Lecture 4:
Eddy Covariance Method, part 2,
Application, Signal Processing and Data Manipulation

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ESPM 228

Spring 2016

Making Eddy Flux Measurements

- Interconverting between Time and Frequency Space
  - Fourier Transforms
- Sampling Frequency
  - Sensor time constant
  - Analog to Digital Conversion and aliasing
- Sampling Duration
- Sensor Placement
  - Ht above surface
  - Proximity of anemometer and scalar sensor
- Signal Attenuation
  - Volume of sensor
  - Tube length and flow rate

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This lecture focuses on how to acquire, process and interpret eddy covariance data

Power Spectrum defines the Frequencies to be Sampled

Power Spectrum

$$\overline{w'w'} = \int_0^\infty S_{ww}(\omega)d\omega$$

Co-Spectrum

$$F = \overline{w'c'} = \int_0^\infty S_{wc}(\omega)d\omega$$

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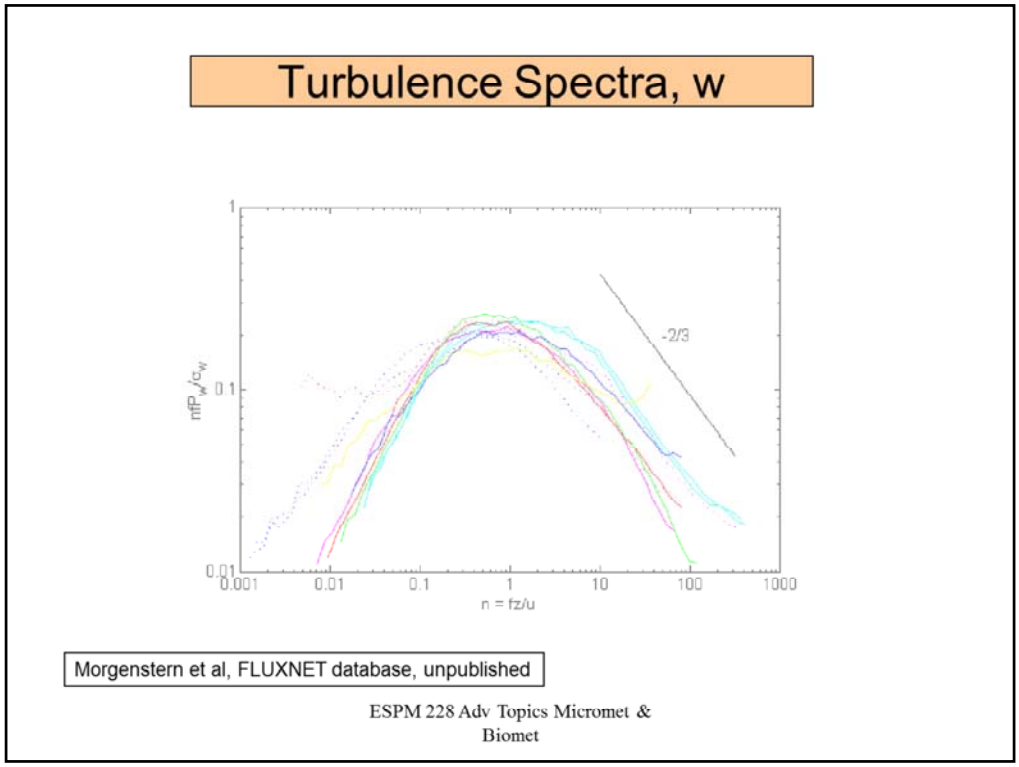
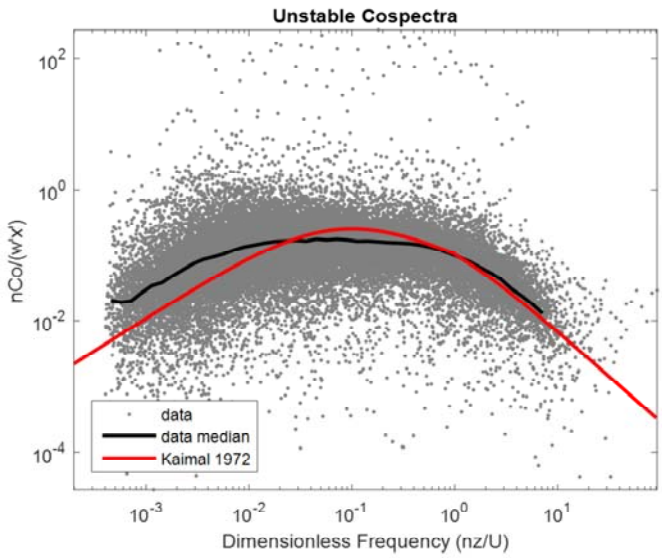
Variances and covariances are the integral of the power spectral or co-spectral density across all the contributing frequencies, or with transformation wave numbers or wavelengths. Here omega (ω) is angular frequency, 2 pi times n, the frequency. This equation tells us you have to sample all the contributing eddies to measure the fluxes and variances correctly.



Here is an example of hundreds of spectra produced from the FLUXNET database for many different canopy heights, densities etc. Note how the spectra converge with normalization (in this case the natural and normalized frequencies, n and f, have been swapped.



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Example of data of the cospectrum for w'T' over rice during unstable atmospheric conditions. Also superimposed is block average means and the theoretical spectrum of Kaimal.

Fourier Transforms can Recreate an Original Time Series through the Super-position of Sine and Cosine functions of varying frequency

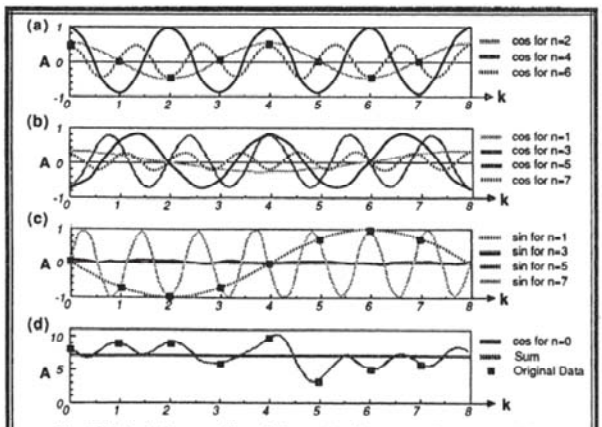

Fig. 8.5 (a-c) Superposition of sine and cosine waves that recreate (d) the original timeseries.

Biomet

From Stull, 1988

Fourier Transforms, F, of time series of periodic function x(t)

$$F_x(\varpi) = \int_{-\infty}^{\infty} x(t)\exp(-i\varpi t)dt$$

$$\exp(ix) = \cos x + i\sin x$$

$$\omega = 2\pi n$$

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Fourier transforms are a powerful way to take a time series and compute is spectrum, a figure that gives information on the power density as a function of frequency. Fourier transform is global compared to wavelets which are local. The Fourier transform pairs exist between the correlation coefficient, F, and the spectral density, S., R is lag correlation coefficient

The transformation between a time series to Fourier spectral space is by recreating that time series by adding sine and cosine waves of various amplitude and wave length.

Power Spectral Density, S

$$S_x(k) = \frac{dt}{N} F_x(k) F_x^*(k)$$

$F_x^*(k)$ Complex conjugate, e.g a + ib and a -ib

Magnitude: $Re^2 + Im^2$

```
 yt= fft(W,nsamp);

FFT.W = yt .* conj(yt)*dt /nsamp;

%FFT.W = 0.25* yt .* conj(yt)*dt./nsamp;
```

Convolution theorem:
the convolution of two time series in temporal space (x(t) equals the products of their Fourier Transforms in frequency space (X(f))

$$x * x = \int_{-\infty}^{\infty} x(\tau)x(t-\tau)d\tau$$

$$x * x = X(f)X(f)$$

$$x * y = \int_{-\infty}^{\infty} x(\tau)y(t-\tau)d\tau$$

$$x * y = X(f)Y(f)$$

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The convolution of time series x and y is the product of their respective Fourier transforms. Fourier transform of the convolution is just the product of the individual Fourier transforms, X and Y. The * stands for the convolution

In other words the convolution of two time series in temporal space equals the products of their Fourier Transforms in frequency spce

Autocorrelation, R_{xx} is:

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t+\tau)x(\tau)dt$$

$$R_{xx} = X(f)X(f)^*$$

$$R_{xx}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(\tau)\exp(i\varpi\tau)d\omega$$

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The convolution of g and h is the product of their respective Fourier transforms. Fourier transform of the convolution is just the product of the an individual Fourier transforms times the complex conjugate of the other transform

Fourier Transform Pairs
Spectral Densities (S) and Lag Correlations (R)

$$S_{xx}(\varpi) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{xx}(\tau)\exp(-i\varpi\tau)d\tau$$

$$R_{xx}(\tau) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{xx}(\tau)\exp(i\varpi\tau)d\omega$$

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Fourier transforms are a powerful way to take a time series and compute is spectrum, a figure that gives information on the power density as a function of frequency. Fourier transform is global compared to wavelets which are local. The Fourier transform pairs exist between the correlation coefficient, F, and the spectral density, S., R is lag correlation coefficient

The transformation between a time series to Fourier spectal space is by recreating that time series by adding sine and cosine waves of various amplitude and wave length.

Correlation of two functions, x and y

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t+\tau)y(\tau)dt$$

$$R_{xy} = X(f)Y(f)^{*}$$

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The convolution of g and h is the product of their respective Fourier transforms. Fourier transform of the convolution is just the product of the an individual Fourier transforms times the complex conjugate of the other transform (Y(f)*)

Cross Spectrum

$$S_{xy}(\varpi) = Co_{xy}(\varpi) + iQ_{xy}(\varpi)$$

Co-Spectrum

$$Co_{xy}(\varpi) = \frac{1}{\pi}\int_{-\infty}^{\infty} R_{xy}(\tau)\cos(\varpi\tau)\,d\tau$$

Quadrature Spectrum

$$Q_{xy}(\varpi) = \frac{1}{\pi}\int_{-\infty}^{\infty} R_{xy}(\tau)\sin(\varpi\tau)\,d\tau$$

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The cross spectrum is a function of the co specturm and the quadrature spectrum. It is the integral of the co-spectrum, not cross spectrum, that produces flux information. The quadrature spectrum tells us how to time series may be in or out of phase with one another. The cross spectrum is a function of the lag correlation between x and y, where tau is the lag time

```
xt= fft(C,nsamp);

yt= fft(W,nsamp);

Cospectrum =  xt .* conj(yt)*dt/nsamp;
```

Coherence

$$Coh(n) = \frac{(Co_{xy}(n)^2 + Q_{xy}(n)^2)}{S_{xx}(n)S_{yy}(n)}$$

Phase Angle

$$\phi_{xy}(n) = \tan^{-1}\left(\frac{Q_{xy}(n)}{Co_{xy}(n)}\right)$$

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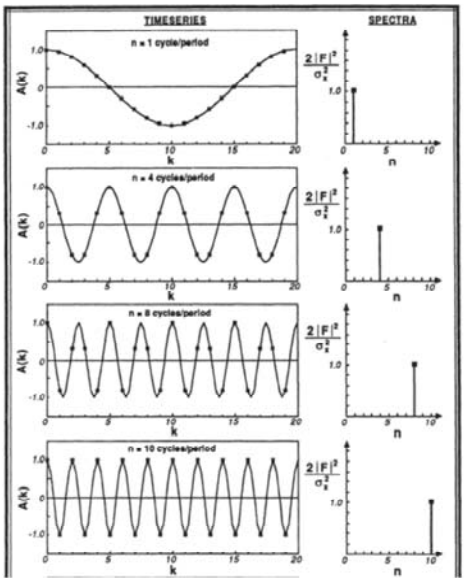
Phase angle produces the lead lag information. Coherence tells us about how well two signals may be correlated as a function of frequency, n

Fourier Transform of Simple Waves



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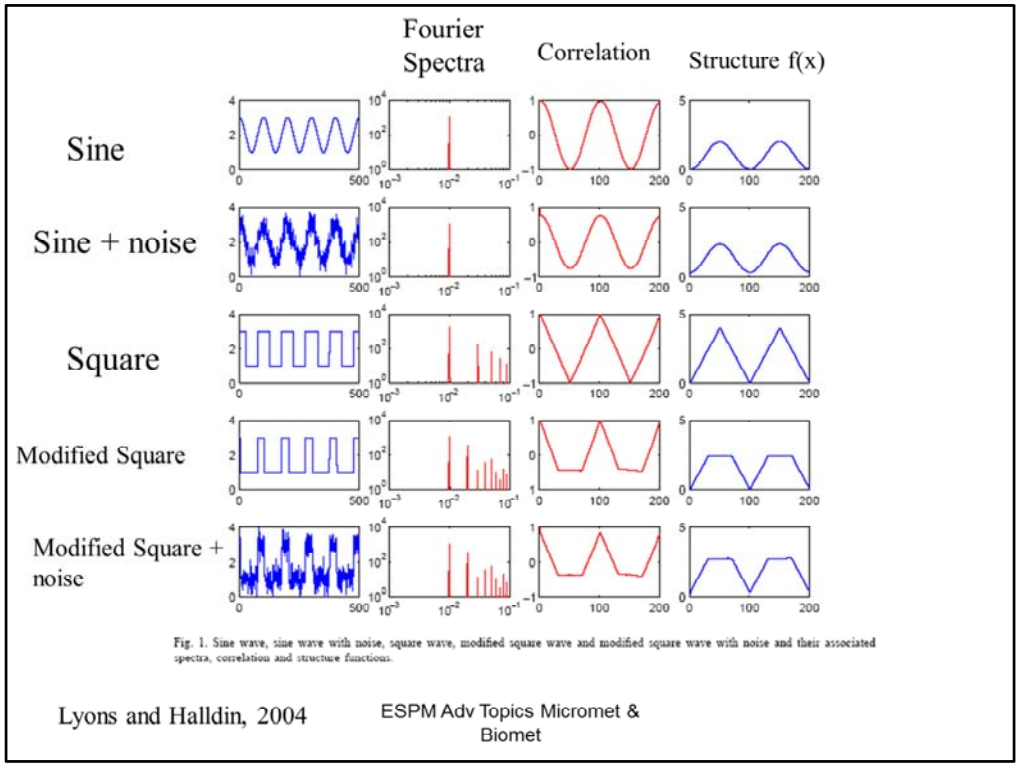
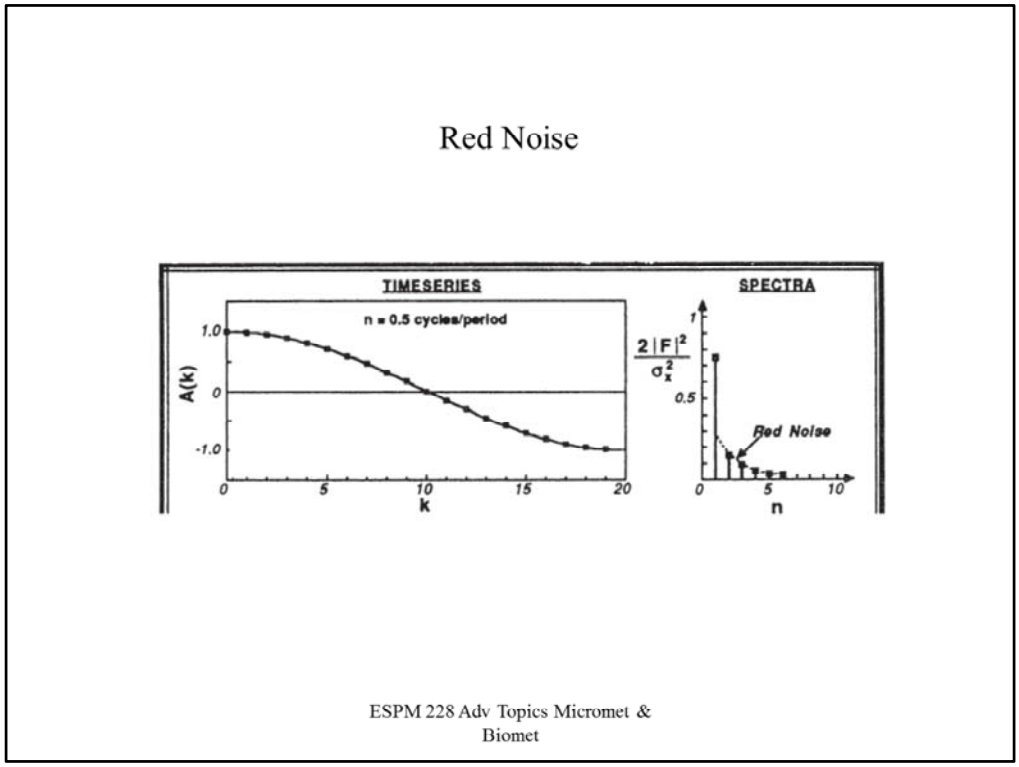
Stull, 1988

Simple waves of different frequency



Examples of how periodic and non periodic time series may affect the Fourier transform. Notice the high frequency noise or ringing on square waves. This does not appear on sinusoidal time series. The simple sine waves are described by a single peak in the spectrum



Unresolved trends can cause red noise. Energy leaks into other frequency bins

From Stull, 1988

2. Signal Attenuation: The Role of Filtering Functions

- High pass filtering
- Low pass filtering
- Digital sampling
- Sensor response time
- Attenuation of signal via sampling
- Line or volume averaging
- Sensor separation
  - Lag and Lead times between w and c

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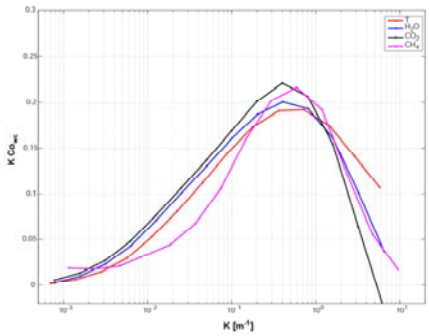
In this next section we will examine different courses of spectral filtering in greater detail. Here is a list of potential causes of spectral filtering.

Sonic T, Open Path q and C sensor, closed path CH4 sensor:
Correct Systems 'Filtering' with an Ideal coSpectrum, e.g.w'T'



1. Develop Transfer Function to Compensate for Signal Attenuation

Co spectra of Detto and Baldocchi

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Different sensors may filter high frequency signals. This is of concern because this filtering affects the computations and accuracy of variance and power spectra. The eddy covariance method is forgiving as two signals are not well correlated in the inertial subrange, so litter information is lost.

$$\overline{w'c'}_{measured} = \int_0^{\infty} H(\varpi) Co_{wc}(\varpi) d\omega$$

Filter Functions

$$H(\varpi) = H_1(\varpi) H_2(\varpi) \ldots H_n(\varpi) = \prod_{n=1}^{N} H_n(\varpi)$$

Moore, 1986 BLM

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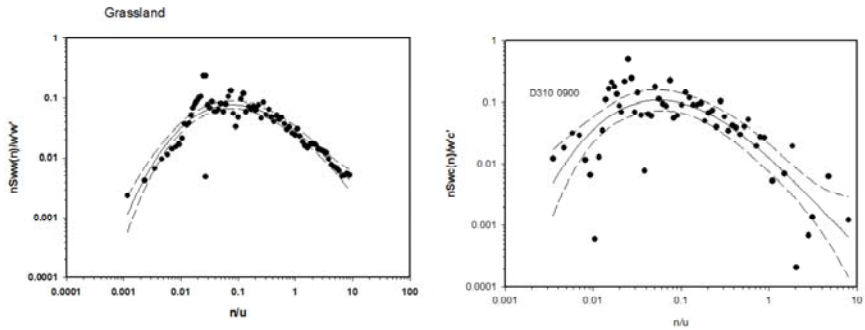
Introduction of filter or transfer coefficient (H) to the measured and ideal cospectra. Ideally we want to correct the measured covariance to compute the more ideal by inverting this equation. Also the filter correction functions are multiplicative.

How Long Should We Sample?

Roles of:
Averaging Time
Time Trends
Low Pass and High Pass Filters
Digital Recursive Filters

Use Spectra to Determine Sampling Time and Frequency



Must Sample Eddies up to 10 times per second for 30 to 60 minutes

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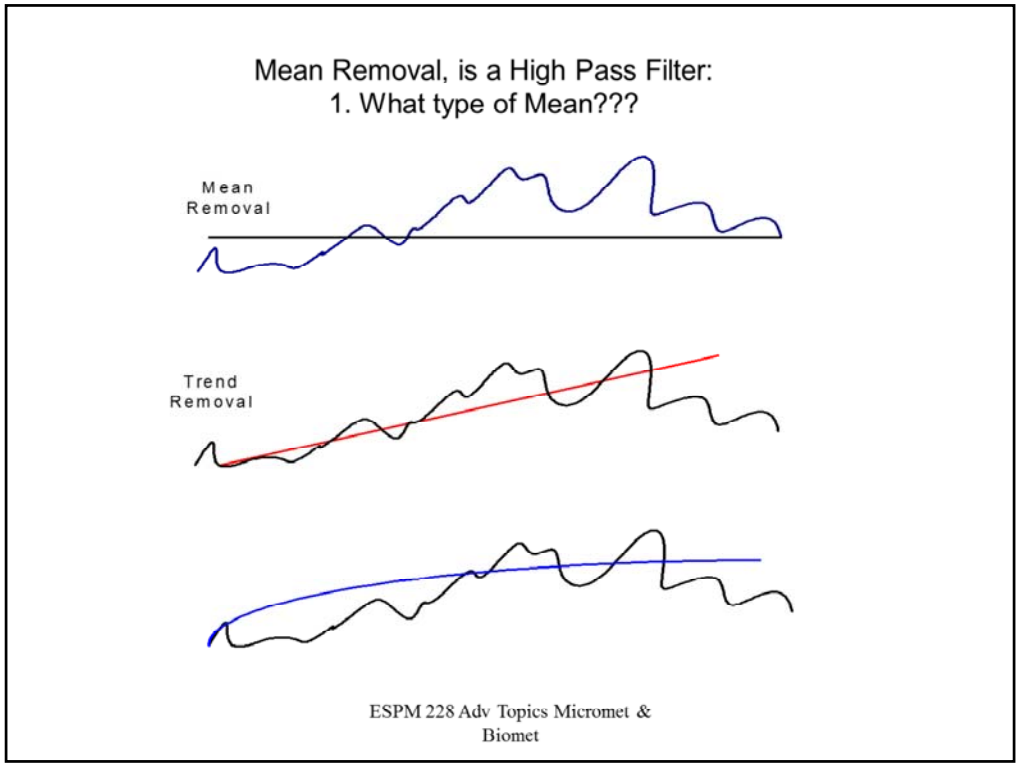
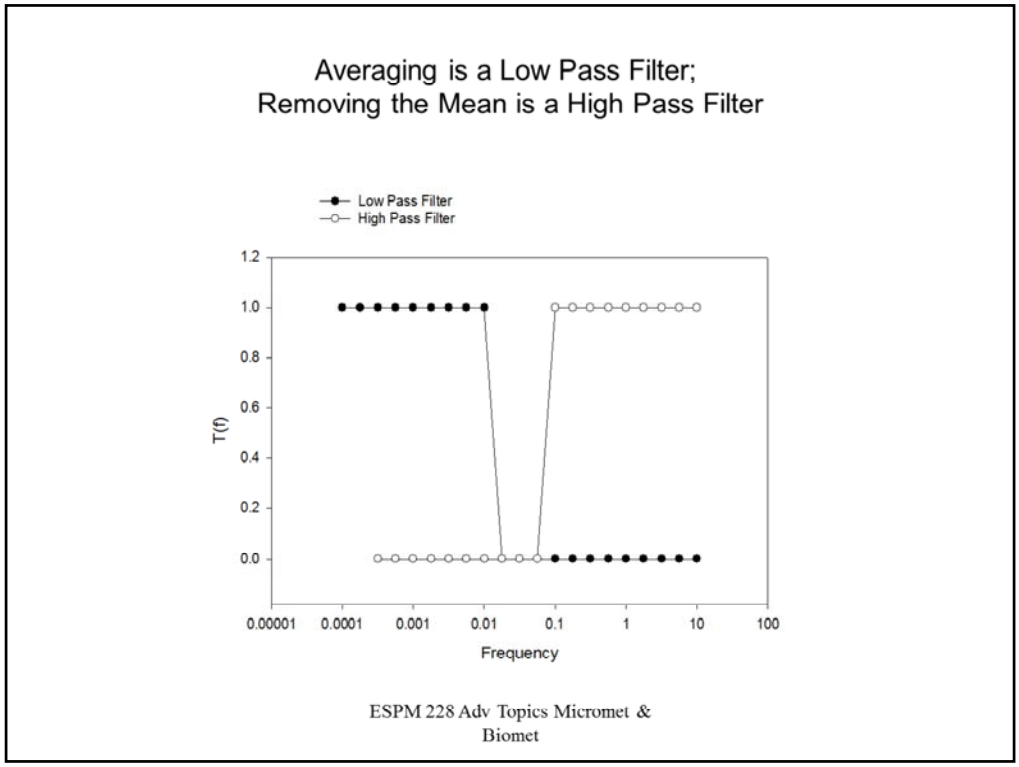
Classic power and co spectra for turbulence time series. There is a spectral peak and an inertial subrange that decreases in a power law fashion. Notice the range of dominant frequencies. In this case most spectral energy is detected by wavenumbers (inverse wavelength, noted by the ratio between natural frequency and wind velocity) ranging between 0.001 and 10. With the wavenumber normalization you can deduce how the spectra may shift with high or lower wind velocities when the spectra are produced in terms of natural frequency; the goal is to find some way to normalize spectra so data from different periods and conditions can be compared.



The act of averaging is a filtering process. So we need to understand its role by the Fourier transform.

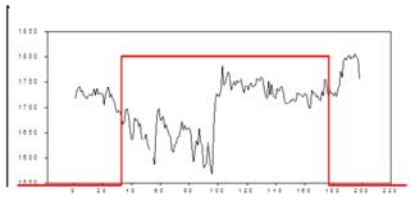
Detrending. Mean removal or low pass filter? Reynolds averaging says nothing about detrending, so arithmetic averaging is the norm. Yet the literature has cases where and when detrending has been applied. Here is something you can investigate with your covariance software and data.



Averaging and mean removal has the impact of band pass filtering. The average is a low pass filter, as well, only low frequency information passes. Conversely, mean removal is a high pass filter, only high frequency information passes.

Filtering by Block Averaging

$$h(t) = 0, \quad -\frac{T}{2} < t > \frac{T}{2}$$



$$h(t) = 1, \quad -\frac{T}{2} > t < \frac{T}{2}$$

Fourier Transform of a Block Average Produces a Sinusoidal Low Pass Filter

$$F(f) = \frac{\sin(\pi \cdot f \cdot T)}{\pi \cdot f \cdot T}$$

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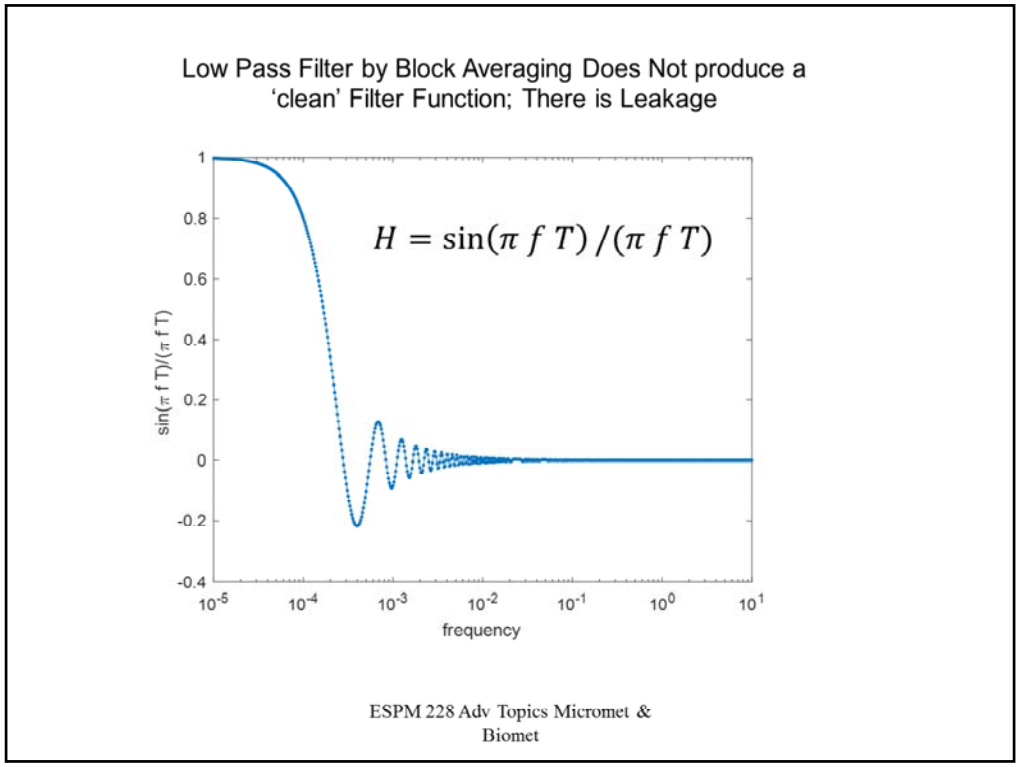
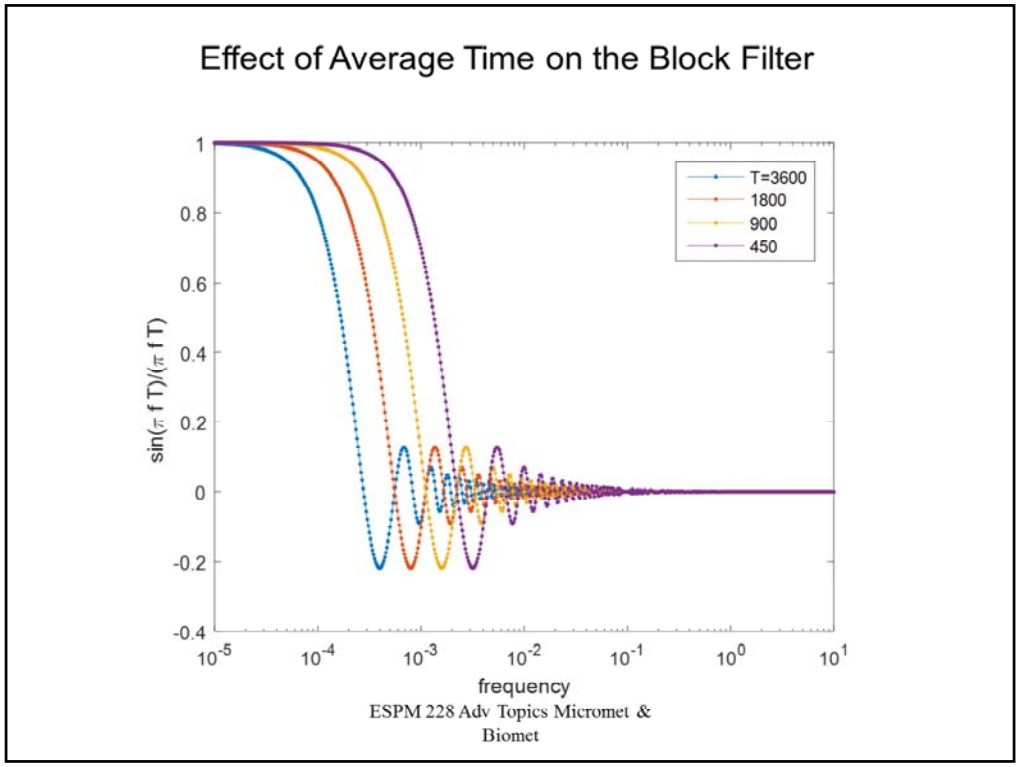
Block averaging is not ideal, it is abrupt and truncated, not sinuosoidal, so it causes sinusoidal ringing. On the contrary, wavelets that have sinusoid properties are used because they are cleaner filters.



This shows the ringing in Fourier space of a simple mean removal and its ringing properties. Point is a block average is not a perfect filter in spectral space. The abrupt edges of the delta function cause this ringing when converted from a time space to a frequency space. It is important to be bi-lingual in looking at turbulence time series in both time and frequency. Both tell us different things about the sensor, the signal and the transfer of mass and energy.



Here is how different averaging times affect the time averaging transfer function.

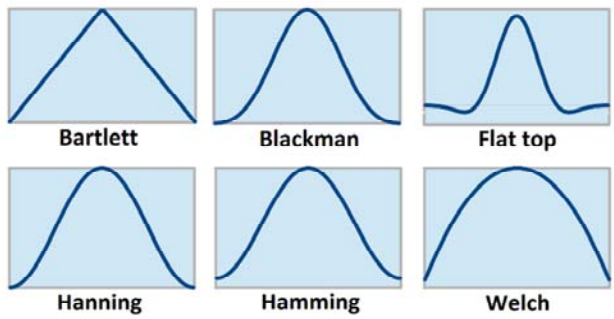
Leakage and Data Windowing

Hamming Window $w(n)=0.54-0.46\cos(2\pi nN), 0\leq n\leq N$

Hanning Window $\frac{1}{2}(1-cos\frac{2\pi j}{N})$

Welch Window

Bartlett Window



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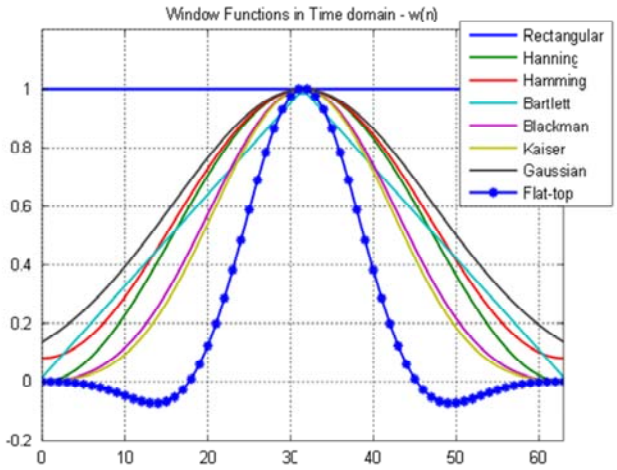
Tapering the time series with weight functions, like Hann, Welch, Bartlett Window
http://www.keuwl.com/SpectrumAnalyser/fftwindowssmall.png



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http://www.gaussianwaves.com/gaussianwaves/wp-content/uploads/2011/02/windowfunction_time_domain.png

When the filter function is applied to correct
a power spectrum or co-spectrum the filter function is squared:
$H(f)=F(f)^2$

$$H_{high\_pass}(f) = (1 - H(f))^2$$

$$\overline{w'c'}_{measured} = \int_0^{\infty} [1 - \frac{\sin^2(\pi \cdot f \cdot T)}{(\pi \cdot f \cdot T)^2}] H(\varpi) Co_{wc}(\varpi) d\omega$$

Moore, 1986 BLM; Kaimal and Finnigan, 1992

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The literature is a mess in terms of how these filter and transfer functions are described and applied. One has to be careful and note when they should or should not be squared. I have tried my best to bring some order, but still not perfect. In this case a high pass filter is 1 minus low pass filter. This term is squared because it is applied to w'c' or w'w'

Real-Time Mean Removal
Digital Recursive Filter

$$\overline{x_i} = \alpha \overline{x_{i-1}} + (1-\alpha) x_i$$

$$\alpha = \exp\left(\frac{-\Delta t}{\tau}\right)$$

$$|H(\omega)| = \frac{[\alpha + \alpha^2][1 - \cos(\omega / f_s)]}{(1 - 2 \cdot \alpha \cdot \cos(\omega / f_s) + \alpha^2)}$$

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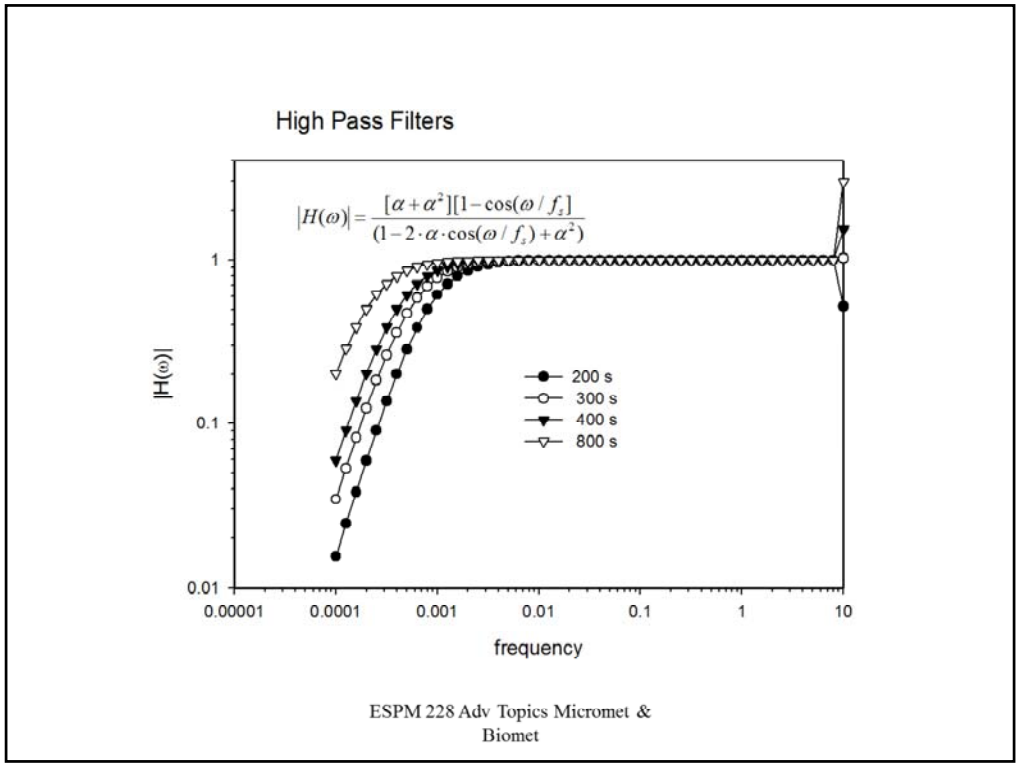
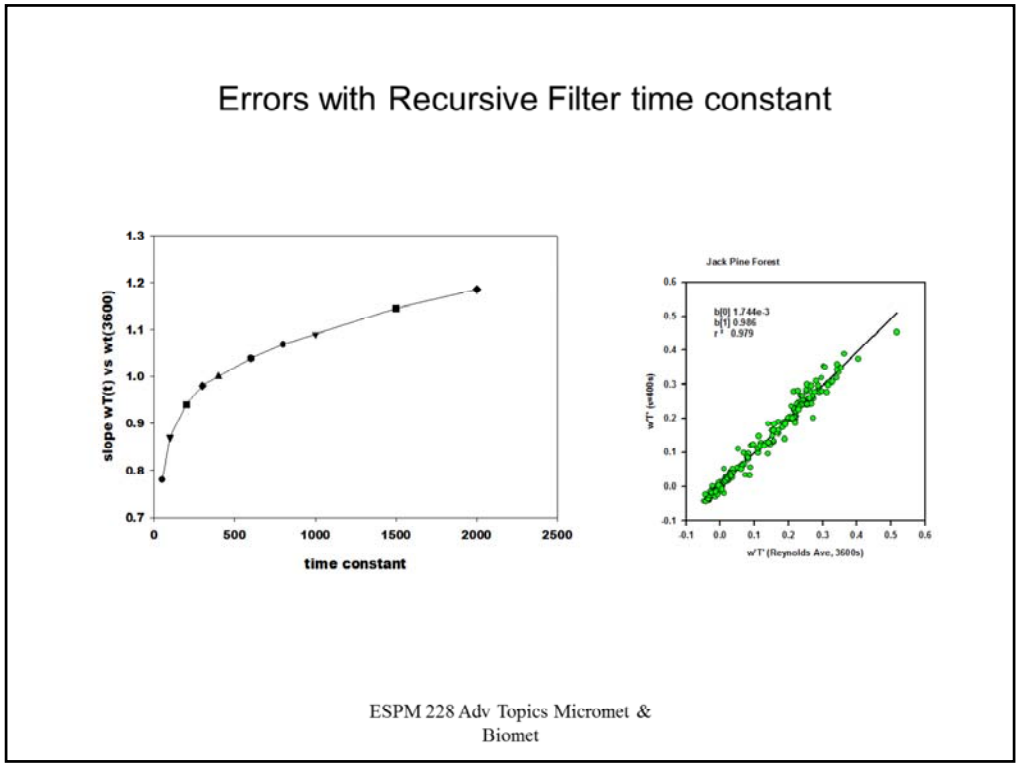
In the early days of eddy covariance we did not have enough computer storage to save all the raw 10 Hz data. So we applied digital recursive filters to detrend the time series. While this approach is not used today, I think the concept of digital recursive filters is interesting and important and may have applications to other problems you may face.

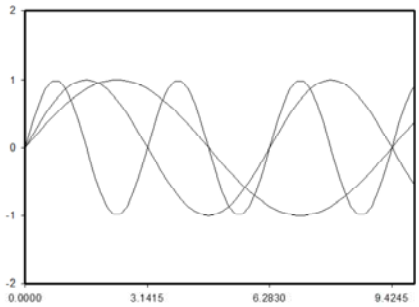


The high pass filters of the digital recursive filter is cleaner and does not have ringing, But the transfer function is not ideally square in spectral space.



What type of errors may arise with a digital recursive filter? This can be asked by comparing fluxes, averaged over 1 hour, with different time constants

How Fast Should we Sample?



Nyquist-Shannon Sampling Theorem and Aliasing

Sample at a rate twice that of the cut off frequency

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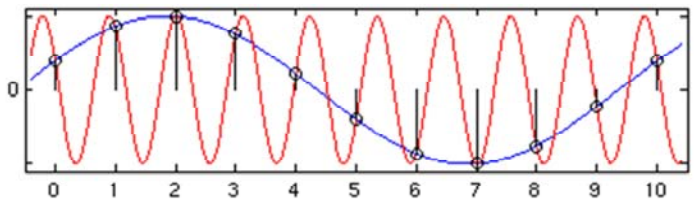
How fast should you sample? This constraint is deduced by Nyquist Shannon sampling theorem. You want to sample twice the rate of the fastest frequency you want to detect. Think about a simple sine wave with a mean of zero. If you sample it twice at its half period, your mean will be zero, too.

If you are not sampling fast enough, High Frequency energy can be folded back onto lower frequency waves



10 Hz Energy Folded onto 1 Hz Signal

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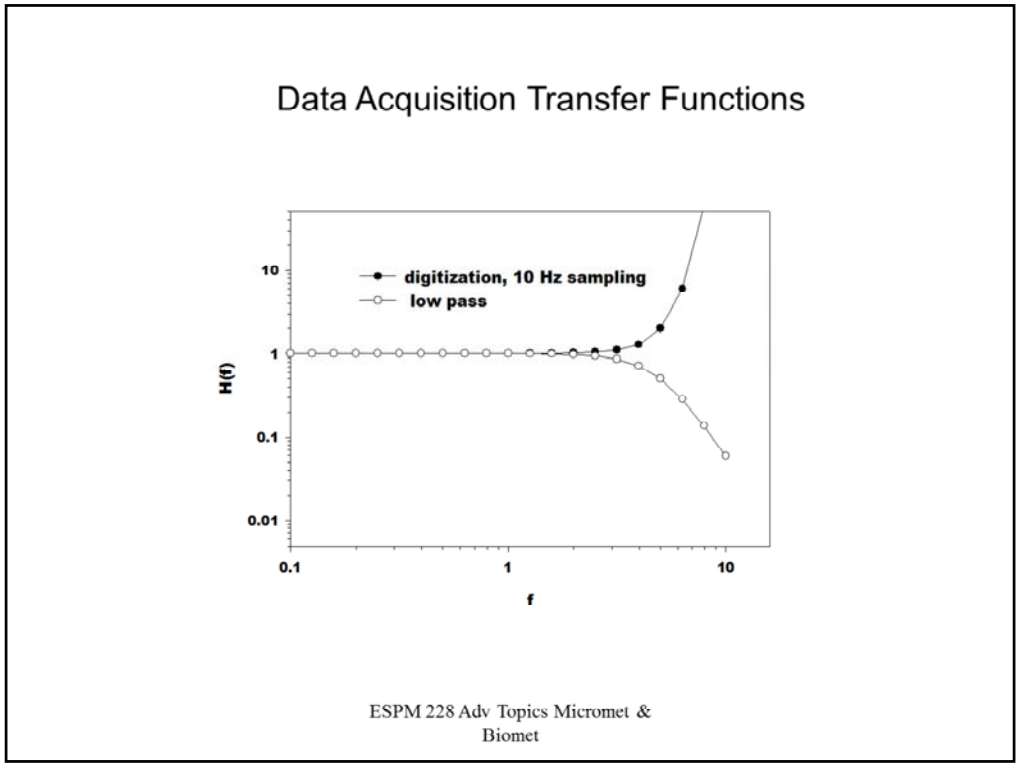
What happens if you sample too slow? Higher frequency contributions will fold back on your energy in the form of aliasing. The classic example is the digitization of rotating wagon wheels in cowboy movies and notice how it looks like the wheels are rotating backwards. In this cartoon you can see how 10 Hz information is folded back on the 1 Hz signal.



This is the net transfer function for data acquisition, considering low pass filtering and discrete digitazation.

Take Home Points

Detrend and Taper Time Series to reduce red noise and leakage

Pad with zeroes to reach power of 2 factor for FFT

Watch for Spurious Spectral Energy from Unresolved Frequencies

Sample Fast Enough to Reduce Aliasing

Sensor Size or Path Length, Response Time, Separation, or Sampling Air through a Tube

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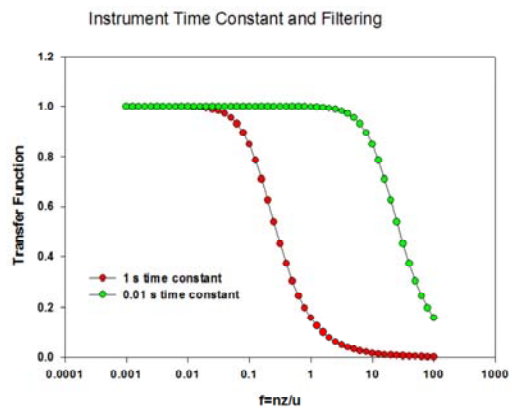
Slow Responding Sensors Filter High Frequency Information

$$H_{sensor\_response}(f) = (1 + (2\pi f)^2 \tau_c^2)^{-1/2}$$

τ: time constant



Moore, 1986 BLM

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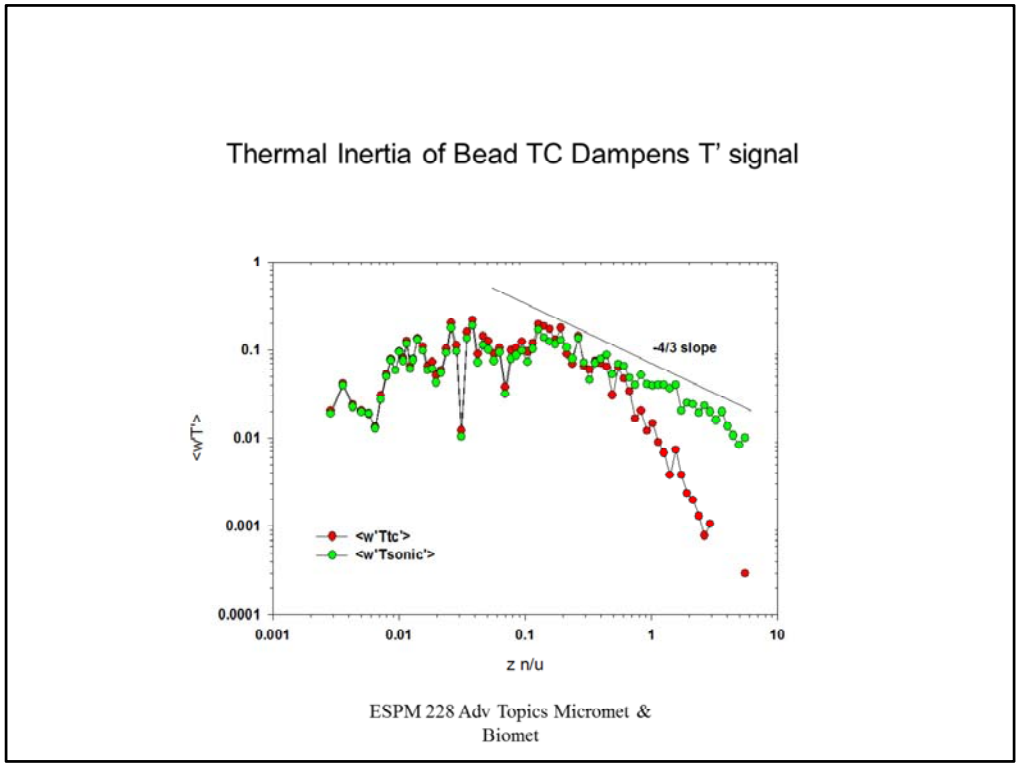
No sensor is perfect So the act of sensing a variable can filter high frequency contributions. Think of the thermal inertia of a bead thermistor that is too big, as noted by the figure above.

Or the line averaging of parcels of air moving through the cell of a spectrometer. In the later case, smaller fluctuations are dampened or smoothed.



Here is some data we collected from a finite-sized bead thermistor and temperature fluctuations from a sonic anemometer. You see the filtering of the w'T' flux covariance at high frequencies. So measuring sensible heat with a temperature sensor has problems. Too big and too much thermal inertia. Too small and it is fragile and breaks often from rain, bugs, dirt..

Sampling air through a tube dampens High Frequency contributions

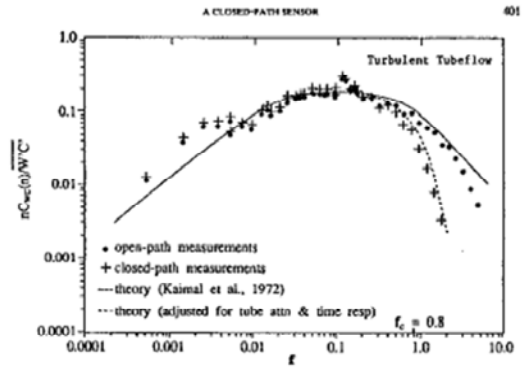


Fig. 3. Normalized logarithmic cospectra ($nC_{w\rho}(n)/\overline{w'\rho'}$) of CO_2 flux as a function of dimensionless frequency ($f = nz/U$). Included are cospectra measured with the open- and closed-path sensors, theoretical cospectrum for a scalar (Kaimal *et al.*, 1972), the adjusted theoretical cospectrum (see Section 4.2.1 for details), and the cutoff frequency (f_c) for turbulent tubeflow (configuration II). August 17, 1990; 12:30–13:00 hr (local time). z = 1.7 m and $\bar{U}$ = 3.9 m s^{-1}.

Suyker and Verma, 1993, BLM

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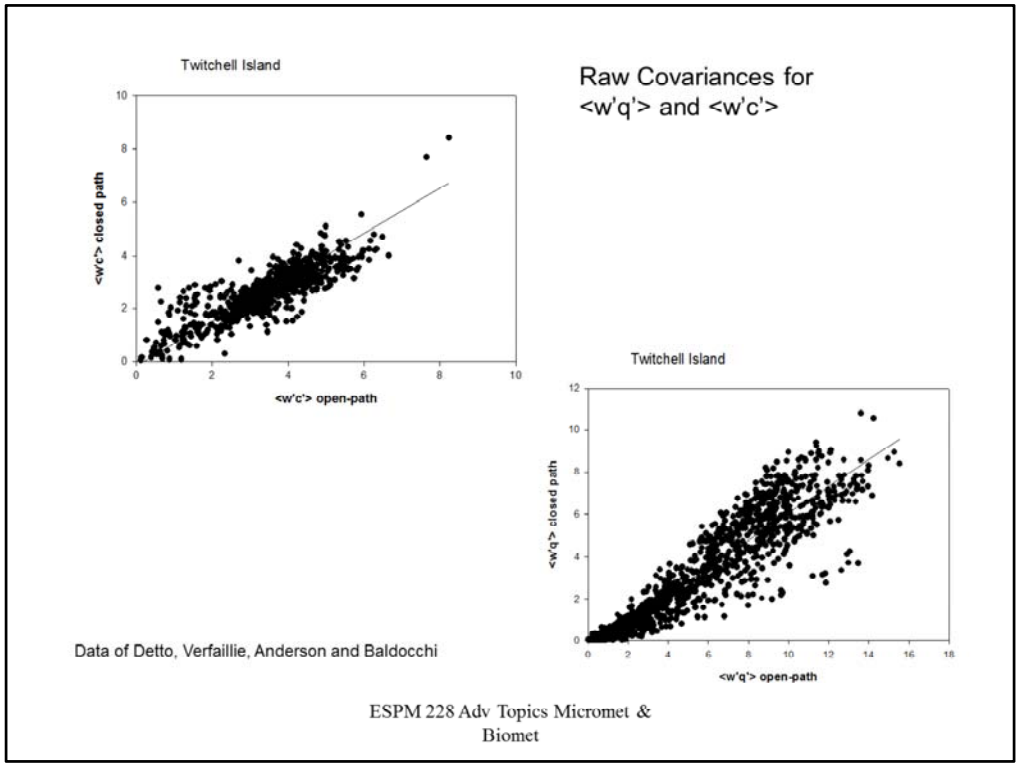
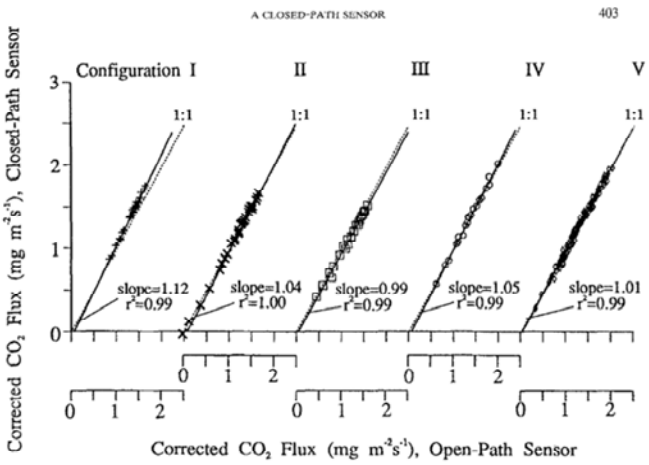
Here is the example of the attenuation of high frequency fluctuations when sampling through a tube. Sampling air through a tube can dampen scalar fluctuations. Interaction with the tube can also remove material from the air stream. This is a notorious problem with hygroscopic dirt particles inside a tube and its effect on water vapor. Fluxes of water vapor with a new tube are different from those with an older tube.



How different may data from closed and open path sensors? The bottom line is if you are careful, good data can be had from both systems. It is important to recognize each has its place. Closed path sensors are probably better in wet environments and where you have ample power to run pumps. Open path sensors are best in remote locations, powered by solar panels. And they suffer less from line attenuation that is greater with water vapor.

With Appropriate Corrections Open and Closed Path IRGAs produce Similar Results



Suyker and Verma, 1993, BLM

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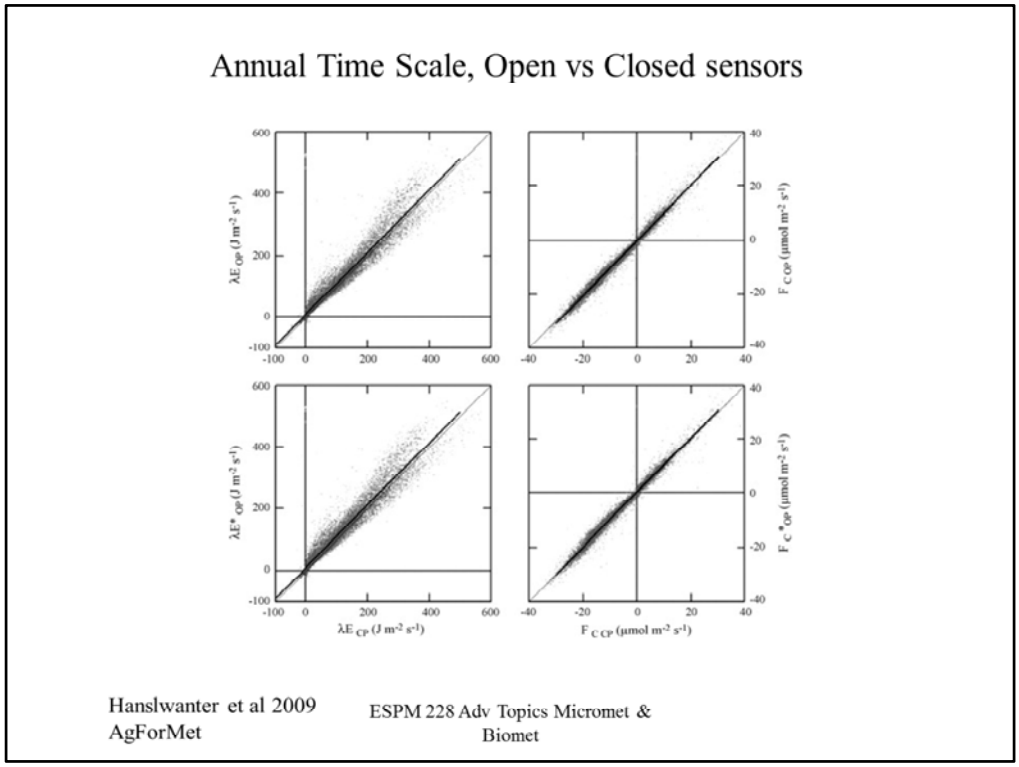
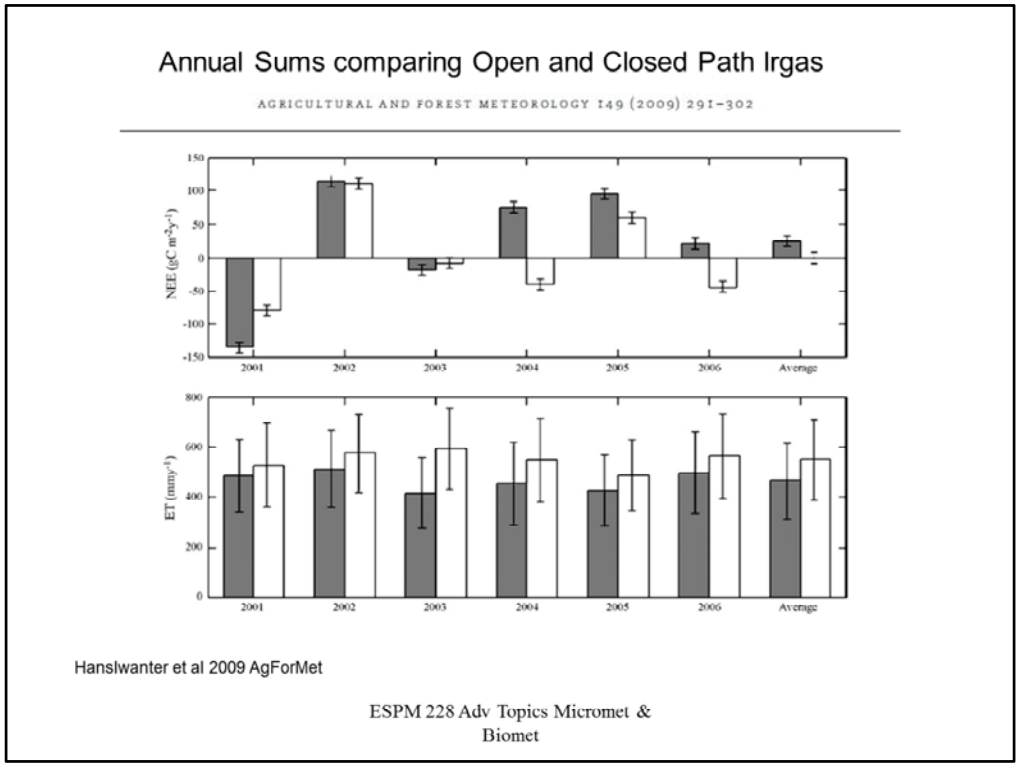
This figure, by Suyker and Verma, show good comparison between open and closed path systems.

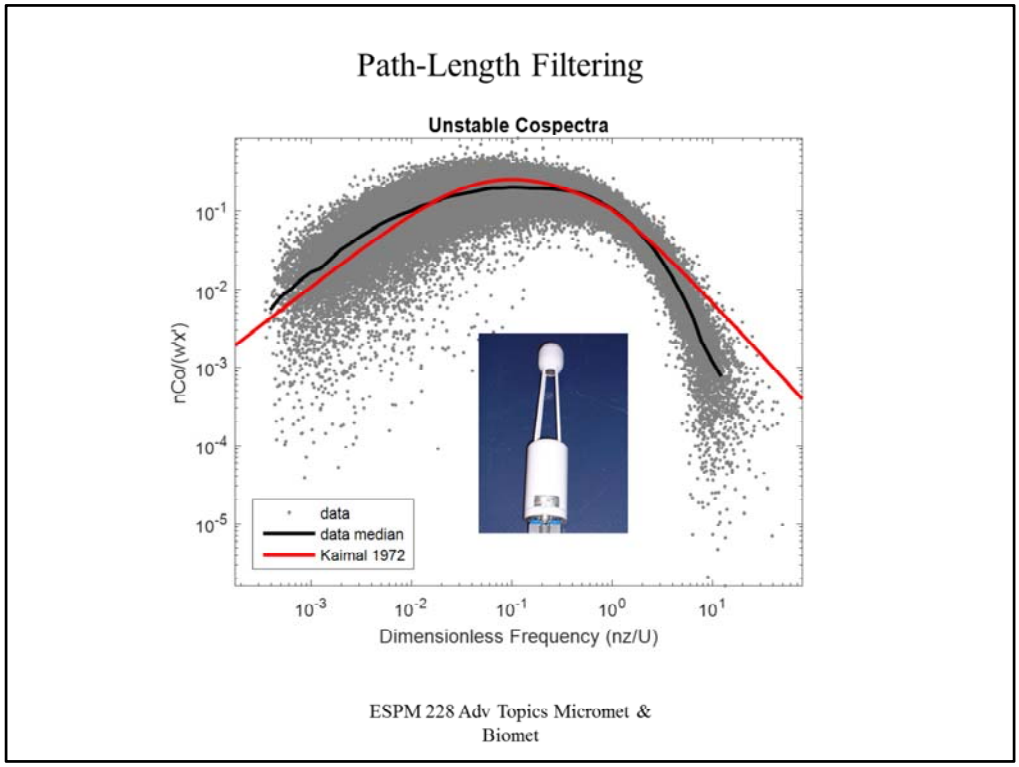
I am agnostic about using open vs closed path CO2 sensors. As mentioned before, know they site. In raining regions or cold places maybe a closed path is better, if you have a short tube and plenty of power to run an ac pump. In dry and clear areas without power an open path can be very good. Plus it is less apt to loose water vapor, as a closed path. As long as the method is tested and validated one should be confident and don't be dogmatic of one time over another. Came to this conclusion with lots of experience and reading a rich literature on this topic



6 years of data from Hanslwanter and Hammerle et al increase our confidence of good data can be had from closed and open path sensors. Again both sensors work well over long time scales



In many cases the annual sums are close, though ET is filtered too much in my opinion by closed by sensors, hence my preference for open path. With small fluxes there may be a swap of sign in regards to CO2 fluxes



Here is a cospectrum where you see some filtering at high frequencies compared to the ideal Kaimal spectrum. This can happen for a variety of reasons associated with sensor size, sampling height, wind speed, sensor separation, etc

Sensors with a Finite Path Dampen small, high frequency Eddies

$$H_{pathave}(f) = \frac{1}{2\pi f}(3 + \exp(-2\pi f) - 4\frac{1 - \exp(-2\pi f)}{2\pi f})$$

f = n pathlength/u

Moore, 1986 BLM

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Sensor separation and path averaging introduces filtering that can be evaluated with this equation

Spatial Separation of a Sonic Anemometer and Gas Sensor Filter Small Eddies

$$H(x)_{sensor\_separation} = \exp(-9.9x^{1.5})$$

Moore, 1986 BLM

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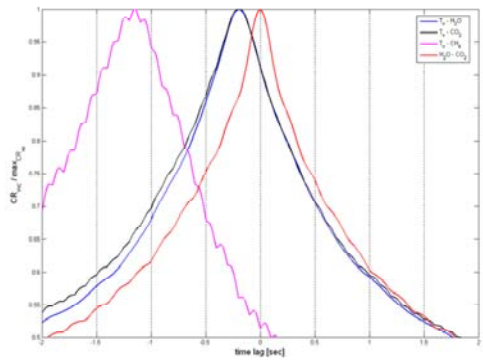
The spatial separation of transducers on sonic anemometers also causes filters

Sensor Separation causes w – c to lag or lead on another



How Accurate Does the Lag time Need to be?
Lag Correlation, w'c'

Data of Detto and Baldocchi

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Sensor separation cause lags and leads between w' and c'. But the lag does not have to be deduced super precisely. The spectral peak spans several seconds. This is another forgiving attribute of eddy covariance.

Lags can be deduced from Fourier transforms. I warn not to compute them for each an every hour as they may be numerically noisy when fluxes are small. So you don't want to chase your tail with introducing a wilding noisy lag when you know the pumps and air flow system through a closed path sensor may be relatively stable. On the other hand it is good to compute these regularly and inspect them to ensure filters are not clogging or pumps are varying due to power loads and supplies.

System Transfer Function

$$T_{data\_acquisition} = H_{aliasing} H_{high\_pass} H_{low\_pass}$$

$$T_{measurement} = H_{sensor\_separation} H_{instrument\_response} \sqrt{H_{path\_ave\_chem\_sensor} H_{path\_ave\_w} H_{tube\_dampening}}$$

$$T_{total}(\varpi) = T_{data\_acquisition}(\varpi) T_{measurement}(\varpi)$$

Moore, 1986 BLM

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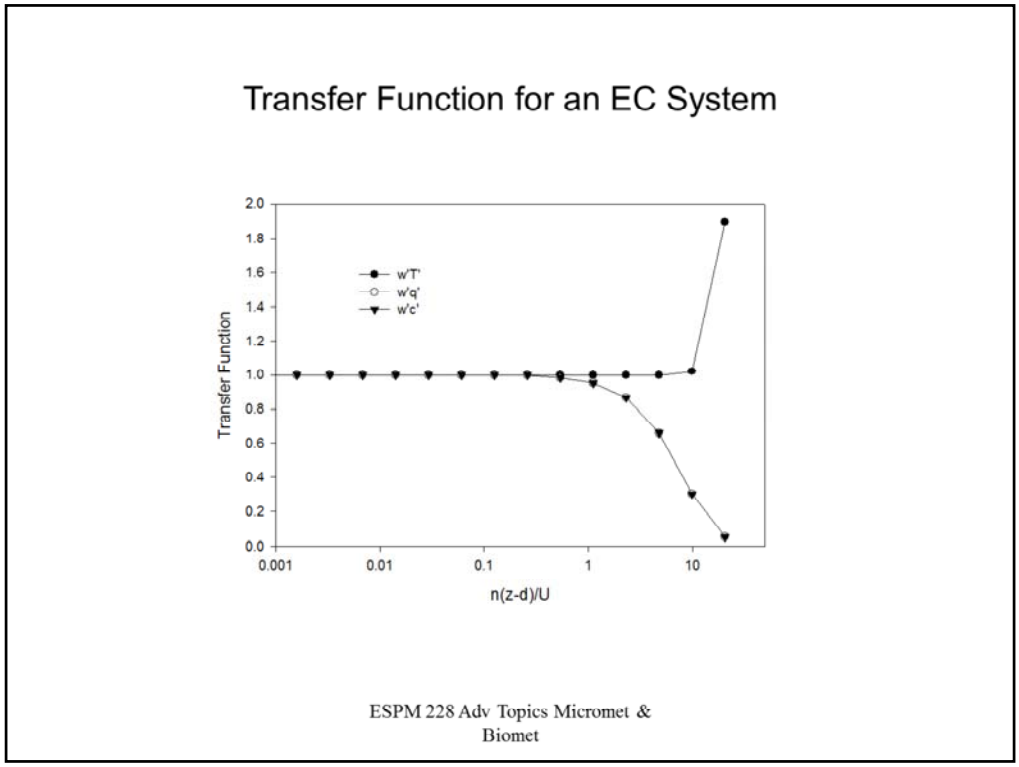
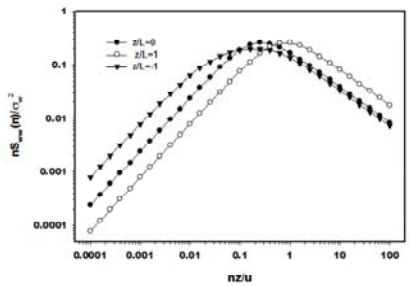
Here is the total transfer function. As I noted before not some H values are squared and others not.



Typical transfer function for an eddy covariance system You can use it to compute the potential error. My philosophy though is not to use these corrections per se, as the models and information injected has errors, too. But to use this information to optimize the design and placement of sensors in the field. How fast should you sample knowing typical wind speeds. How close should the sensors be? How high should they be?

Correction is a Function of the CoSpectral Function, which depends on Stability

$$\overline{w'c'}_{measured} = \int_0^{\infty} H(\varpi) Co_{wc}(\varpi)\, d\omega$$



Analytical Model of Massman (2000)

$$\frac{\overline{w'c'}_m}{\overline{w'c'}} = \left[\frac{ab}{(a+1)(b+1)}\right]\left[\frac{ab}{(a+p)(b+p)}\right]\left[\frac{1}{(p+1)}\right]\left[1-\frac{p}{(a+1)(a+p)}\right]$$

$a = 2\pi f_x \tau_h$ Detrending time constant

$b = 2\pi f_x \tau_b$ Block average mean removal

$p = 2\pi f_x \tau_e$ 1st order time constant

$f_x = \frac{n_x u}{z}$

$n_x = 0.085$; $z/L < 0$

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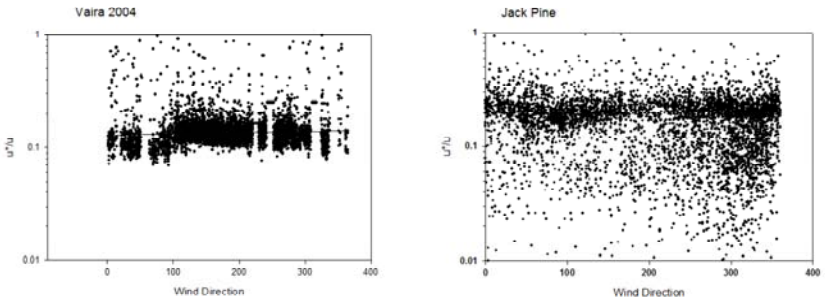
Massman put together a nice analytical model for correcting fluxes. Again it has inherent assumptions.

Flow Distortion by Towers and Wind Direction



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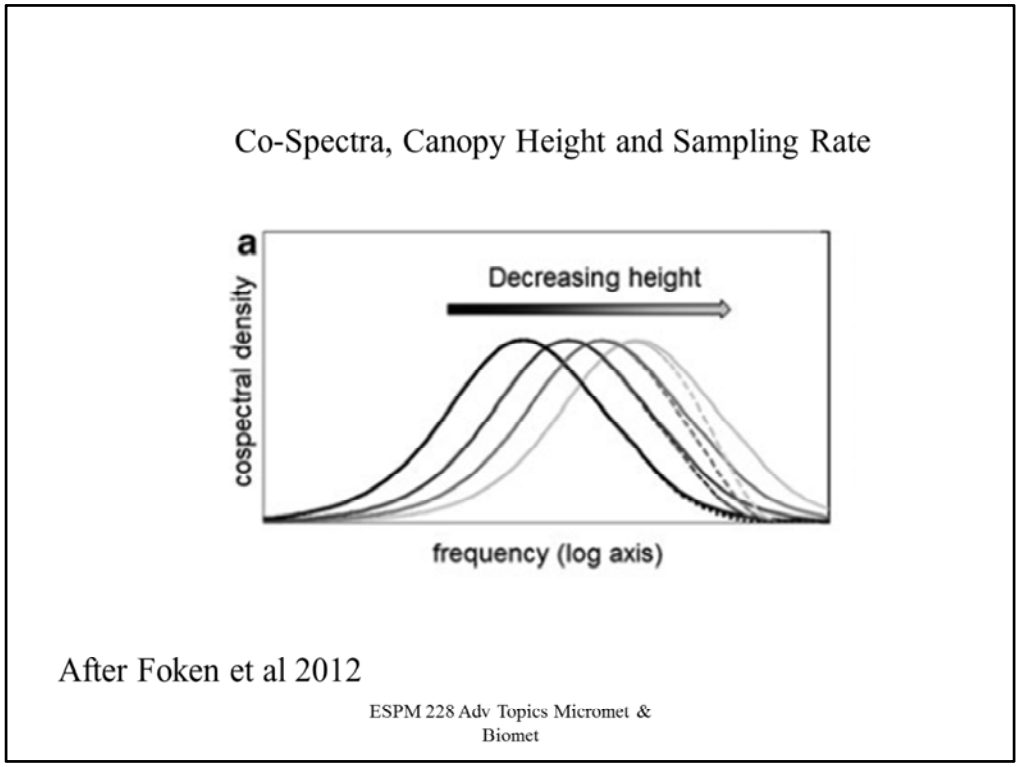
Wind distortion may occur with less preferred wind directions, or if wind is flowing through the structure of a tower, as occurred at the jack pine site. We deployed a horizontal sonic anemometer on a 3 m boom. But when winds came from the back side direction, through a double wide walk up tower, as from the North, you can see the change in u* over U



The spectra shifts towards high frequencies as you get closer to the ground. So if you have slow responding instruments you may want to deploy a taller tower. But then you need Greater Fetch

Where to Place the Sensor: Optimize Height and Sampling Frequency

$$f_{crit} \sim 10 = \frac{nz}{u}$$

$$u = \frac{u^*}{k}\ln(\frac{z}{z_0})$$

$$10 = \frac{nz}{\frac{u^*}{k}\ln(\frac{z}{z_0})}$$

$$\frac{10u^*}{zk}(\ln(z) - \ln(z_0)) = n$$

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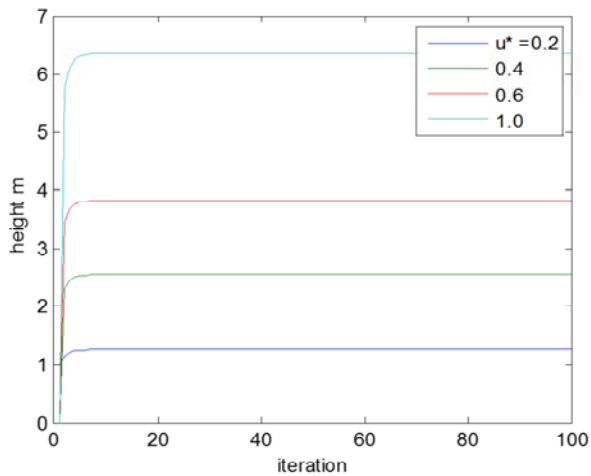
Simple analytical equation can be derived to calcuate the ideal height to place a sensor, given height

Iterative Solution for Optimal $Z_{measurement}$



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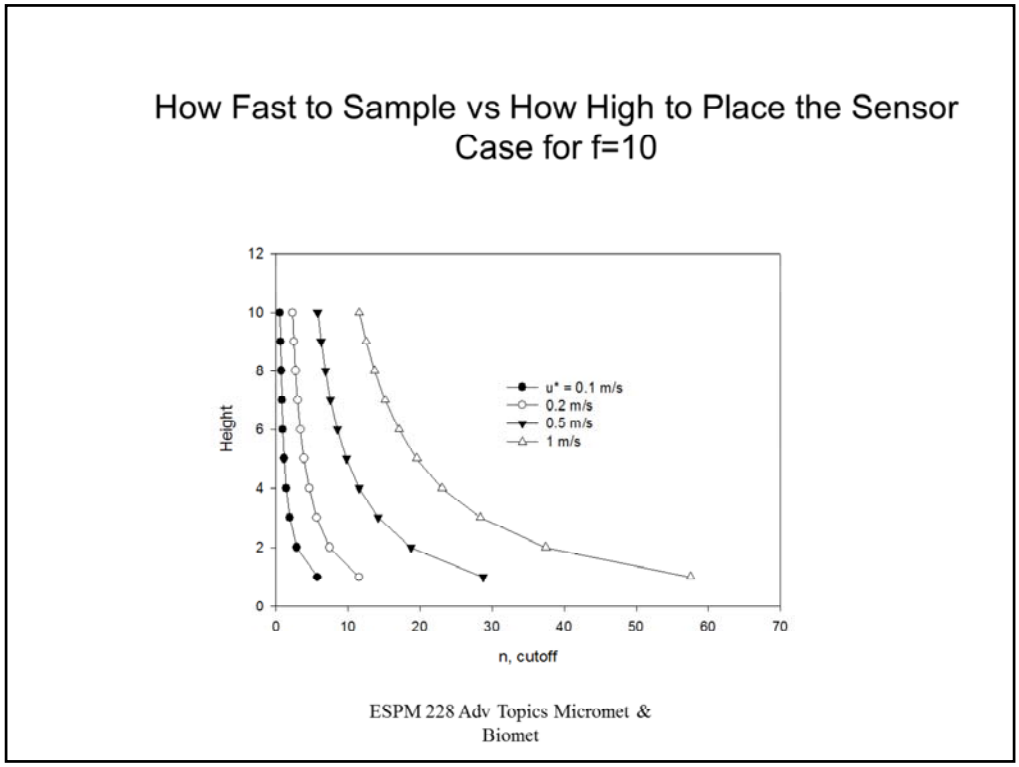

Given different heights and friction velocity you an see how fast you should sample

HomeWork

Apply Matlab Transfer Function Code and Explore Errors in the Eddy Covariance for various

1. Wind speeds
2. Sampling Heights
3. Stability
4. Sampling Rates

Sensor Placement:
How much fetch do We Need/Have?;
How deep is the IBL?

IBL height

$$\delta = 0.1 \cdot z_o \cdot 0.75 \cdot (x / z_o)^{0.8}$$

Fetch @ 90%

$$x_f = \frac{z \cdot (ln(z / z_o) - 1 + z_o / z)}{k^2 \cdot ln(0.90)}$$

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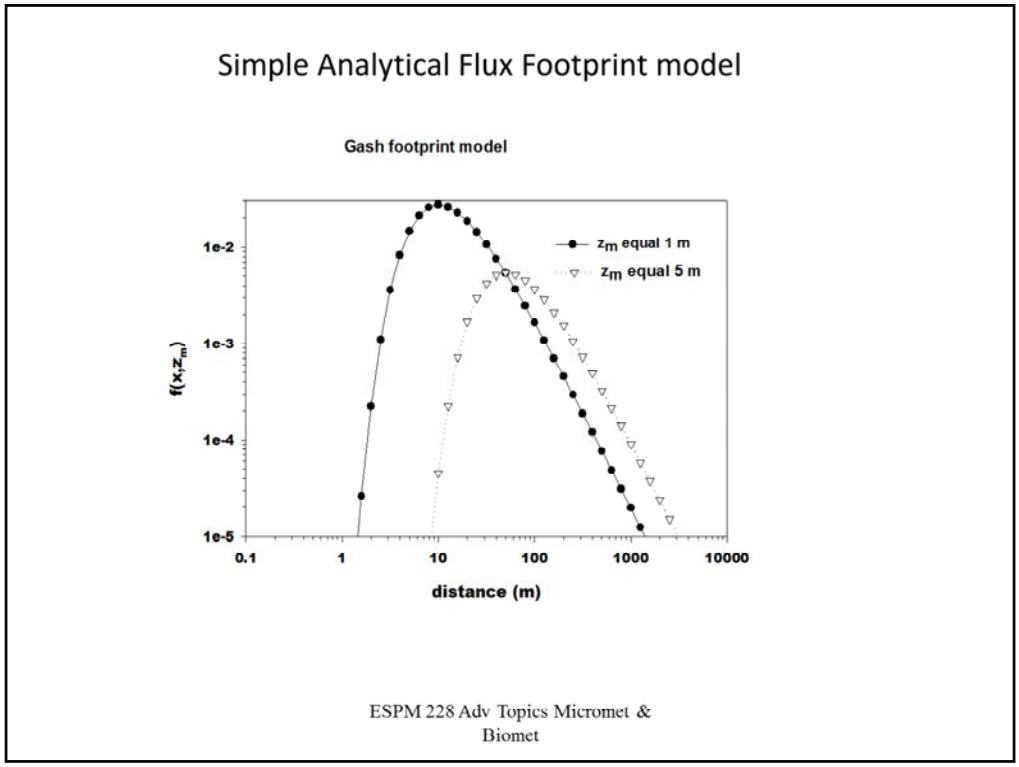
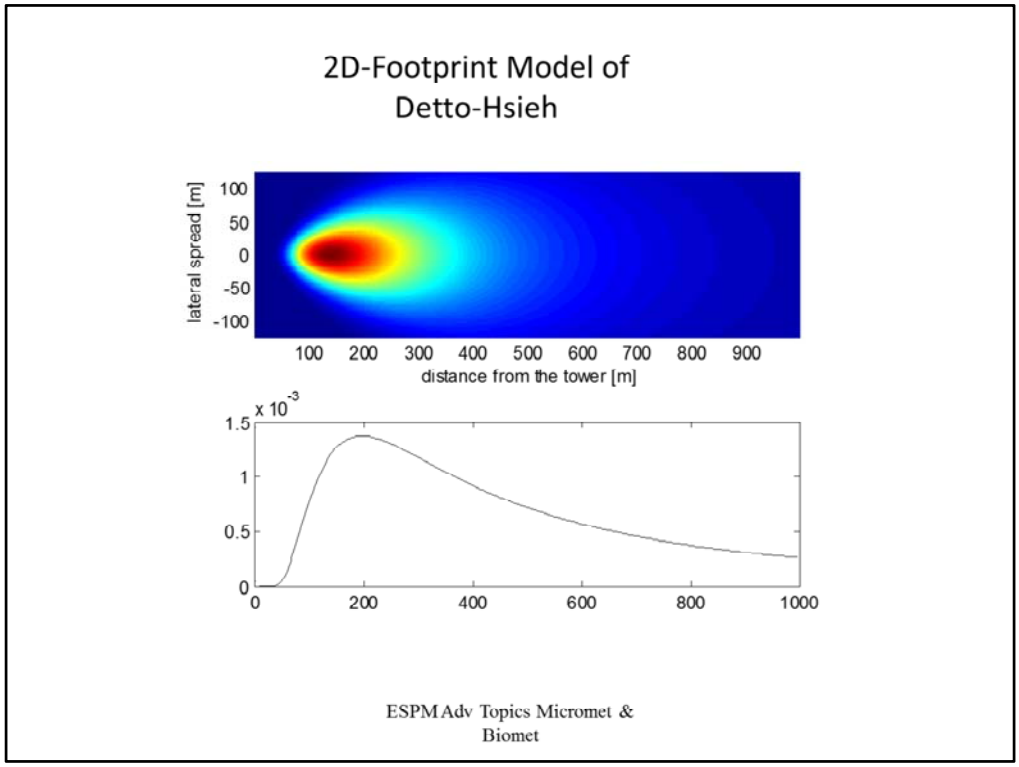
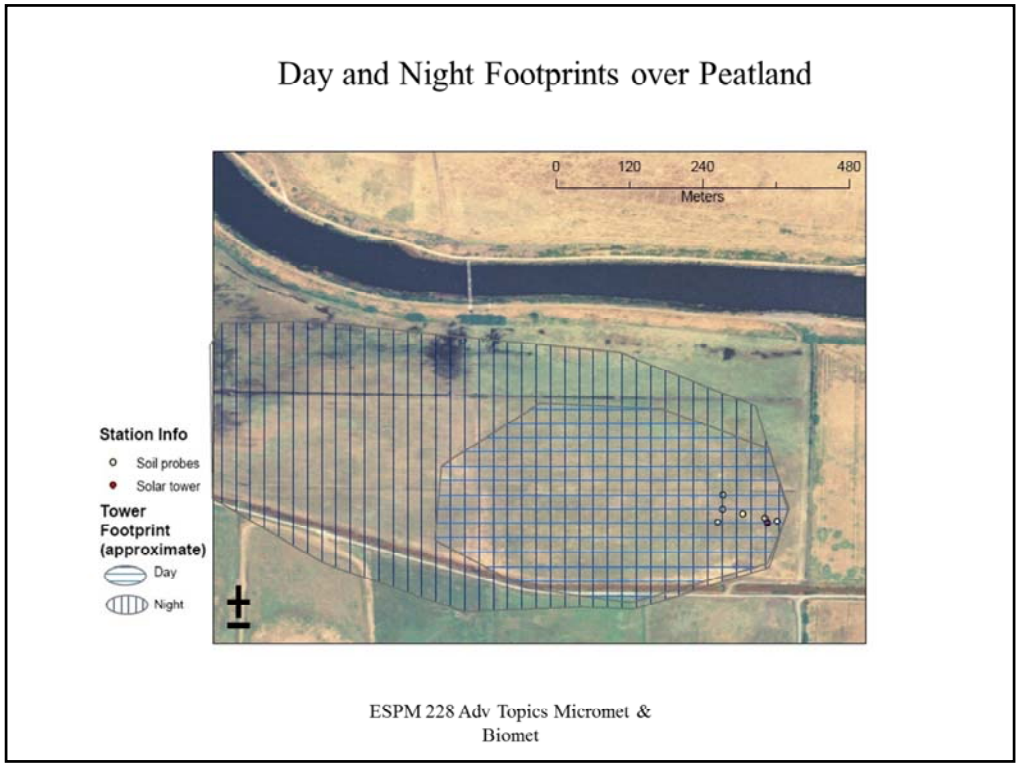
Fetch is another important topic. How big should the field you are studying be? Old rule of thumb 100:1 fetch to height ratio. Fluxes are valid only in the internal boundary layer, which is in the well developed surface boundary layer. You want to minimize advection among landscape tiles. In this case we assume a 90% fetch. It is impractical to assess a 100% fetch due to the asymptotic nature of the flux cumulative probability distribution.



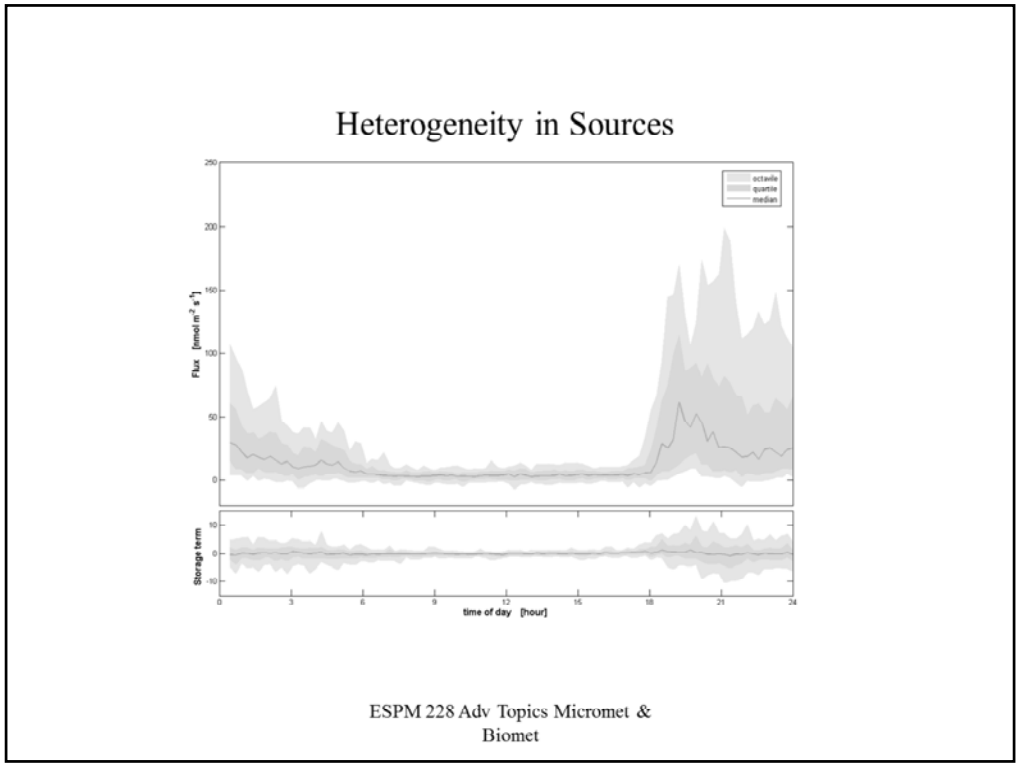
Pdf of a flux footprint. Simple analytical model of John Gash. Key point is the extent of the footprint extends with sampling height



Here is the 2d probability distribution of flux source with the model of Detto and Hsieh. In later lectures we will cover flux footprint theory. Suffice to say, here we need to know the correspondence between the dimension of the field under survey and the size of the flux footprint



Footprints during day and night at our methane flux site. Here we see different flux footprints day and night. During the day the footprint is small as turbulence is convective and well mixed. In this case most source air comes from the drier portion of the field where methane fluxes are small. At night the flux footprint is elongated under stable stratification. Then we are sampling wetter spots in the field, which are strong methane sources. This diel switch in the elongation of the flux footprint is responsible for producing a strong diel pattern of methane emissions with the largest values at night, rather than during the day, when it is more productive, warmer and convective.



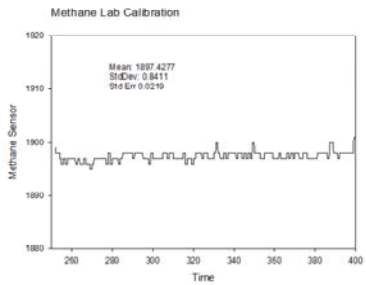
Extended footprint at night over wet methane producing spots lead to anomalous high fluxes at night.

Flux detection limit

$$F = \overline{w'c'} = r_{wc}\sigma_w\sigma_c$$

$r_{wc} \sim 0.5$
$\sigma_w \sim 0.125$ m s^{-1}
$\sigma_c \sim 0.84$ ppb

minimum methane flux density on the order of 4.68 nmol m^{-2} s^{-1} (0.27 mg CH_4 m^{-2} h^{-1}),



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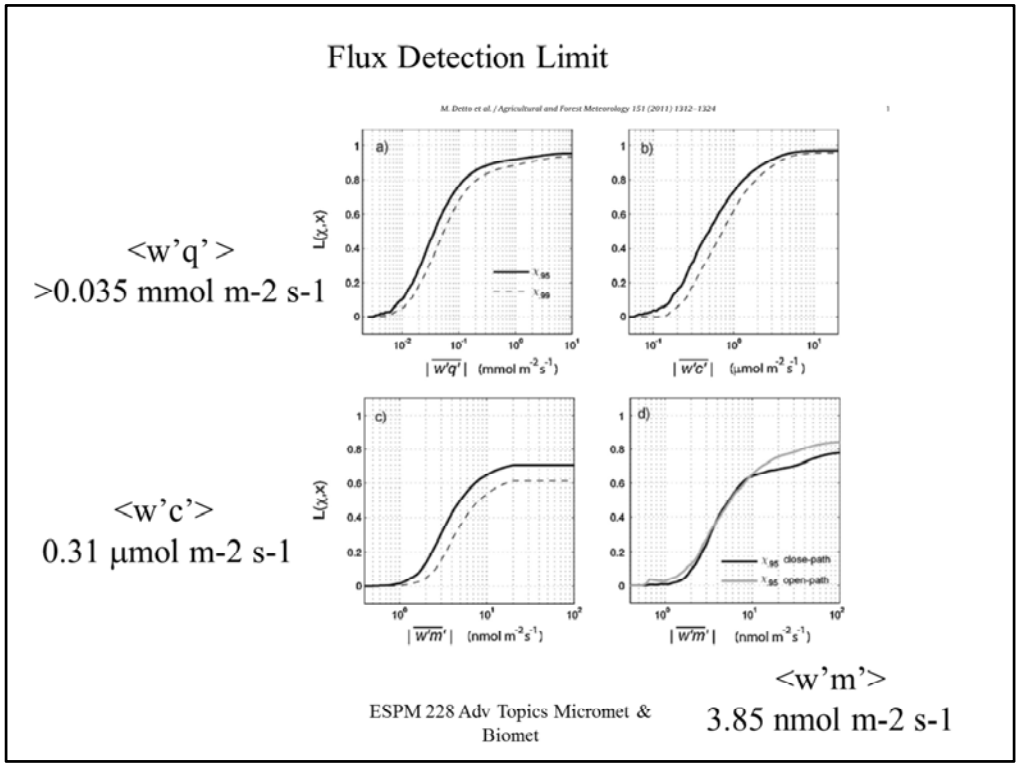
We also need to know how small a flux we can detect with noisy signals, from natural turbulence and the sensors. This is the simple approach I use. Sigma w is 1.25 times u* for neutral conditions. Rwc is about 0.5. sigma c can come from instrument specifications



Sensors can be noisy, hence we need to know when we are measuring fluxes that are significantly different than zero. We also do not want to bias our sums by filtering fluxes of biologically wrong signs if the flux is within the zero noise zone.

Matteo Detto used likelihood theory to define the flux detection limit. Using several years of data he produced a cumulative probability distribution of values that are different than zero. At the P 0.05 level, near the inflection of the cumulative distribution, we can quantify these values with statistical certainty.

Relative Sampling Error in Flux

$$\frac{\sigma_F}{F} = \left(\frac{2\tau}{T}\right)^{1/2} \left(\frac{1 + r_{wc}^2}{r_{wc}^2}\right)^{1/2}$$

Lenschow et al. 1994 J Atmos Ocean Tech

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Lenschow developed theory for sampling errors in noisy signals. This is the relative sampling error given a noisy sensor

Sampling Error, How Long to Sample?

$$< \sigma_x^2 >= \frac{2\sigma_x^2 \tau}{T}$$

$$T_x = \frac{< x^2 > - < x >^2}{< x >^2} \frac{2 \cdot \tau}{\varepsilon^2} \qquad T \sim \frac{2\sigma_x^2 \tau}{\overline{x}^2 \varepsilon^2}$$

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Knowing how noisy a signal is leads to information on how long to sample, too. Again we don't want to sample too long as then we invalidate the non stationarity assumption.

variable	Sampling time, 10% error (min)	Sampling time, 20% error (min)
$\overline{u'^2}$	12.1	3
$\overline{w'^2}$	3.4	.9
$\overline{T'^2}$	18.1	4.5
$\overline{q'^2}$	10.8	2.7
$\overline{u'^4}$	53.1	13.3
$\overline{w'^4}$	17.6	4.4
$\overline{T'^4}$	56.2	14.1
$\overline{q'^4}$	36.8	9.2
$\overline{(w'u')^2}$	20.4	5.1
$\overline{(w'T')^2}$	15.8	3.9
$\overline{(w'q')^2}$	10.3	2.57

Sreenivasan et al 1978, BLM

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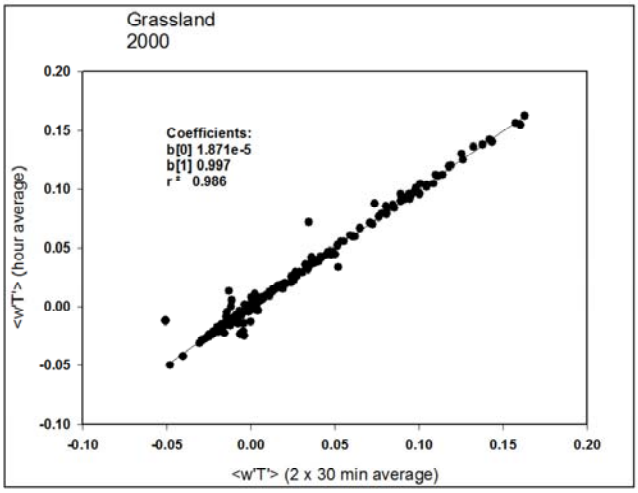
Higher order moments need longer sampling times, like kurtosis

30 and 60 min Averages Give Comparable Results



If you sample tooo long you risk suffering from red noise due to the daily march of the sun

30 and 60 minute averaging times yield comparable results.

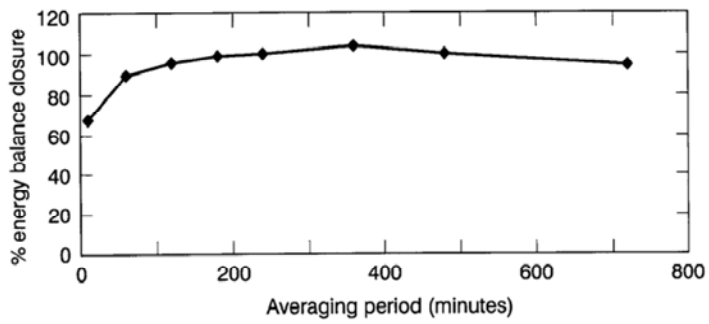
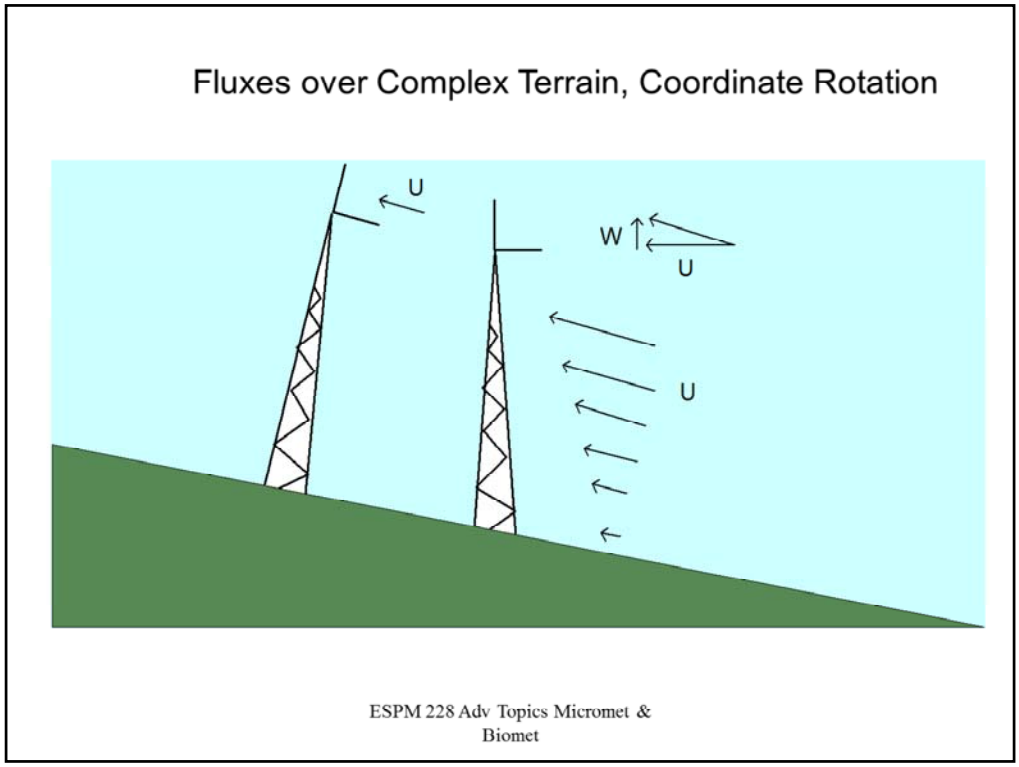


Figure 14. Ratio of total energy flux, $H + LE$ to available radiant energy, $Rn - G$ at Manaus for different averaging and rotation periods.

But be Careful about using Too Long of Averaging Times, e.g. Lack of Stationarity As the Sun moves across the sky

Finnigan et al, 2003 BLM

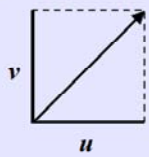
I am critical of an analysis of energy balance closure by Finnigan that was based on 15 minute averaging time. None of the fluxnet community uses such a short sampling time, leading to some red herring arguments in that paper. And while this paper suggests longer sampling times, they are non stationary due to diel changes in the sun and temperature



Orientation of sensors relative to terrain is the next topic to cover. Here we introduce coordinate rotation. We want to measure fluxes perpendicular to mean streamlines, not the geopotential.

Coordinate Rotation

1st Rotation

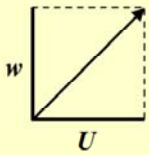


$$U = \left(u^2 + v^2\right)^{1/2}$$

$$\cos\varepsilon = \frac{u}{\left(u^2 + v^2\right)^{1/2}}$$

$$\sin\varepsilon = \frac{v}{\left(u^2 + v^2\right)^{1/2}}$$

2nd Rotation



$$S = \left(u^2 + v^2 + w^2\right)^{1/2}$$

$$\cos\theta = \frac{U}{\left(u^2 + v^2 + w^2\right)^{1/2}}$$

$$\sin\theta = \frac{w}{\left(u^2 + v^2 + w^2\right)^{1/2}}$$

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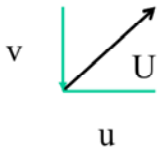
The first rotation is horizontal and aligns the u component along with the wind. The second rotation is vertical and removes mean vertical motion.

$$\overline{w'c'} = \overline{W'C'}_{raw} \cdot \cos\theta - \overline{U'C'}_{raw} \cdot \sin\theta \cdot \cos\varepsilon - \overline{V'C'}_{raw} \cdot \sin\theta \cdot \sin\varepsilon$$

First Rotation, X-Y plane

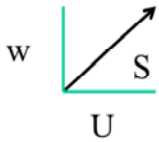


$$U = (u^2 + v^2)^{1/2}$$

$$\cos\varepsilon = \frac{u}{(u^2 + v^2)^{1/2}}$$

$$\sin\varepsilon = \frac{v}{(u^2 + v^2)^{1/2}}$$

Second Rotation, X-Y vs Z plane



$$S = (u^2 + v^2 + w^2)^{1/2}$$

$$\cos\theta = \frac{U}{(u^2 + v^2 + w^2)^{1/2}}$$

$$\sin\theta = \frac{w}{(u^2 + v^2 + w^2)^{1/2}}$$

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Basics behind coordinate rotation

Concept

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = R_{x,y,z} \begin{bmatrix} x_m \\ y_m \\ z_m \end{bmatrix}$$

1) Rotate around the vertical axis, z

$$\overrightarrow{U_1} = \overrightarrow{R_1} \times \overrightarrow{U_m}$$

$$\begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix} = \begin{bmatrix} \cos\varepsilon & \sin\varepsilon & 0 \\ -\sin\varepsilon & \cos\varepsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} u_m \\ v_m \\ w_m \end{bmatrix}$$

$$\tan(\varepsilon) = \frac{\sin(\varepsilon)}{\cos(\varepsilon)} = \frac{\overline{V}}{\overline{U}}$$

Rotate about y

$$\overrightarrow{U_2} = \overrightarrow{R_2} \times \overrightarrow{U_1}$$

$$\begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \times \begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix}$$

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{\overline{W}}{(\overline{U}^2 + \overline{V}^2)^{1/2}}$$

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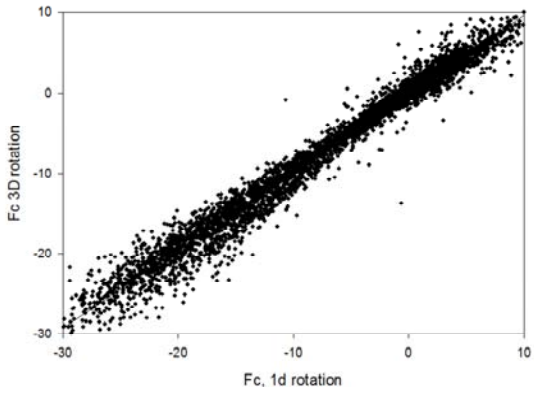
Matrix algebra yields computationally efficient schemes for coordinate rotation. This makes coordinate rotation computations very quick and easy in matlab

Coordinate Rotation over a Deciduous Forest



2% difference or 15 gC m^{-2} y^{-1}

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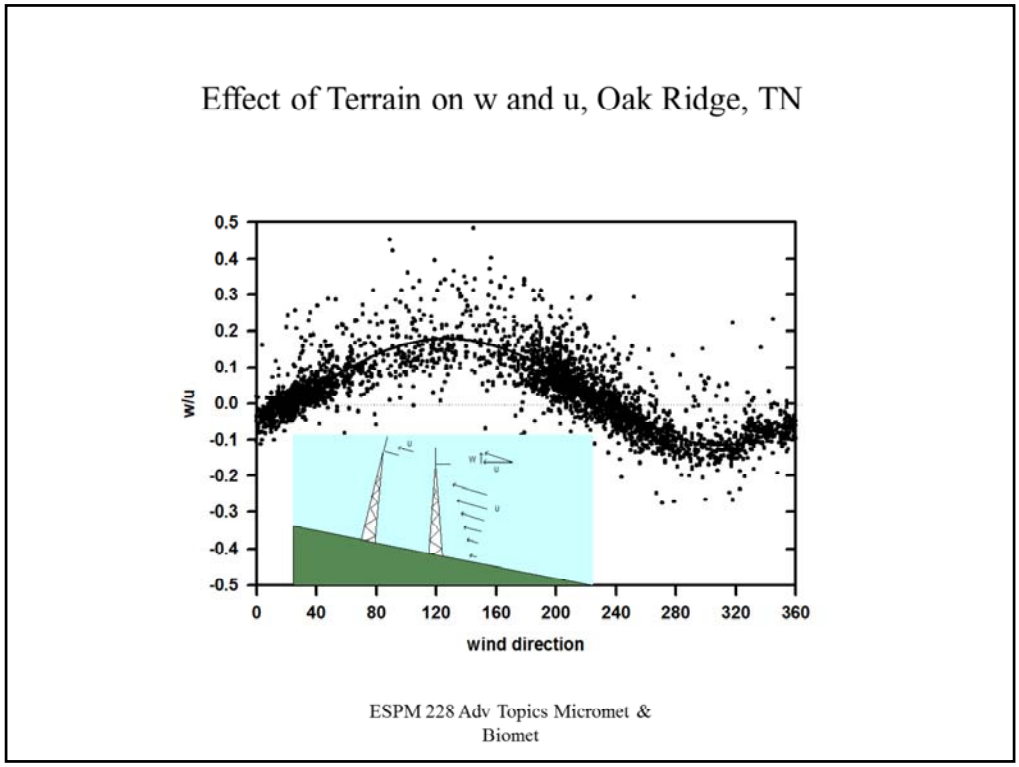
Does one need 1 or 3 dimensional rotations? How sensitive is a short cut. Over this rough forest in hilly terrain the 1D vs 3d rotation yielded a small difference, 2% or a bias in the CO2 flux of 15 gC m-2 y-1



What is w over complex terrain?. Here the planar rotation idea is introduced. Terrain an introduce a complex azimuthal pattern for raw w. As wind moves up slope it adds to w and vice versa as wind moves down slope.

Mean measured w is a function of a random component, w_r, and the terrain component, w_t

$$\overline{w_m} = \overline{w_r} + \overline{w_T}$$

The Terrain induced velocity, w, can be computed with a planar fit to u_m and v_m

$$\overline{w_T} = b_0 + b_1\overline{u_m} + b_2\overline{v_m}$$

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We are interested in rotating the random w to zero. The random component is the mean minus the terrain component. With the planar fit this can be computed as a linear combination of u and v, or use lookup tables.