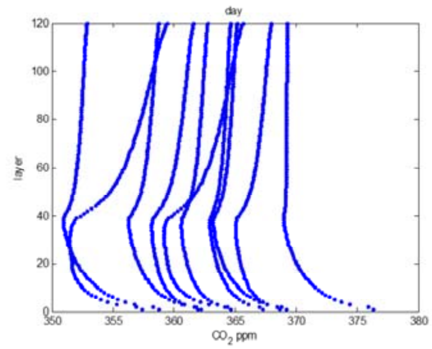
Fs is directly measured with eddy covariance or lysimeter

When is this method Useful???

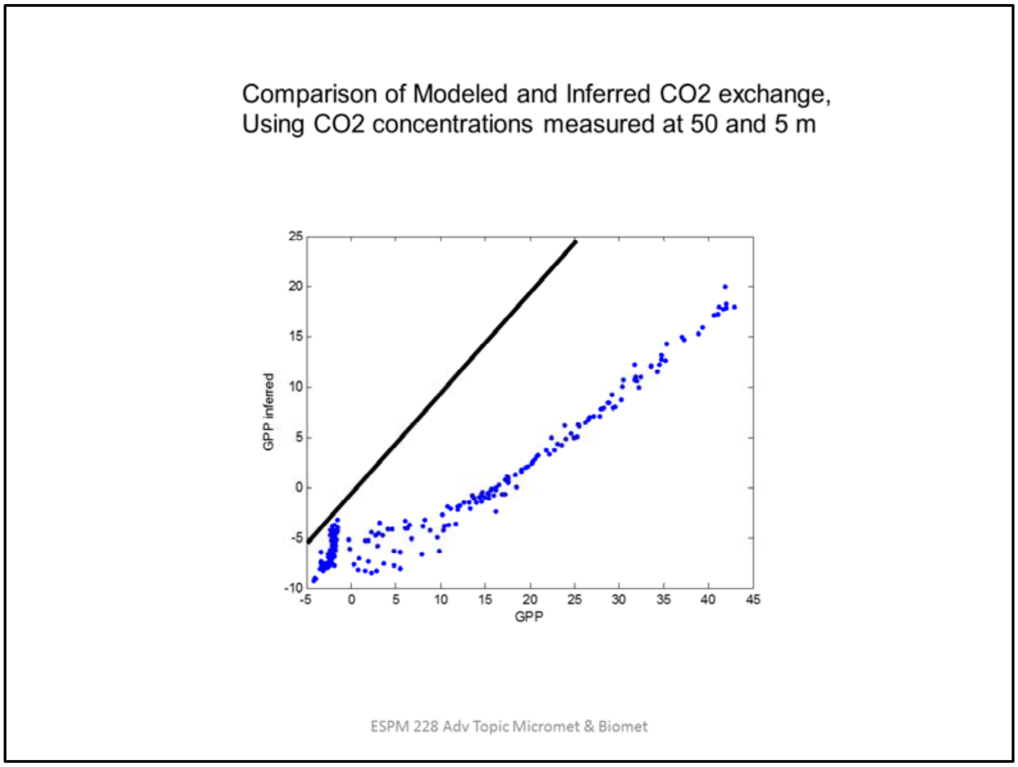
The indirect method is a good way of deducing K when you have reliable flux measurements of another scalar, eg sensible heat with a sonic anemomter, or evaporation with a lysimeter

Fallacy of computing fluxes with gradients measured above and within the canopy

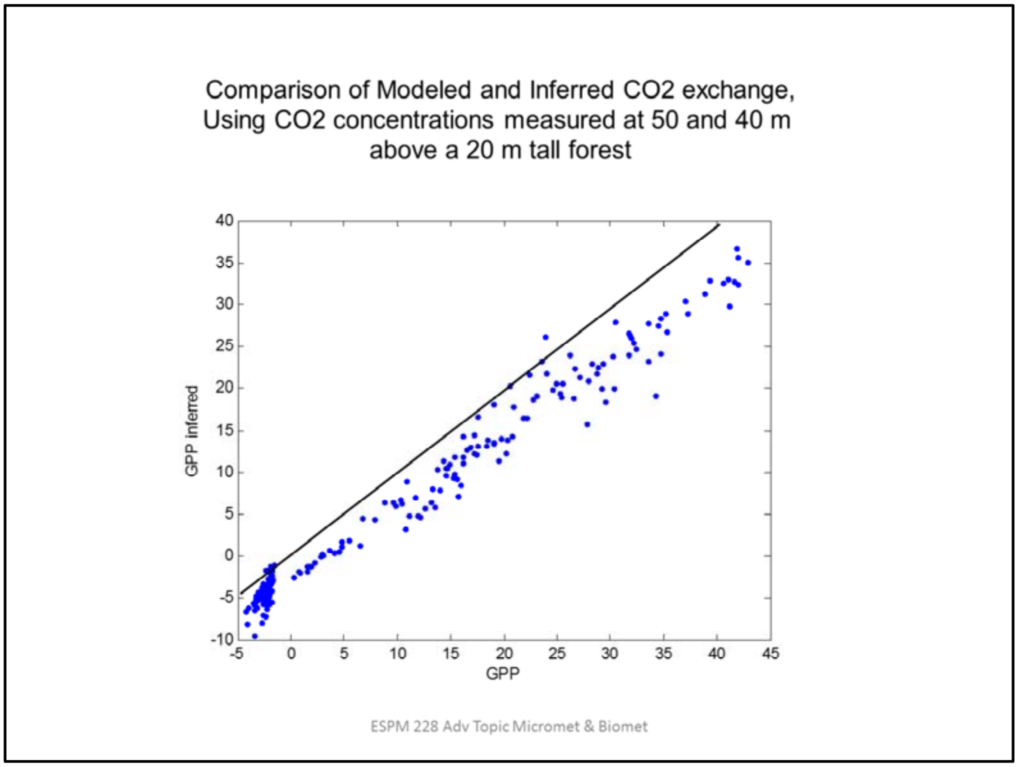
$$F_c = \frac{\Delta c}{\Delta z} F_a \frac{\Delta z}{\Delta a}$$



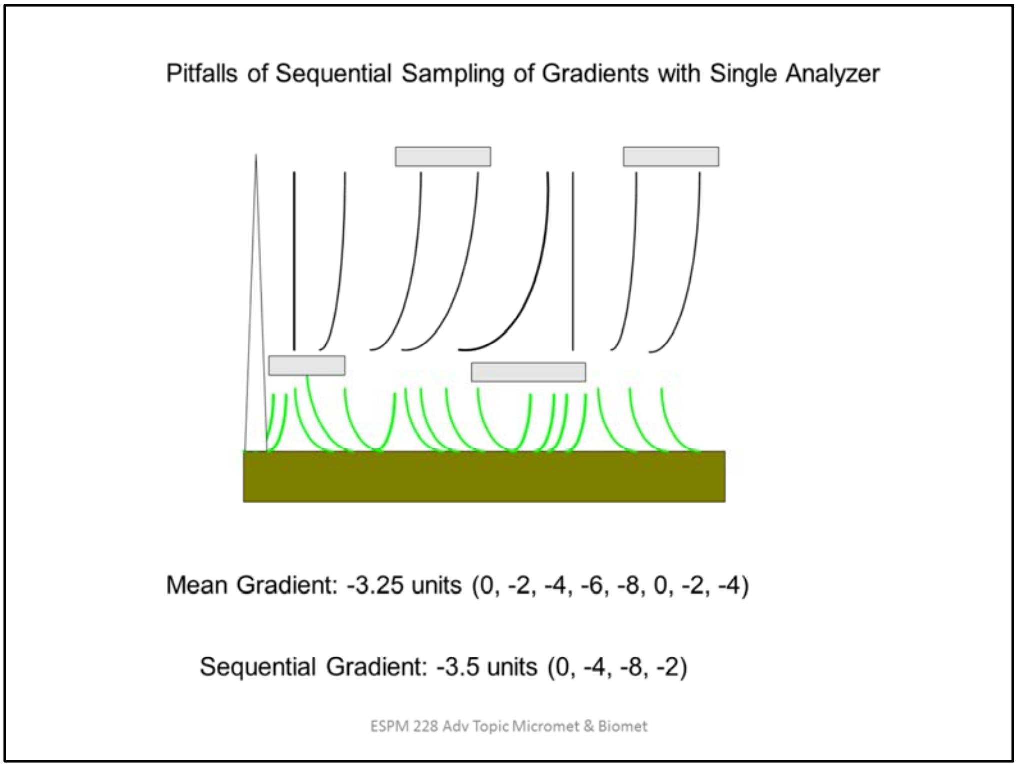
Too often I have seen micromet methods misused and abused by ecologists and biogeochemists. They have some notion that one can derive a gradient by measuring the scalar in the forest and above it. This is wrong



I used my CANOAK model to test this approach. The model explicitly computes gross primary productivity and concentration profiles. So we can test how well we can reconstruct fluxes by computing them with gradiens within and above the canopy. The error is great.



In comparison here is how well we compute fluxes with gradients above the canopy..Big difference, eh?



Redo next time..Take a 100 s time series in u* and compute dC/dz from eddy flux and sample…

Sampling Protocol: Resolving Gradients

$$\frac{\Delta C}{C} = -\frac{F}{ku_* C}\ln\left(\frac{z_2}{z_1}\right)$$

Gradient is weaker when Turbulent Mixing Increases or the Flux decreases

Must know sensor resolution, precision and accuracy

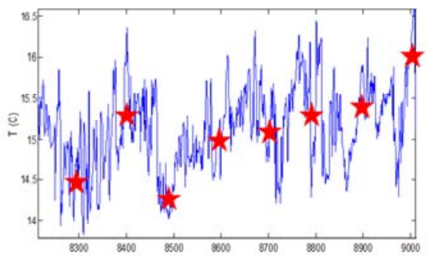
Table L1 The measurement requirements for assessing relative concentration gradients as a function of turbulent mixing the the underlying flux density. Values of ΔC/C for near neutral conditions (after Wesely et al., 1989). These computations are for near neutral stratification.

u*	V_d = 0.1 cm/s	V_d= 0.5 cm/s	V_d= 1.0 cm/s
	ΔC/C	ΔC/C	ΔC/C
0.1	1.7%	8.7%	17.3%
0.2	0.9	4.3	8.7
0.3	0.6	2.9	5.8
0.6	0.3	1.4	2.9

$$V_d = \frac{F}{C}$$

How Often one Samples (T_c) the Profile depends on the Time Scale of Turbulent Mixing

$$\varepsilon = 6\left(\frac{T_c}{\tau}\right)^{0.8}$$



ε□□□□, error	T_c	τ
7	60	50
4	60	100
3	60	200
44	600	50
25	600	100
14	600	200
105	1800	50
60	1800	100
35	1800	200

Smaller Errors if you Sample more often

Error Analysis

$$\frac{\sigma F_c}{F_c} = \sqrt{(\frac{\sigma K}{K})^2 + (\frac{\sigma(\Delta\rho_c)}{\Delta\rho_c})^2}$$

Error in K from Energy Balance Method

$$\frac{\sigma K}{K} \approx \sqrt{(\frac{\sigma(Rn-G)}{Rn-G})^2 + (\frac{\sigma(\Delta\rho_v)}{\Delta T + \Delta\rho_v})^2 + (\frac{\sigma(\Delta T)}{\Delta T + \Delta\rho_v})^2}$$

Discussion Points.

What are the practical limits to sampling scalar profiles often?

How do we apply K-theory to design an Experiment?
How far should the sensors be placed above one-another?;
What is the error tolerance of the sensor vs the detectable gradient?

When and where is a micrometeorological Flux tower representative?