Canopy Modeling: Eulerian and Lagrangian Closure Schemes

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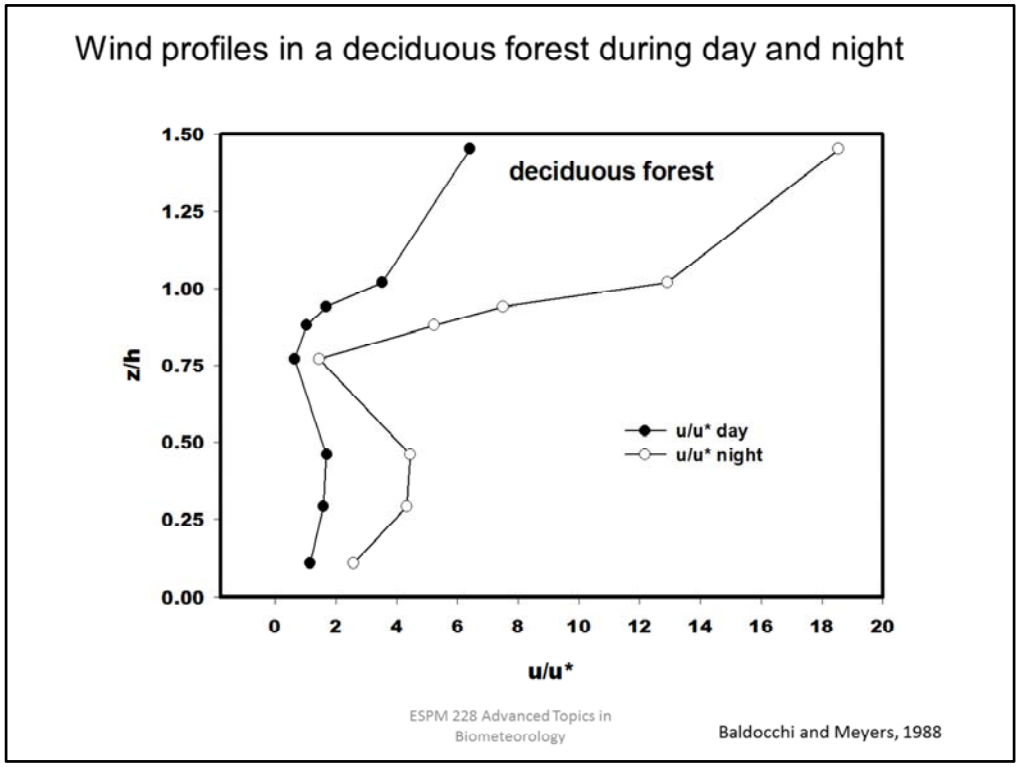
Outline

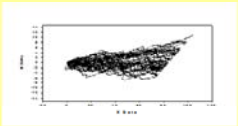
- Roles of Turbulence
- Eulerian Closure Schemes
- Lagrangian Turbulence Schemes



Wind and turbulence experience strong gradients in vegetation. How do we deal with this? We have to predict wind and turbulence profiles through the canopy.

Turbulence Closure Schemes

- Lagrangian



- Eulerian
  - Zero Order, c(z)=constant
  - First Order, F=K dc/dz
  - Second Order and ++ (dc/dt, dw'c'/dt)
  - Large Eddy Simulation, LES

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Next we discuss turbulence in the canopy. We have two options. Lagrangian or Eulerian frameworks

Turbulence Frameworks

- Lagrangian, dc/dt
  - It is a moving framework that follows a trajectory of an ensemble of fluid parcels, like viewed from a drone that is traveling with a smoke plume
- Eulerian,
  - It is a static framework that studies the local derivative of c as time passes a fixed point, like viewed from a tower. It has advective and higher order terms

Unique features of canopy turbulence that must be accounted for by a competent theory:

- momentum is absorbed by the ground and elevated plant elements
- a hydrodynamically unstable inflexion occurs in the wind profile at the canopy atmospheric interface.
- Turbulence in the vegetation is vertically inhomogeneous.
- mean kinetic energy is converted to turbulent kinetic energy in the wake of plants, which accelerates the eddy cascade.

Turbulence is multiscaled

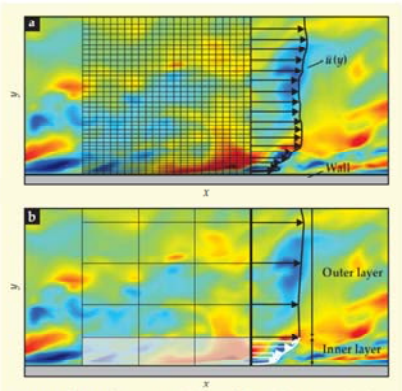

Figure 5. The grid spacing in a large eddy simulation must be fine enough to resolve all but the smallest length scales in a turbulent flow. In this simulated boundary-layer flow, red and blue areas indicate eddies, and the arrows indicate the mean streamwise velocity $\bar{u}$ at various distances y from the wall. **(a)** For standard large eddy simulation, the necessary grid spacing is exceedingly fine due to the very small scales in the near-wall region. **(b)** A coarser grid can be used if the inner layer near the wall, shaded white, is described instead by an empirical model. (Adapted from ref. 7.)

Smits and Maruvic 2013 Physics Today

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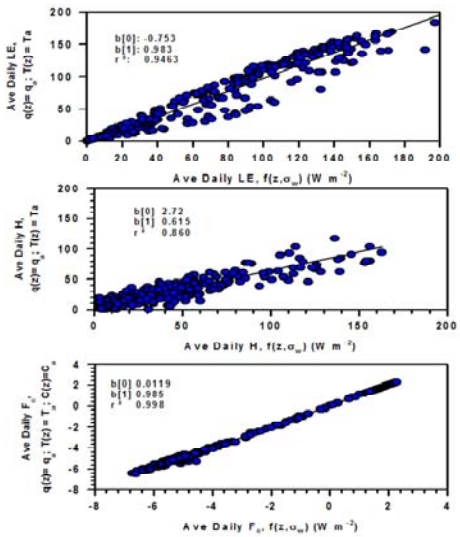
Turbulence is quite complicated and is one of the great unsolved problems of physics. It is multiscaled yielding different answers at different scales

Zero Order Closure Test



First Order Closure,
1 Equation, 2 unknown (c, w'c')

budget

$$\frac{dc}{dt} \approx \frac{\partial \overline{w'c'}}{\partial z}$$

closure

$$\overline{w'c'} = F(c) = -K\frac{\partial c}{\partial z}$$

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We face the central problem of closure because the budget equation for c, dc/dt, has one equation and two unknowns, c and w'c', the flux. First order closure solves the problem by applying K theory. This works fine above a canopy in the surface boundary layer. It has problems in canopy and within 2 times canopy height.

K Theory in the Surface Layer

$$K_m(z) = \frac{\overline{w'u'}(z)}{\rho\, \partial\bar{u}/\partial z}$$

Short vegetation

$$K_m(z) = \frac{u_*^2}{\partial\bar{u}/\partial z} = \frac{u_*^2}{u_* / k\, z} = u_* k\, z$$

Tall vegetation

$$K_m(z) = \frac{u_*^2}{\partial\bar{u}/\partial z} = \frac{u_*^2}{u_* / k(z-d)} = u_* k(z-d)$$

K Theory in the Plant Canopy

$$K_m(z) = \frac{\overline{w'u'}(z)}{\rho\, \partial\bar{u}/\partial z}$$

$$K_m(z) \approx \tau_l(z)\sigma_w^2(z)$$

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K theory is ill advised in canopies because counter gradient transfer occurs, leading to negative K's which are non physical.

Stanley Corrsin (1974) wrote that several conditions must hold to apply K-theory:

- the length scales of the turbulent transfer must be less than the length scales associated with the curvature of the concentration gradient of the scalar.

- the turbulent length scale must be constant over the distance where the concentration gradient changes significantly.

$$\overline{w'c'} \sim K_c \frac{\partial \bar{c}}{\partial z} + TT$$

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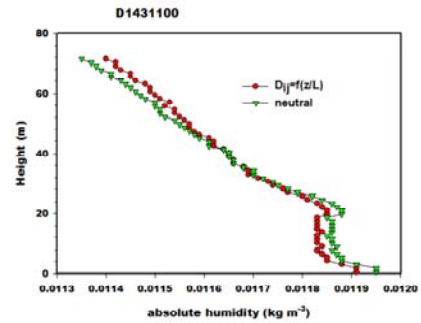
Many observations show strong shear in the canopy, secondary wind maxima, and kinks in scalar profiles, all evidence of non local transport. Hence K theory tends to fail

Counter Gradient 'Diffusion' from Non-Local Transport

$$\frac{\partial c}{\partial z} = [\frac{\overline{w'c'}}{\tau} - \frac{\overline{\partial w'w'c'}}{\partial z}] / \overline{w'w'}$$



$$\overline{w'c'} = \tau(\overline{w'^2}\frac{\partial \overline{c}}{\partial z} + \frac{\partial \overline{w'w'c'}}{\partial z})$$

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Theoreticians Who Advanced Higher Order Closure Modeling to Plant Canopies

Roger Shaw

John Finnigan

K T Paw U

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Second Order Budget

$$\overline{u'u'}, \overline{w'w'}, \overline{v'v'}, \overline{w'u'}$$

3rd Order Unknowns

$$\overline{u'u'u'}, \overline{w'w'w'}, \overline{w'u'u'}, \overline{u'w'w'}$$

closure

$$\overline{w'w'u'} = -K \frac{\partial \overline{w'u'}}{\partial z}$$

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2nd order closure tries a work around by defining budget equations for the second moment terms. But . If we solve for 2nd order terms, they produce equations with 3rd order unknowns

9 unknowns are associated with 2nd order budgets, *u'w'*, *u'u'*, v'v', w'w', w'u'w', w'u'u', w'v'v', w'w'w'

Third Order Closure

Assorted Budgets

$$\overline{w'w'T'}, \overline{w'T'q'}, \overline{u'w'T'}, \overline{w'T'T'}, \overline{w'w'q'}$$

4th Order Terms

$$\overline{w'T'w'T'}, \overline{w'T'T'T'}, \overline{w'w'w'q'}$$

Quasi-Gaussian Approximation

$$\overline{w'w'w'w'} = 3\overline{w'w'} \cdot \overline{w'w'}$$

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And third order moments produce 4th order terms

Higher Order Closure Equations and Unknowns

Order of Closure	Turbulence Budget Equations	Unknowns	Scalar Budget Equations $(\frac{\partial c}{\partial t})$	Unknowns	Closure Scheme
zero		$\overline{u}, \overline{w}$		$\overline{T}, \overline{q}, \overline{C}$	$\overline{c} = f(t, x, y, z)$
First Order	$\overline{u}$	$\overline{u'u'}, \overline{w'w'}, \overline{w'u'}$	$\overline{T}, \overline{q}, \overline{C}$	$\overline{T'^2}, \overline{q'^2}, \overline{C'^2}, \overline{w'T'}$ $\overline{w'q'}, \overline{w'C'}$	$\overline{w'c'} = -K\frac{\partial \overline{c}}{\partial z}$
Second Order	$\overline{u'u'}, \overline{w'w'}, \overline{w'u'}$ $\overline{v'v'}$	$\overline{u'u'u'}, \overline{w'w'w'},$ $\overline{w'u'u'}, \overline{u'w'w'}$	$\overline{T'^2}, \overline{q'^2}, \overline{w'T'},$ $\overline{w'q'}, \overline{T'q'}, \overline{q'T'}$	Numerous third order moments	$\overline{w'w'c'} = -K\frac{\partial \overline{w'c'}}{\partial z}$
Third Order	$\overline{w'w'u'}, \overline{w'w'w'}$ $\overline{w'u'u'}, \overline{w'v'v'}$		$\overline{w'w'T'}, \overline{w'T'q'},$ $\overline{u'w'T'}, \overline{w'T'T'},$ $\overline{w'w'q'}$	Numerous fourth order moments	$\overline{w'w'w'w'} =$ $3\overline{w'w'} \cdot \overline{w'w'}$ quasi-Gaussian approximation

Instantaneous Navier Stokes Eq

Fluid Velocity Budget or Fluid Acceleration (force) equation

$$\frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{g\theta}{T}\delta_{i3} + \upsilon \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

Einstein Notation

Summation implied with repeated indices

$$a_{ij} A_j = a_{i1} A_1 + a_{i2} A_2 + a_{i3} A_3$$

$$u_j \frac{\partial u_i}{\partial x_j} = u_1 \frac{\partial u_i}{\partial x_1} + u_2 \frac{\partial u_i}{\partial x_2} + u_3 \frac{\partial u_i}{\partial x_3}$$

Expansion of Total Derivative

$$\frac{dc(t,x,y,z)}{dt} =$$

$$\frac{\partial c}{\partial t} + \frac{dx}{dt}\frac{\partial c}{\partial x} + \frac{dy}{dt}\frac{\partial c}{\partial y} + \frac{dz}{dt}\frac{\partial c}{\partial z} =$$

$$\frac{\partial c}{\partial t} + u\frac{\partial c}{\partial x} + v\frac{\partial c}{\partial y} + w\frac{\partial c}{\partial z}$$

Mean Budget Equations and Reynolds Averaging

Time Partial Derivative

$$\overline{\frac{\partial u_i}{\partial t}} = \overline{\frac{\partial(\overline{u}+u')}{\partial t}} = \overline{\frac{\partial \overline{u}}{\partial t}} + \overline{\frac{\partial u'}{\partial t}} = \frac{\partial \overline{u}}{\partial t}$$

Advection Term

$$\overline{u_j \frac{\partial u_i}{\partial x_j}} = \overline{(\bar{u} + u')}\overline{(\frac{\partial \bar{u}}{\partial x} + \frac{\partial u'}{\partial x})} =$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \overline{u'\frac{\partial \bar{u}}{\partial x}} + \overline{\bar{u}\frac{\partial u'}{\partial x}} + \overline{u'\frac{\partial u'}{\partial x}}$$

$$= \overline{u}\frac{\partial \bar{u}}{\partial x} + \overline{u'\frac{\partial u'}{\partial x}}$$

Time Averaging and 2nd order Terms

$$u_j \frac{\partial u_i}{\partial x_j}$$

$$\frac{\partial \overline{u_i' u_j'}}{\partial x_j} = \overline{u_i' \frac{\partial u_j'}{\partial x_j}} + \overline{u_j' \frac{\partial u_i'}{\partial x_j}}$$

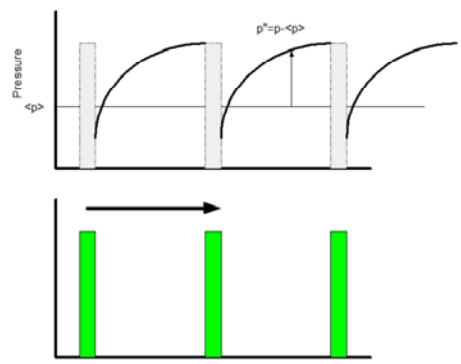
Continuity Eq $\quad \frac{\partial u_j'}{\partial x_j} = 0 = \frac{\partial u_j}{\partial x_j} = \frac{\partial \overline{u_j}}{\partial x_j}$

$$\frac{\partial \overline{u_i' u_j'}}{\partial x_j} = \overline{u_j' \frac{\partial u_i'}{\partial x_j}}$$

New Term arises at 2nd order:
Crux of the turbulence closure problem

Pressure Gradient

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial \overline{p}}{\partial x_i} + \overline{\frac{\partial p'}{\partial x_i}}$$



Viscous Drag

$$\overline{\upsilon \frac{\partial^2 u}{\partial x_j \partial x_j}} = \upsilon \frac{\partial^2 \overline{u}}{\partial x_j \partial x_j} + \overline{\upsilon \frac{\partial^2 u'}{\partial x_j \partial x_j}}$$

$$\overline{\upsilon \frac{\partial^2 u'}{\partial x_j \partial x_j}} \sim 0$$

Simplified Mean Budget for u

$$\frac{\partial \overline{u}}{\partial t} = 0 = -\frac{\partial \overline{u' w'}}{\partial z} - \frac{\overline{\partial p'}}{\rho \partial x} - \frac{\partial \overline{p}}{\rho \partial x} + \upsilon \frac{\partial^2 \overline{u}}{\partial z \partial z}$$

2nd Order Equation

$$\frac{\partial \overline{u_i' u_k'}}{\partial t} + \overline{u}_j \frac{\partial \overline{u_i' u_k'}}{\partial x_j} + \overline{u_k' u_j'} \frac{\partial \overline{u}_i}{\partial x_j} + \overline{u_i' u_j'} \frac{\partial \overline{u}_k}{\partial x_j} + \frac{\partial \overline{u_i' u_k' u_j'}}{\partial x_j} =$$

$$-\left[\frac{\overline{\partial u_k' p'}}{\partial x_i} + \frac{\overline{\partial u_i' p'}}{\partial x_k}\right] + \left[\overline{u}_k \frac{\overline{\partial p'}}{\partial x_i} + \overline{u}_i \frac{\overline{\partial p'}}{\partial x_k}\right] + \overline{p'\left[\frac{\partial u_k'}{\partial x_i} + \frac{\partial u_i'}{\partial x_k}\right]} + \frac{g}{T}\left[\overline{\theta' u_i'}\delta_{k3} + \overline{\theta' u_k'}\delta_{i3}\right] + \upsilon \frac{\partial^2 \overline{u_i' u_k'}}{\partial x_j \partial x_j} - 2\upsilon \frac{\overline{\partial u_i'}}{\partial x_j} \frac{\overline{\partial u_k'}}{\partial x_j}$$

TKE Budget

$$\bar{e} = \frac{1}{2}\left(\overline{u'u'} + \overline{v'v'} + \overline{w'w'}\right)$$

transport

dissipation

$$\frac{\partial \bar{e}}{\partial t} = 0 = -\overline{w'u'}\frac{\partial \bar{u}}{\partial z} + \frac{g\overline{w'\theta'}}{\bar{\theta}} - \frac{\partial \overline{w'e'}}{\partial z} - \frac{\overline{\partial p' w'}}{\rho \partial z} - \varepsilon$$

Shear Production

Pressure Production

Return to Isotropy Via P-V Interactions

Variance Budget

transport

viscous dissipation

$$\frac{\partial \overline{u'u'}}{\partial t} = 0 = -2\overline{w'u'}\frac{\partial \overline{u}}{\partial z} - \frac{\partial \overline{w'u'u'}}{\partial z} - 2\frac{\overline{u'\partial p'}}{\rho \partial x} - 2\frac{\overline{u}\overline{\partial p'}}{\rho \partial x} - \frac{2}{3}\varepsilon$$

Shear Production

Pressure interaction

Examples of 2nd Order Budget Equations

$$\frac{\partial \overline{u'u'}}{\partial t} = 0 = -2\overline{w'u'}\frac{\partial \bar{u}}{\partial z} - \frac{\partial \overline{w'u'u'}}{\partial z} - 2\frac{\overline{u'\partial p'}}{\rho \partial x} - 2\frac{\bar{u}\overline{\partial p'}}{\rho \partial x} - \frac{2}{3}\varepsilon$$

$$\frac{\partial \overline{w'w'}}{\partial t} = 0 = -\frac{\partial \overline{w'w'w'}}{\partial z} - 2\frac{\overline{w'\partial p'}}{\rho \partial z} - 2\frac{g}{\theta}\overline{w'\theta_v'} - \frac{2}{3}\varepsilon$$

$$\frac{\partial \overline{u'w'}}{\partial t} = 0 = -\overline{w'w'}\frac{\partial \bar{u}}{\partial z} - \frac{\partial \overline{w'u'u'}}{\partial z} + \left(\frac{\overline{p'\partial u'}}{\partial z} + \frac{\overline{p'\partial w'}}{\partial x}\right)$$

$$\frac{\partial \overline{w'q'}}{\partial t} = 0 = -\overline{w'w'}\frac{\partial \bar{q}}{\partial z} - \frac{\partial \overline{w'w'q'}}{\partial z} - \frac{\overline{q'\partial p'}}{\partial z} + g\beta\overline{\theta'_v q'}$$

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Specific 2nd order budget equations for turbulence variance and covariances, including Reynolds stress and water vapor. There are shear production terms, non local transport and pressure interaction terms.

Scalar Conservation Budget Equations:
What the Terms Mean

$$\frac{\partial \overline{u}}{\partial t} = 0 = -\frac{\partial \overline{u'w'}}{\partial z} - \frac{\overline{\partial p'}}{\rho \partial x} - \frac{\partial \overline{p}}{\rho \partial x} + \upsilon \frac{\partial^2 \overline{u}}{\partial z \partial z}$$

$$\frac{\partial \overline{q}}{\partial t} = 0 = -\frac{\partial \overline{w'q'}}{\partial z} + S_q$$

Flux Divergence

Time Rate of Change

Source/Sink

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Higher order closure models are one way to work around the problems with K theory. These can be derived starting with Navier Stokes and applying Reynolds averaging rules. Nothing magic. But assumptions needed to solve for terms like dp'/dx

Pressure Fluctuation is a function of the drag coefficient

$$\overline{\frac{\partial p'}{\partial x_i}} = C_d a(z) \overline{u_i}^2$$

$$\frac{\partial \overline{u'w'}}{\partial z} = C_d a(z) \overline{u_i}^2$$

$$\overline{u(z)} = u_h \exp(-\alpha(1 - \frac{z}{h}))$$

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Flux Conservation Budget:
What the Terms Mean

$$\frac{\partial \overline{w'q'}}{\partial t} = 0 = -\overline{w'w'}\frac{\partial \overline{q}}{\partial z} - \frac{\partial \overline{w'w'q'}}{\partial z} - \frac{\overline{q'\partial p'}}{\partial z} + g\beta\overline{\theta'_v q'}$$

Non-local Transport

Buoyancy prod

Shear Prod

Pressure Interactions

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Form of Third Order Budget Equation

$$\frac{\partial \overline{a'a'b'}}{\partial t} = 0 = -\overline{w'a'a'}\frac{\partial \overline{b}}{\partial z} - 2\overline{w'a'b'}\frac{\partial \overline{a}}{\partial z} - \overline{w'b'}\frac{\partial \overline{a'a'}}{\partial z} - 2\overline{w'a'}\frac{\partial \overline{a'b'}}{\partial z} - \varepsilon_{aab}$$

Simulation of Turbulence Statistics

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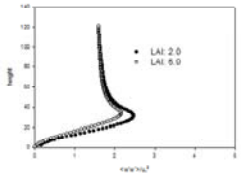

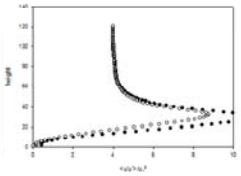

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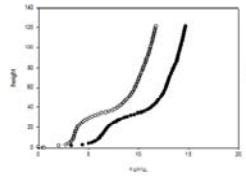

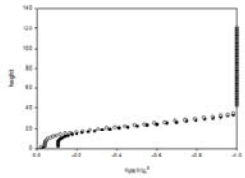

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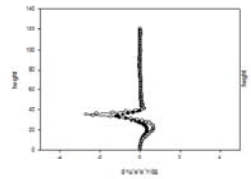

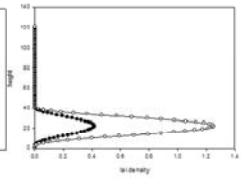

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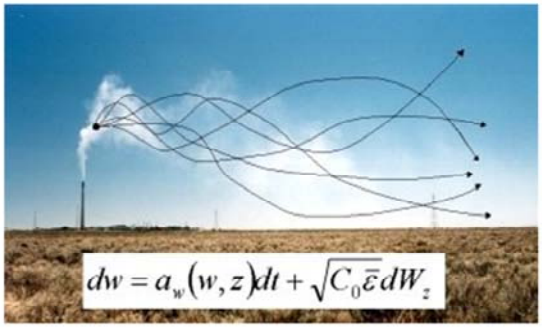
Lagrangian Theory



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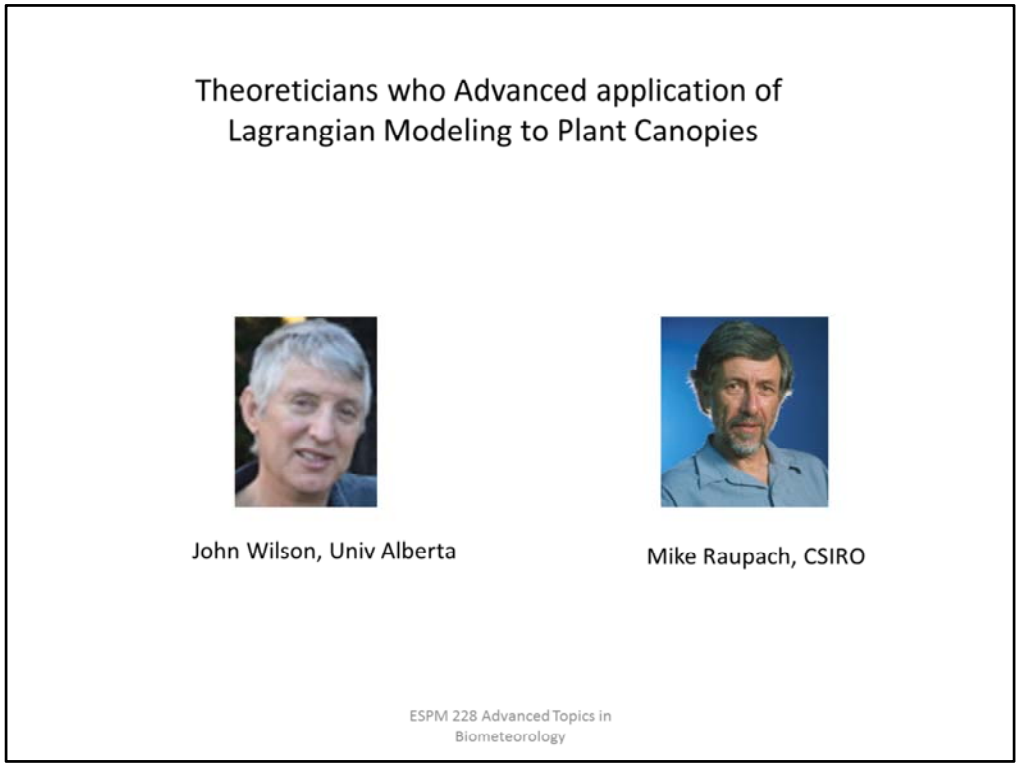
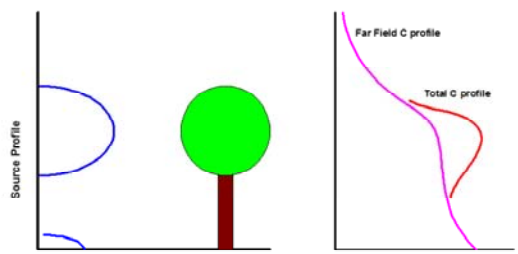
http://www.cmar.csiro.au/airquality/images/LagrangianPlumeModelling.jpg



Theoreticians advancing theory to plant canopies

On the basis of Lagrangian theory Raupach (1987) explains counter-gradient transfer:

'(because) scalar from nearby elementary sources is dispersing in a near-field regime
*...its contribution to the overall **gradient** (is)*
*much greater than its contribution to the overall **flux density**.*
Just below a fairly localized and intense source in the canopy,
the near-field gradient contribution is large and positive; when this is combined
with the upward flux of scalar required by conservation of scalar mass, a countergradient flux
is obtained'.



$$C(r,t) = \frac{n(r,t)}{V}$$

$$c(z,t) = \int_0^t \int_0^z P(z,t \mid z_o, t_o)\, S(z_o, t_o)\, dz_o\, dt_o$$

Concentration is function of conditional probability and Source Strength

e.g. the probability of a source released at z0 and t0 will travel to z at t

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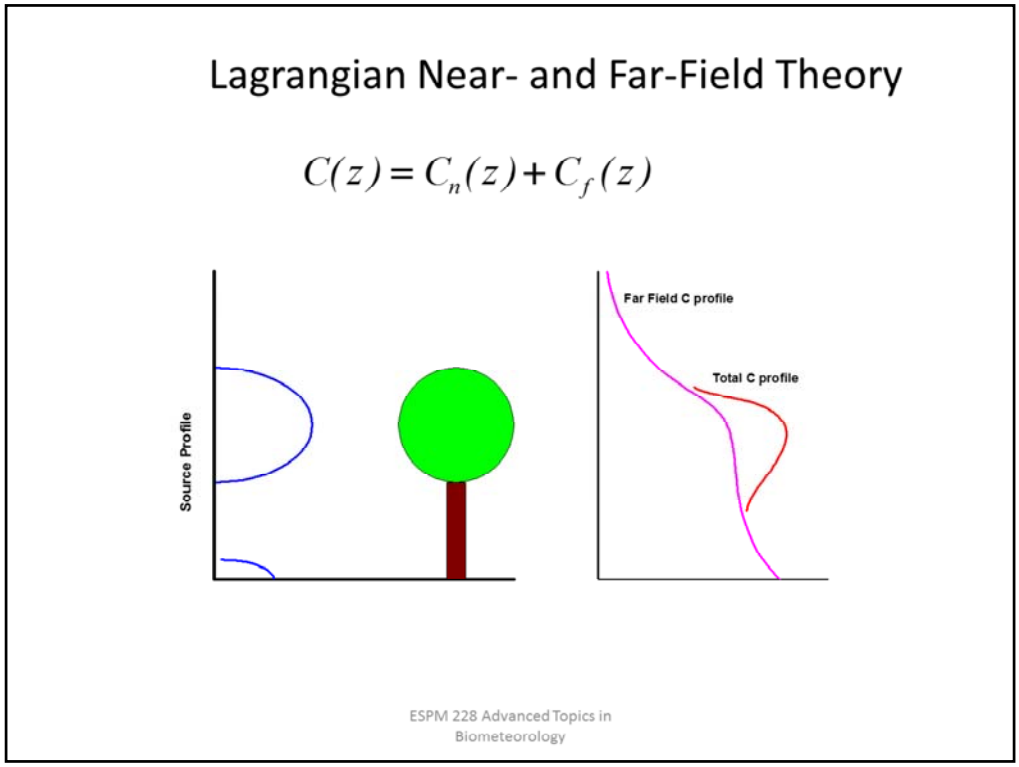
We study the probability of the trajectory of an ensemble of fluid parcels that are transported and dispersed in the atmosphere. It is a function of the source at a given location and time and the conditional probability of receiving a parcel at a time t and position z, given it was released at z0 and time t0. The concentration is the superposition of these trajectories



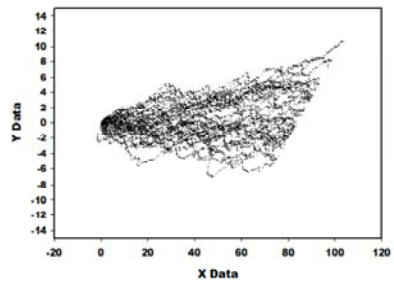
Lagrangian theory also accounts for non local transport and counter gradient transport. It is the method I use in Can Veg.

For a recent history
Thomson, D.J. and Wilson, J.D., 2013. History of Lagrangian Stochastic Models for Turbulent Dispersion, Lagrangian Modeling of the Atmosphere. American Geophysical Union, pp. 19-36.

Probability of Particle Position, P(z,t|zo,to)

Compute position with a random walk Markov model

$$dx = u_L \cdot dt$$



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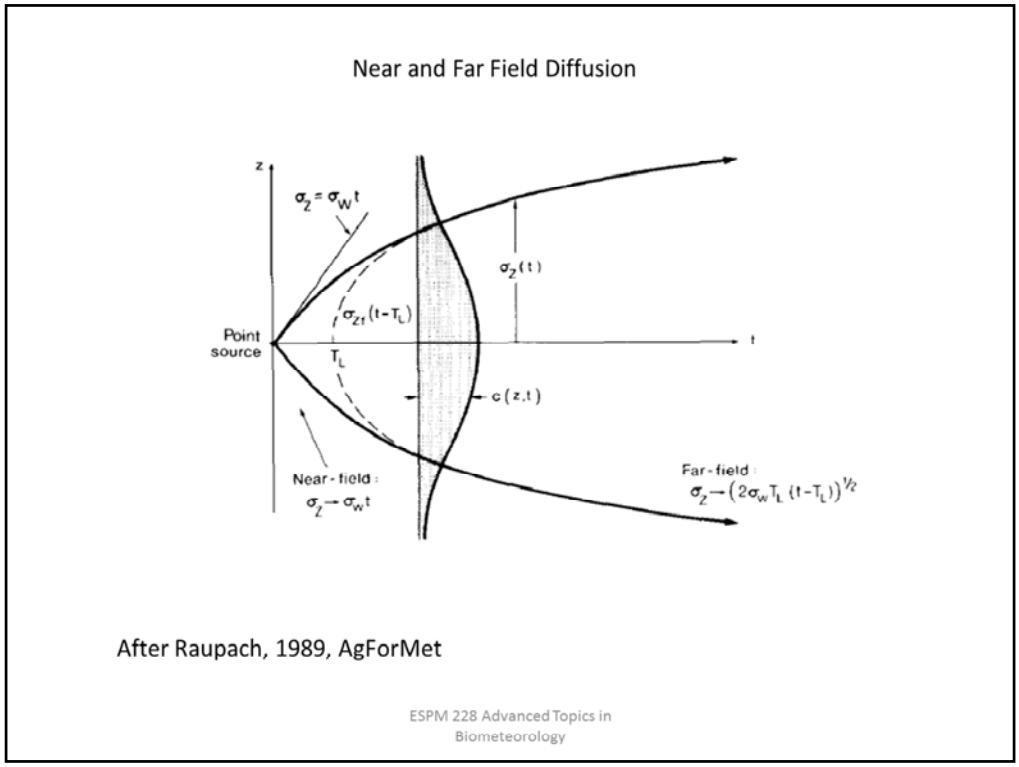
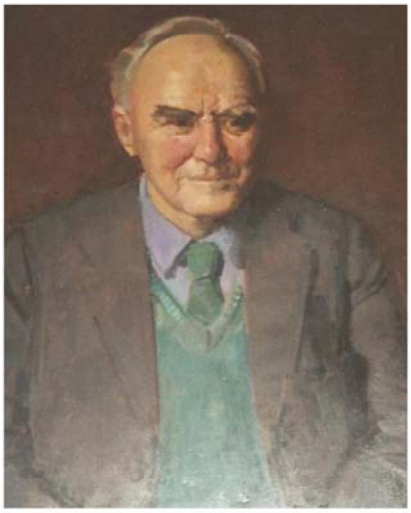
This is how a parcel moves an increment dx given its velocity over a time step dt



If you look at a plume it will have two limits a near and far field condition

What is near and far field diffusion? Why do we care? How do we estimate its limits? We will discuss this next

GI Taylor, 1886-1975



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GI Taylor, a professor at Cambridge, was one of the leading fluid mechanics of the 20th century. He laid out some of the key theories on plumes

Statistical Theory of Turbulence

G. I. Taylor

Proc. R. Soc. Lond. A 1935 151 421-444; DOI: 10.1098/rspa.1935.0158. Published 2 September 1935

Taylor Diffusion, Homogeneous Turbulence

$$Z = \int_0^t W\,dt$$

Size of plume

$$\frac{dZ^2}{dt} = 2Z\frac{dZ}{dt}$$

Time rate of change of spread of plume

$$\frac{dZ}{dt} = W$$

Lagrangian velocity

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Lets first look at basics from GI Taylor. His simple derivation is a good way to learn to think about solving problems. Z is height, W is vertical velocity. This is valid for homogeneous turbulence, where this is no gradients in turbulence statistics

Redefinition of plume spread

$$\frac{dZ^2}{dt} = 2Z\frac{dZ}{dt} = 2ZW = 2\int_0^t WW\,dt$$

$$WW = \sigma_W^2 r_L$$

$$\frac{d\sigma_z^2}{dt} = 2\sigma_w^2 \int_0^t r_L(\tau)\,d\tau$$

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We can derive the variance of the spread of the plume along z as a function of the variance in w and the integral of the Lagrangian correlation coefficient, a measure of persistence of motion. W is Lagrangian velocity, Z is Lagrangian height

Limit, Near Field Diffusion

Integral time scale

$$T_L = \int_0^\infty r_L(\tau)\,d\tau$$

$$r_L = 1$$

$$\int_0^t r_L(\tau)\,d\tau = t$$

$$\frac{d\sigma_z^2}{dt} = 2\sigma_w^2 \int_0^t r_L(\tau)\,d\tau$$

$$r_L = \frac{< W'(t)W'(t+\tau) >}{< W'^2 >}$$

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Integrate the correlation coefficient gives the Lagrangian time scale. For short time scales, r =1 and Tl is t, time

Near field diffusion, cont

$$\frac{d\sigma_z^2}{dt} = 2\sigma_w^2 t$$

$$\int d\sigma_z^2 = \int 2\sigma_w^2 t\, dt$$

$$\sigma_Z^2 = \sigma_W^2 t^2$$

$$\sigma_Z \sim \sigma_W t, t << T_L$$

Diffusion scale is proportional to travel time

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This allows us to define the spread of the plume in the near field when t <<< TL. It is proportional to the standard deviation in W times time, t

Limit, Far Field Diffusion

$$\frac{d\sigma_z^2}{dt} = 2\sigma_w^2 \int_0^t r_L(\tau)\,d\tau$$

$$t \rightarrow \infty$$

$$\int_0^\infty r_L(\tau)\,d\tau = T_L$$

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Far field is when time goes to infiniity

Far field diffusion, cont

$$\int d\sigma_z^2 = \int 2\sigma_w^2 T_L \, dt$$

$$\sigma_Z^2 = 2\sigma_W^2 T_L t$$

$$\sigma_Z = \sigma_W \sqrt{2T_L t}$$

Diffusion is proportional to the square root of travel time

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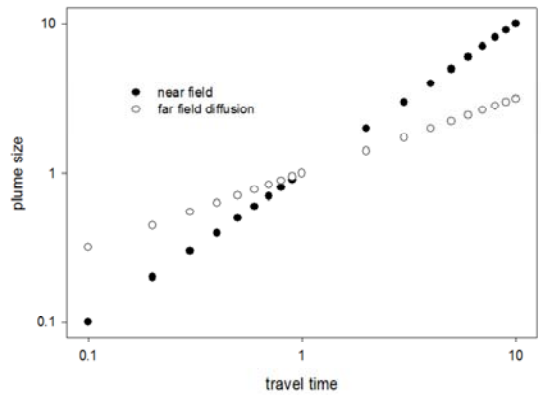
Here spread is proportional to the square root of time at the far field limit



Plume is tight near source, concentrations elevated

Velocity increments: Langevin Eq

$$\frac{du}{dt} = -a_1 u + b\xi(t)$$

ξ Random forcing term

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Langevin equation is the foundation for studying parcel movement. It has a persistent (-a u) and random term. The next task is to define the a and b coefficients

ξ Gaussian White Noise

- Uniform spectral density
- Gaussian pdf
- Mean of zero
- Variance of dt
- Uniform spectral density
- Discontinuous

$$W(T) = \int_0^T \xi(t)\,dt$$

The integral is continuous and defined

$$\frac{dW(t)}{dt}$$

The time derivative of the random component is not defined

$$dW(t) = \xi(t)dt$$

The increment is defined as an incremental Weiner process

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Rules of Weiner Process

$$\langle dW(t) \rangle = 0$$

$$\langle |dW(t)|^2 \rangle = dt$$

$$\langle (W(t) - w_0) \rangle = t - t_0$$

Defining Coefficients of Langevin Equ

$$\frac{du}{dt} = -a_1 u + b\xi(t)$$

$$\alpha = \frac{1}{T_L} \qquad b = \sigma_w \sqrt{2a_1} = \sigma_w \sqrt{2/T_L}$$

$$dw = -\frac{w}{T_L} dt + \sqrt{\frac{2\sigma_w^{\ 2}}{T_L}} \xi(t) dt$$

Form of Langevin's Equation for Heterogeneous turbulence

$$dw = -a_1(z,t,w)dt + b(z,t,w)\xi(t)dt$$

$$dw_L = \left(-\frac{w_L}{T_L} + \frac{1}{2}\left[1 + \frac{w_L^{\ 2}}{\sigma_w^2}\right]\frac{\partial \sigma_W^2}{\partial z}\right)dt + \sqrt{\frac{2\sigma_W^2}{T_L}}\xi(t)dt$$

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If turbulence is heterogeneous an extra term arises and is necessary. This is what happens in canopies where there are strong gradients in variance, time scales, shear, etc. If you apply Langevin equation in a canopy it will numerically violate laws of physics. A well mixed field should remain well mixed. But with the Langevin equation there will be a build up of particles near the floor as the drop down more then up due to asymmetric turbulence. So new terms need to be found

Thomson's Theory

- the well-mixed criterion must hold: if particles are initially well-mixed they will remain so in turbulent flow
- Eulerian models derived from the Lagrangian model must be compatible with Eulerian equations that are derived from first principles.
- the model will reduce to the diffusion equation when the Lagrangian time scale approaches zero
- the forward and reverse formulations of dispersion are consistent
- the small time behavior of the velocity distribution of particles must be correct.

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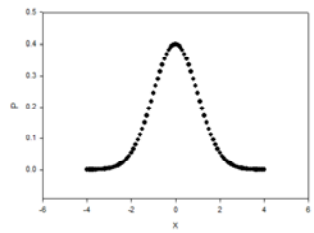
Thomson, D.J., 1987. Criteria for the Selection of Stochastic-Models of Particle Trajectories in Turbulent Flows. Journal of Fluid Mechanics, 180: 529-556.

Thomson developed one of the leading theories for applying the Lagrangian model in heterogeneous turbulence using the Fokker Planck Equation

Fokker-Planck Eq

$$\frac{\partial P(x,u,t)}{\partial t} + u\frac{\partial P(x,u,t\,|\,x_0,u_0,t_0)}{\partial x} =$$
$$-\frac{\partial (a_0 - a_1 u)P(x,u,t\,|\,x_0,u_0,t_0)}{\partial u} + \frac{1}{2}\frac{\partial^2\left[b^2 P(x,u,t\,|\,x_0,u_0,t_0)\right]}{\partial u^2}$$



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This is a partial differential equation for a conditional probability. This type of stochastic partial differential equation is used in the ED and ED2 model for forest dynamics

Gaussian Plume Model

$$\frac{\partial P(u,t|u_0,t_0)}{\partial t} = \frac{1}{2}\frac{\partial^2\left[P(u,t|u_0,t_0)\right]}{\partial u^2}$$

Initial condition $$P(u,t|u_0,t_0) = \delta(u-u_0)$$

$$P(u,t\,|\,u_0,t_0) = 2\pi(t-t_0)^{-1/2}\exp\left(-\frac{(u-u_0)^2}{2(t-t_0)}\right)$$

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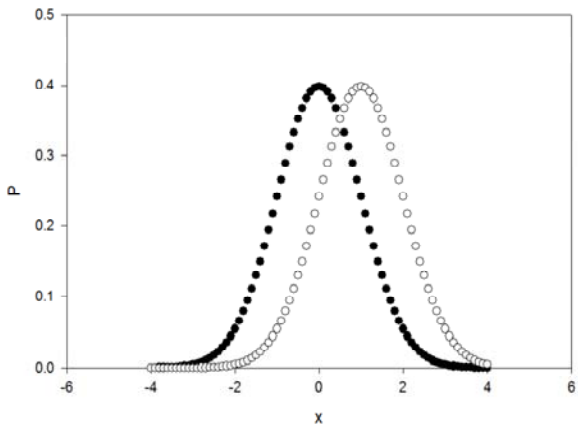
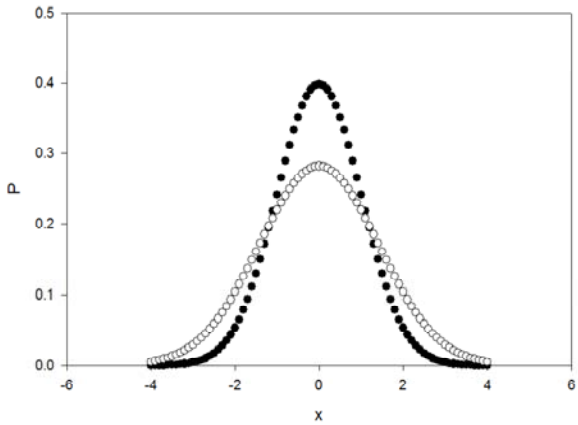
Simple version of Fokker Planck limits leads to Gaussian plume

Translation



spreading



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Dispersion Matrix

$$c_i - c_r = \sum_{j=1}^{N} S_j(c_j) D_{i,j} \Delta z_j$$

$$D_{ij} = \frac{C_i - C_r}{S_j \Delta z_j}$$

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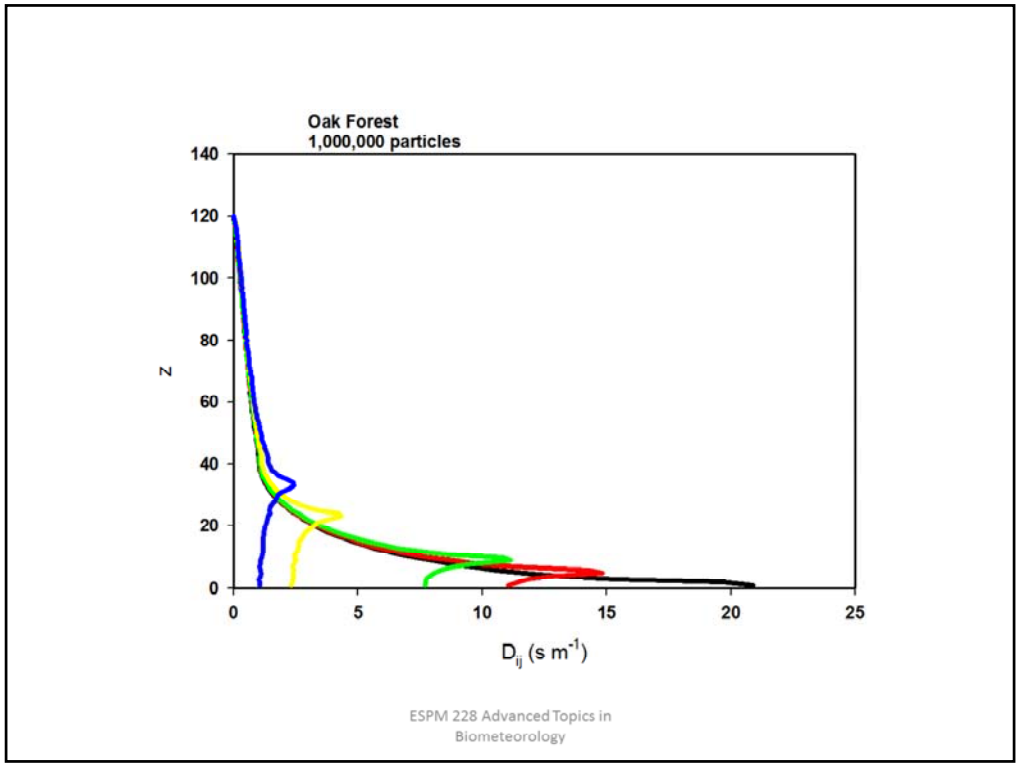
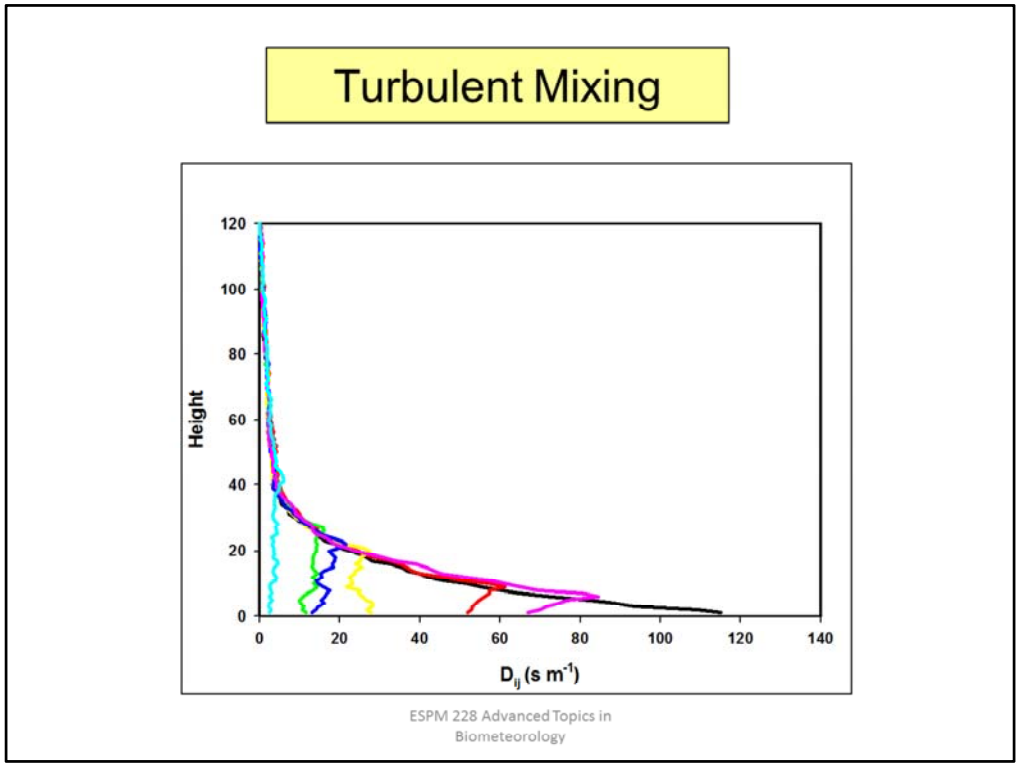
We can compute Dij for a known source of unit strength by calculating the C fields with a Lagrangian model by releasing parcels from different heights. Then apply it to compute C fields by iterating with S(c) for real conditions.



Here is example of Dispersion matric



Dij is noisy with fewer parcels

Analytical Near/Far Field Theory of Raupach

$$C(z) = C_n(z) + C_f(z)$$

$$\frac{\partial}{\partial z}(K(z)\frac{\partial C_f(z)}{\partial z} = -S_c(z)$$

$$F(z) = -K(z)\frac{\partial C_f}{\partial z} = F_c + F_s \qquad F_c(z) = \int_0^z S(z')dz'$$

$$K(z) = \sigma_w^2(z)T_L(z)$$

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Analytical models of Raupach

Raupach, M.R., 1989. A Practical Lagrangian Method for Relating Scalar Concentrations to Source Distributions in Vegetation Canopies. Quarterly Journal of the Royal Meteorological Society, 115(487): 609-632.

Simple Empirical Model

$$\frac{\sigma_w}{u_*} = \begin{cases} a_1, z > h \\ a_0 + (a_1 - a_0)\frac{z}{h}; z \le h \end{cases}$$

$$\frac{T_L u_*}{h} = \begin{cases} C_o; \frac{k}{a_1^2 h}(z-d) \le C_0 \\ \frac{k}{a_1^2 h}(z-d); \quad \frac{k}{a_1^2 h}(z-d) \ge C_o \end{cases}$$

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Simple assumptions of canopy turbulence. One can use empirical fits to data or use a higher order closure model top produce estimates of turbulence statistics given the canopy leaf area index, height, leaf size, for a hybrid Eulerian-Lagrangian model.

$$C_f(z_r) = C(z_r) - C_n(z_r)$$

$$C_f(z) - C_f(z_r) = \int_z^{z_r} \frac{F(z)}{K_f} dz$$

$$C_f(z) - C_f(z_r) = \int_z^{z_r} \frac{1}{\sigma_w^2(z')T_L(z')} [\int_0^{z'} S(z'')dz'' + F_s] dz'$$

$$C_n(z) = \int_0^{\infty} \frac{S(z')}{\sigma_w'} \{k_n \frac{z - z'}{\sigma_w(z')T_L(z')} + k_n \frac{z + z'}{\sigma_w(z')T_L(z')}\} dz'$$

$$k_n(x) \approx -0.3984 \ln(1 - \exp(-|x|) - 0.1562 \exp(-|x|)$$

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Improvements Warland and Thurtell

$$\frac{\partial c}{\partial z} = \int_0^\infty \frac{-q}{\sqrt{2\pi\sigma_z^2}} \frac{(z-z_s)}{\sigma_z^2} \exp\left(\frac{-(z-z_s)^2}{2\sigma_z^2}\right)$$

$$\frac{\partial c}{\partial z}|ff = \frac{-q}{2\sigma_w L_L} \qquad \frac{\partial c}{\partial z}|nf = \frac{-q}{\sqrt{2\pi}\sigma_w |z-z_s|}$$

$$\frac{\partial c}{\partial z} = Dq$$

$$D_{ff} = \begin{Bmatrix} \frac{-1}{2\sigma_w L_L}; z > z_s \\ \frac{1}{2\sigma_w L_L}; z < z_s \end{Bmatrix} \qquad D_{nf} = \begin{Bmatrix} \frac{-1}{\sqrt{2\pi}\sigma_w (z-z_s)}; z > z_s \\ \frac{1}{\sqrt{2\pi}\sigma_w (z_s - z)}; z < z_s \end{Bmatrix}$$

$$D = \begin{cases} \dfrac{-1}{2\sigma_w L_L[1-\exp(-\sqrt{\frac{\pi}{2}}(\frac{(z-z_s)}{L_L})]} ; z > z_s \\ \dfrac{1}{2\sigma_w L_L[1-\exp(-\sqrt{\frac{\pi}{2}}(\frac{(z_s-z)}{L_L})]} ; z < z_s \end{cases}$$

THE RELATIONSHIP BETWEEN A DISTRIBUTED SOURCE AND CONCENTRATION PROFILE 467

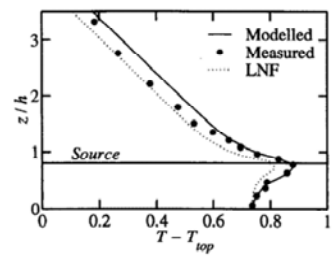

Figure 6. Comparison of CRL86's wind-tunnel data and the present model using Eulerian scales in place of the Lagrangian. Results from localized near-field theory also shown. Data are shown as the difference from the highest measured point T_{top} at $z/h \approx 6.3$.

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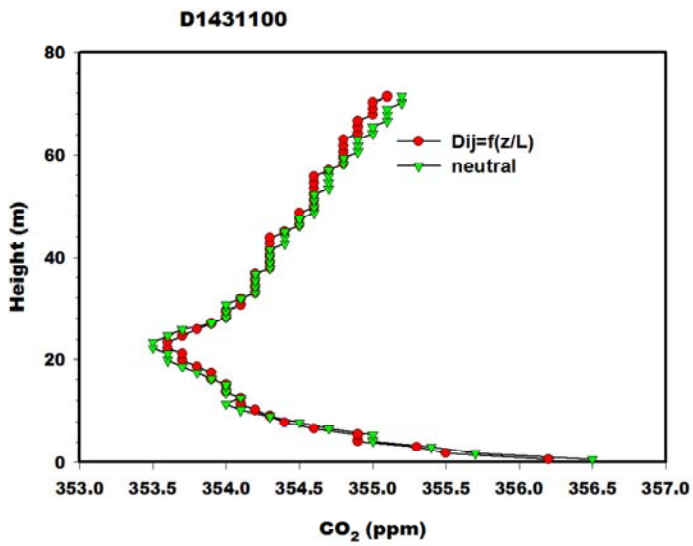
Warland, J.S. and Thurtell, G.W., 2000. A Lagrangian solution to the relationship between distributed source and concentration profile. Boundary Layer Meteorology, 96: 453-471.

Vertical Gradients in CO_2



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