Analysis of feedback mechanisms in land-atmosphere interaction

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Abstract. The initiation of a hydrologic drought may depend on large-scale or teleconnective causes; however, local positive feedbacks in the land-atmosphere system are believed to contribute to the observed persistence and intensification of droughts. In this study a basic linearization technique is combined with a nonlinear stochastic model of land-atmosphere interaction to analyze and, more importantly, quantify feedback mechanisms that arise in the coupled water and energy balances at the land surface. The model describes land-atmosphere interaction by four coupled stochastic ordinary differential equations in soil moisture, soil temperature, mixed-layer humidity, and mixed-layer potential temperature. The solution is a physically consistent joint probability distribution. The steady and perturbation-induced parts of the model equations are decomposed into the dependence of each component physical process upon each model state. Because of the negative correlation between soil moisture and soil temperature, the physical mechanisms that serve to restore each state individually (largely soil moisture control of evaporation and temperature dependence of saturation specific humidity) act as significant anomaly-reinforcing mechanisms for the other state.

1. Introduction

The major challenge in investigations of land-atmosphere interaction is to decipher the principal influences and feedbacks when the many state variables and physical processes in water and energy balance are all interconnected in complicated ways. Isolating the individual (partial) dependencies and ranking them according to the magnitude of their influences are important steps in understanding climatic variability and predicting seasonally persistent anomalies. In this paper a simple model of coupled land and near-surface atmosphere water and energy balance is used to determine the magnitude of some basic feedback mechanisms influencing anomalies in soil moisture and temperature.

The energy and moisture budgets at the land surface are principally linked by evaporation, which is an expenditure of both energy and water mass. As a result of this and other important linkages in land-atmosphere interaction, the soil heat and moisture states tend to be negatively correlated. When the evaporation rate responds to an anomalous state, this negative correlation puts the water and energy budgets into competition. In a warm/dry anomaly the elevated temperature creates a demand for increased evaporation to cool the surface; at the same time, the low moisture state restricts evaporation. This is the most basic and conceptually well-known feedback mechanism. Our key objective here is to quantify and assess the relative magnitude of the competition between restoring forces in components of the water energy balance in returning the surface water and energy states from warm/dry or cool/moist anomalies.

Numerous other forms of land-atmosphere coupling also result in important feedback mechanisms. For example, the atmospheric demand for evaporation depends on the temperature and humidity of the near-surface air and is thus dependent on the evaporation that has already occurred within the region [Bouchet, 1963]. Another important feedback mechanism develops when moist soil provides water for high evaporation when energy is available, which may in turn enhance precipitation through moisture supply and reinforce moist anomalies; in the case of dry soil, suppressed evaporation may reduce precipitation, leading to further drying. This precipitation recycling mechanism [Brubaker et al., 1993; Entekhabi et al., 1992] constitutes an example of a positive (anomaly-enhancing) feedback for the soil moisture state. Increased albedo due to drying of the soil or removal of vegetation contributes to a net radiative heat loss, which may enhance sinking motion and further inhibit precipitation, leading to further drying in regions such as the Sahel [Charney, 1975]. Synoptic observations over the United States [Namias, 1988] indicate that large-scale soil moisture deficits may inhibit precipitation because the elevated surface temperature deepens the adiabatically mixed air layer, entraining dry air aloft, and intensifies the midcontinental high-pressure ridge. Thus although the initiation or termination of a hydrologic drought may depend on large-scale (global or hemispheric) causes, such as persistent circulation patterns or teleconnections [Namias, 1983; McNab and Karl, 1989], local positive feedbacks in the land-atmosphere system may contribute to the observed persistence and intensification of droughts.

A number of statistical studies of the observational record reflect the influence of land-atmosphere interaction on climate variability [van den Dool, 1984; Huang and van den Dool, 1993; Zhao and Khalil, 1993; Kemp et al., 1994]. Other investigations of land-atmosphere interactions are based on numerical models of the atmosphere that integrate the atmospheric primitive equations (equation of state; conservation of mass, energy, and
model of land-atmosphere interaction. Feedback mechanisms that are not apparent if soil moisture is considered alone become apparent when the surface water and energy budgets in both land and atmosphere are allowed to affect one another. In the present study the model equations are decomposed to isolate the physical processes responsible for these feedbacks, as well as to determine the sign and magnitude of their competing contributions.

This mathematical model provides a unique opportunity to quantify positive and negative feedbacks in the coupled water and energy budgets at the land surface. It allows us to dissect and, more importantly, quantify the “arrows” (Tables 4 through 6, 9 and 10, and Figures 2 and 3) in conceptual diagrams such as Figure 1.

2. Model Summary

The land-atmosphere interaction model used in this paper is developed, and its equilibrium properties explored, by Brubaker and Entekhabi [1995]; a summary appears here; for further details, the reader is referred to the original paper.

2.1. Basic Model Equations

The model is a four-state area-averaged balance for a continental region and the overlying turbulently mixed atmospheric boundary layer. The soil layer and the near-surface atmosphere are treated as reservoirs with storage capacities for heat and water, with the transfers between them regulated by four states: \( s \), depth-averaged relative soil saturation (soil moisture, dimensionless); \( q_w \), specific humidity in the mixed layer (grams H\(_2\)O per kilogram air); \( T_g \), soil temperature (degrees Celsius); and \( \theta_w \), potential temperature in the mixed layer (degrees Celsius). The radiative and turbulent heat fluxes that couple these states are explicitly represented. The major simplifying assumptions in the model are complete dry adiabatic mixing in the boundary layer and parameterization of warm dry air entrainment in lieu of variations in boundary layer height. The horizontal extent of the region is conceptually equivalent to the length scale over which generally homogeneous heat and moisture conditions are present, or over which advective and radiative effects can equilibrate. In midcontinental regions without marked orography, such an area may cover up to \( 10^4 \) to \( 10^5 \) km\(^2\).

The soil layer is assigned an active depth and a porosity. The atmospheric reservoir is treated as a developed, vertically mixed turbulent boundary layer with height \( h \), on the order of 1 km. In this idealized mixed layer, specific humidity and potential temperature, defined as \( \theta = T(p_{ref}/p)^{R_t/C_{pa}} \) are invariant with height. (Here, \( T \) is the thermodynamic temperature, \( p \) the pressure, \( p_{ref} \) a reference pressure, and \( R_t \) and \( C_{pa} \) dry-air gas constant and specific heat under constant pressure.) Potential temperature is, by definition, conserved under adiabatic pressure change; it is therefore the appropriate conserved temperature quantity in a well mixed, unsaturated layer. Specific humidity (grams H\(_2\)O per kilogram air) is also conserved under adiabatic mixing. The model assumes a gradient in both humidity and potential temperature in the surface sublayer. The mixed-layer height \( (h) \) is invariant in the model; the layer-top entrainment of warm dry air is parameterized in lieu of time-varying \( h \). Precipitation is derived from the fluxes of moisture into the atmospheric control volume (lateral advection \( (Q_{in}) \) and surface evaporation \( (E) \)) according to a partitioning parameter, \( b \), such that
\[ P = (1 - b)Q_w + E. \]  

In (1) the model partitioning parameter \( b \) divides the incoming water vapor (advection and evaporation) between moistening the mixed-layer control volume (fraction \( b \)) and condensing as precipitation (fraction \( 1 - b \)). This approach is consistent with the model used by Rodriguez-Iiturbe et al. [1991] and Entekhabi et al. [1992], which assumes that precipitation is derived from advected and evaporated water masses in proportion to their respective fractions of the total water mass in a well-mixed atmospheric column. The infiltration of precipitation is controlled by soil moisture through the function \( \phi(x) \), the moisture-dependent runoff ratio; a fraction \((1 - \phi)\) of precipitation infiltrates into the soil layer.

The model’s four states evolve in time according to the following system of coupled ordinary differential equations:

\[
\begin{bmatrix}
\frac{ds}{dt}
\frac{dq_m}{dt}
\frac{dT}{dt}
\frac{d\theta_m}{dt}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{\rho_p \mu Z_a}[(1-\phi)(1-b)Q_m + [(1-\phi)(1-b)-1] \beta E_p] \\
\frac{1}{(p_s - p_b)/g} \{ \beta [Q_m + \beta E_p] - Q_{out} \} \\
\frac{1}{C_{out} \rho_s} \left[ RS(1-a) + RL_{wd}(1-v_{out}) + RL_{rd} - RL_{wu} - H - \lambda \beta E_p \right] \\
\frac{1}{C_{ps}(p_s - p_b)/g} \left[ (RL_{wd} + RL_{wu}) v_{out} - RL_{wu} - RL_{rd} + \left[1 + A_{sup}\right]H + H_m - H_{out} \right]
\end{bmatrix}
\]

(2)

In (2), \( Q_{in,out} \) and \( H_{in,out} \) represent the large-scale advection of moisture and sensible heat into and out of the region. In this application, atmospheric water vapor convergence (\( Q_m \) and \( Q_{out} \)) is included, as it is necessary to maintain a regional hydrologic cycle. Sensible heat convergence due to lateral temperature gradients (\( H_m \) and \( H_{out} \)) is neglected; for a large land area this term is small, relative to the large radiative exchanges, particularly the shortwave.

The functional forms of the terms in (2) are given in Table 1, and the model variables are listed in the notation section.

2.2. System of Stochastic Differential Equations

The model is forced by solar radiation at the top of the atmosphere and by mixed-layer wind speed, which advects moisture and determines the magnitude of the transfer coefficient for the surface turbulent fluxes. The regional wind speed is taken to be composed of a mean component, plus (zero-mean, serially independent, normally distributed) perturbations with variance \( \sigma^2_u \):

\[ U_z = \bar{U} + \sigma_u \int dw_f \]

\[ E[dw_f] = 0, \quad E[dw_fdw_{f*}] = \delta(t - v) \]

where \( E[ \ ] \) is the expectation operator and \( \delta(\ ) \) is the Dirac delta function. (Here noise, \( dw_f \), is the infinitesimal of a Wiener process, and \( \int dw_f \) represents the integrated white noise process over a finite increment of time.) The humidity of air advected by this wind into the region is fixed. Noise is placed in the wind speed because it is an important physical parameter, affecting moisture advection and turbulent conductivity, and because it varies on shorter timescales than the model states.

The white noise assumption is, of course, physically unrealistic; actual wind speed is temporally autocorrelated. However, a major purpose of this model is to isolate the intercorrelations among the state variables that result from physical interactions between the variables, not from their simultaneous response to an autocorrelated forcing.

The functions that are affected by Equation (3) are the regional moisture advection terms \( Q_m \) and \( Q_{out} \) as well as the turbulent heat fluxes \( \lambda E \) and \( H \). The moisture advection terms are given by

\[
\begin{bmatrix}
Q_m \\
Q_{out}
\end{bmatrix} = \frac{(p_s - p_b)/g}{L} \begin{bmatrix}
q_m \\
q_{in}
\end{bmatrix} \mathbf{U}_z
\]

(4)

where \( p_s \) and \( p_b \) are the air pressure at the land surface and at the top of the mixed layer (invariant in this model), \( L \) is the length scale of the region, and \( g \) is the acceleration of gravity. The surface turbulent fluxes are given [after Stull, 1994] as

\[ H = \rho_c \rho_{s0} C_u U_z + C_{2w} w_h (T - \theta_m) \]

(5)

\[ \lambda E = \lambda \beta \lambda (C_u U_z + C_{2w} w_h) [q^*(T) - q_m] \]

(6)

where \( w_h \) is a buoyancy velocity scale and \( C_{1,2} \) are empirical constants for forced and free convection, respectively. The evaporation efficiency function \( \beta(s) \), represents the soil control on evaporation; it is the ratio of the allowed evaporative flux to the energy-limited potential value. The variables in (4)–(6) are defined in the notation section.

After substituting (4)–(6) into (2), together with this definition, the time evolution of the system is described by a continuous stochastic differential equation that may be compactly written as

\[ dx = G(x)dt + G(x)dw_x \]

(7)

where the time-varying state vector \( x = [s \ q_m \ T \ \theta_m] \) has been defined. Equation (7) is obtained from the corresponding ordinary differential equations by separating the mathematical terms containing the mean wind speed from those containing its fluctuating part.

The vector function \( G(x) \) represents the physical forcing of the state variables through the radiative fluxes as well as the steady (mean) part of wind speed in incremental time \( dt \). \( G(x) \) is sometimes called the “drift” function because it describes how the four states would evolve in the absence of noise. The four components of this steady-forcing function are as follows:

\[ G_1 = \frac{1}{\rho_p \mu Z_a} \left[ (1-\phi)(1-b)Mq_w \bar{U} + [(1-\phi)(1-b)-1] \right] \]

\[ \cdot \beta(C_u \bar{U} + C_{2w} w_h) [q^*(T) - q_m] \]

(8)

\[ G_2 = \frac{1}{(p_s - p_b)/g} \left[ b(Mq_m \bar{U} + \beta(C_u \bar{U} + C_{2w} w_h) \right] \]

\[ \cdot \rho [q^*(T) - q_m] - Mq_m \bar{U} \]

(9)
Table 1. Functional Forms of Terms in the Land-Atmosphere Model

<table>
<thead>
<tr>
<th>Term</th>
<th>Form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albedo</td>
<td>$a = a(x)$</td>
</tr>
<tr>
<td>Evaporation efficiency</td>
<td>$\beta = \sigma$</td>
</tr>
<tr>
<td>Runoff ratio (runoff/precipitation)</td>
<td>$\varphi = \sigma^2$</td>
</tr>
<tr>
<td>Air mass</td>
<td>$M = \frac{(p_a - p_h)}{\rho}$</td>
</tr>
<tr>
<td>Sensible heat*</td>
<td>$H = (C_w U + C_x w)\rho C_p(T - \theta_m)$</td>
</tr>
<tr>
<td>Potential evaporation*</td>
<td>$E_p = (C_w U + C_x w)\rho q^*(T_p, p_1) - q_m$</td>
</tr>
<tr>
<td>Buoyancy velocity scale*</td>
<td>$w_B = \frac{g}{\theta_m} h (T - \theta_m)^{1/2}$</td>
</tr>
<tr>
<td>Mixed-layer LW (up)†</td>
<td>$RL_{ua} = \left[0.107 \left(\frac{2q_m p_h}{3 g} \right)^{1/7} F \left(\frac{p_h}{p_1}\right) \sigma \theta_m^{1/2} \right]$</td>
</tr>
<tr>
<td>Mixed-layer LW (down)†</td>
<td>$RL_{ud} = \left[0.107 \left(\frac{2q_m p_h}{3 g} \right)^{1/7} F \left(\frac{p_h}{p_1}\right) \sigma \theta_m^{1/2} \right]$</td>
</tr>
<tr>
<td>LW from above†</td>
<td>$RL_{ch} = 1.24 \left[\frac{q_m p_h}{\theta_m \rho \theta_m} \left(\frac{p_h}{p_1}\right) \sigma \theta_m^{1/2} \right]$</td>
</tr>
<tr>
<td>Mixed-layer bulk LW emissivity†</td>
<td>$e_{col} = 0.75 \left{\frac{2}{3} \frac{p_h}{\rho_c} \left[1 - \left(\frac{p_h}{p_1}\right)^{3/2}\right] \right}^{1/7}$</td>
</tr>
<tr>
<td>Soil LW radiation</td>
<td>$RL_{p_s} = \sigma T_e^4$</td>
</tr>
<tr>
<td>LW cloud correction</td>
<td>$X_c = 1 + \frac{c_1 N_c}{e_{col}}$</td>
</tr>
<tr>
<td>SW cloud correction</td>
<td>$Y_c = \frac{c_2 + c_3 N_c}{e_{col}}$</td>
</tr>
</tbody>
</table>
| Saturation specific humidity| $q^*(T, p) = \frac{e_{s0}}{p} e_{s0} \exp \left\{\frac{\lambda}{R_e^0 \left(1 - \frac{T}{T_0} - \frac{1}{T}\right)}\right\}$  

*After Stull [1994].†After Brutsaert [1982].

\[
G_3 = \frac{1}{C_{soil}} \left\{ RS(1 - \alpha) + RL_{sa}(1 - e_{col}) + RL_{sa} - RL_{pu} \right. \\
- \rho C_{pu}(C_1 U + C_x w_1)(T - \theta_m) \\
- \lambda \beta (C_1 U + C_x w_1) \rho q^*(T_p, p_1) - q_m \} \right\} (10)
\]

\[
G_4 = \frac{1}{C_{pu}(p_a - p_h)/g} \left\{ (RL_{ud} + RL_{pu}) e_{col} - RL_{su} - RL_{ud} \right. \\
+ (1 + A_{bhp}) \rho C_{pu}(C_1 U + C_x w_1)(T - \theta_m) \\
+ MC_{pu} \theta_m U - MC_{pu} \theta_m U \} \right\} (11)
\]

where $M$ represents the mixed-layer column air mass per unit width, $M = (p_a - p_h)/g$.

The fluctuating part of wind speed affects the system in proportion to the deterministic function $g(x)$. The wind speed is a physical parameter of the system; therefore the influence of the random fluctuations is modulated by the state of the system at the time of the event (see (4)-(6)). The state dependence of this perturbation-forcing term is an important property of this model that distinguishes it from, for example, autoregressive models. Because it contributes to a spreading of probability mass in state-space, $g(x)$ is also known as the diffusion function. The four components of the perturbation-forcing function are as follows:

\[
g_1 = \frac{1}{\rho C_{pu}} \left\{ (1 - \varphi)(1 - b) M q_m + [(1 - \varphi)(1 - b) - 1] \right. \\
\cdot \beta C_p \rho q^*(T_p, p_1) - q_m \} \sigma_u \right\} (12)
\]

\[
g_2 = \frac{1}{(p_a - p_h)/g} \left\{ b (M q_m + \beta C_p \rho q^*(T_p, p_1) - q_m)) - M q_m \} \sigma_u \right\} (13)
\]

\[
g_3 = \frac{1}{C_{soil}} \left\{ (1 + A_{bhp}) \rho C_{pu} C_1(T - \theta_m) - \lambda \beta C_p \rho q^*(T_p, p_1) - q_m \} \sigma_u \right\} (14)
\]

\[
g_4 = \frac{1}{C_{pu}(p_a - p_h)/g} \left\{ (1 + A_{bhp}) \rho C_{pu} C_1(T - \theta_m) \\
+ MC_{pu} \theta_m U - MC_{pu} \theta_m U \} \right\} (15)
\]

The notations $G_i(x)$ and $g_i(x)$ are used here to emphasize that each component is a function of all four model states.

Although the system is forced by serially independent white noise, the output of the model is serially dependent with some statistical memory, due to the storage and interactions in the system. The random fluctuations in wind speed that drive the system are routed and distributed among components of the system through the state-dependent fluxes of energy and water.
mass. Any resulting temporal covariability among the model states results from the physical linkages that are implicit in the model, not from preassigning a correlation structure to the variables. The basic model parameters used in this study are listed in Table 2.

The solution to (7) is a joint probability distribution for the four model states. This distribution arises from the physical interrelationships described by the model equations and is not preassigned. Entekhabi and Brubaker [1995] explore the statistical behavior of the model by integrating the four-state stochastic differential equation (7); they present marginal probability distributions of the states as well as the components of the surface water and energy balance (such as net radiation, latent heat flux, sensible heat flux, etc.). They also demonstrate the role of two-way land-atmosphere interaction on the lagged covariance of the model states.

3. Definitions and Methodology

The four-equation system is integrated in time using an Euler-type discretization [Pandolfi and Talay, 1984] of (7); solution statistics are computed from the resulting stochastic time series. Because of the differential formulation of this stochastic equation, its integration in time is independent of the numerical time step. The analysis in this paper is based on the specific stochastic solution described by Entekhabi and Brubaker [1995]; the conditions of this solution are a continental region at latitude 45°N, forced by daily-average solar radiation at perpetual summer solstice. The basic statistics for the model states in this stochastic solution are given in Table 3, together with the equilibrium solution \( \mathbf{x}^* \) resulting from \( \mathbf{G}(\mathbf{x}) = 0 \). Comparison of the model equilibrium \( \mathbf{x}^* \) and the statistical expectation \( \mathbf{x} \) illustrates how the covariability of heat and moisture in the land and atmosphere contribute to the definition of regional climate. An important implication of the near equivalence between the equilibrium and statistical expectation of the model states is that a linearized version of the model may be effectively used to explore the magnitude of the restoring and antirestoring forces and factors in the model.

### Table 2. Parameter Values

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h )</td>
<td>height of mixed layer, m</td>
<td>1000</td>
</tr>
<tr>
<td>( p_s )</td>
<td>surface pressure, mbar</td>
<td>1000</td>
</tr>
<tr>
<td>( p_h )</td>
<td>pressure at slab top, mbar</td>
<td>880</td>
</tr>
<tr>
<td>( A_{top} )</td>
<td>entrainment parameter for sensible heat at slab top, ( ^\circ )C</td>
<td>0.2</td>
</tr>
<tr>
<td>( b )</td>
<td>moistening parameter, dimensionless</td>
<td>0.3</td>
</tr>
<tr>
<td>( Z_h )</td>
<td>hydrologically active soil depth, m</td>
<td>0.2</td>
</tr>
<tr>
<td>( Z_t )</td>
<td>thermally active soil depth, m</td>
<td>0.4</td>
</tr>
<tr>
<td>( n )</td>
<td>soil porosity, dimensionless</td>
<td>0.25</td>
</tr>
<tr>
<td>( U_z )</td>
<td>mean mixed-layer wind speed, m s(^{-1} )</td>
<td>4.0</td>
</tr>
<tr>
<td>( \sigma_n )</td>
<td>standard deviation of mixed-layer wind speed, m s(^{-1} )</td>
<td>1.5</td>
</tr>
<tr>
<td>( L )</td>
<td>length scale of region, km</td>
<td>500</td>
</tr>
<tr>
<td>( q_{in} )</td>
<td>specific humidity of incoming air, (g H(_2)O) (kg air(^{-1} ))</td>
<td>8.0</td>
</tr>
</tbody>
</table>

*Tennekes [1973].

### Table 3. Statistics of State Variables

<table>
<thead>
<tr>
<th>Index</th>
<th>Variable</th>
<th>Equilibrium Solution ( \mathbf{x}^* )</th>
<th>Stochastic Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( s ), dimensionless</td>
<td>0.613</td>
<td>0.611</td>
</tr>
<tr>
<td>2</td>
<td>( q_{in} ), g kg(^{-1} )</td>
<td>4.27</td>
<td>4.25</td>
</tr>
<tr>
<td>3</td>
<td>( T_g ), °C</td>
<td>20.7</td>
<td>20.6</td>
</tr>
<tr>
<td>4</td>
<td>( \theta_m ), °C</td>
<td>15.3</td>
<td>15.5</td>
</tr>
<tr>
<td>5</td>
<td>( T_g - \theta_m ), °C</td>
<td>5.4</td>
<td>5.3</td>
</tr>
</tbody>
</table>

3.1. The Steady-Forcing Function: Tendency to Equilibrate

If the white noise term \( dw_i \) is removed from the right-hand side of (7), the system is reduced to a set of four coupled ordinary differential equations that describe the interconnected water and energy balances of the land-atmosphere system when subjected to a steady wind speed:

\[
\frac{d\mathbf{x}}{dt} = \mathbf{G}(\mathbf{x}).
\]

Brubaker and Entekhabi [1995] explore the equilibrium behavior \( \mathbf{G}(\mathbf{x}) = 0 \) of this deterministic form of the model and its sensitivity to several important parameters. The model is an open system, for both energy and water mass, so that the equilibrium solution reflects the existence of temperature and moisture states that balance the specified external insolation and water mass convergence. At this four-variable equilibrium state, the model’s air and soil stores are coupled such that the net incoming solar radiation is exactly balanced by longwave back radiation and latent heat advection; moisture convergence is exactly balanced by runoff; and the net exchange between air and soil is zero for both water mass and energy. This does not imply that the fluxes themselves are 0 (they are not), but that they are in balance. From any initial state that is physically realistic for the conceptual continental climate represented by this particular parameter set, in the absence of wind speed fluctuations, the \( \mathbf{G}(\mathbf{x}) \) function will drive the system to \( \mathbf{x}^* \).

Because \( \mathbf{x}^* \) is a stable equilibrium of the system, it is natural to analyze the deterministic behavior of \( \mathbf{x}(t) \) near \( \mathbf{x}^* \). The components of the \( \mathbf{G}(\mathbf{x}) \) function are expanded in truncated Taylor series around \( \mathbf{x}^* \), for \( i = 1, 4 \):

\[
\mathbf{G}(\mathbf{x}) = \mathbf{G}(\mathbf{x}^*) + \left[ \frac{\partial \mathbf{G}_i}{\partial s} \right]_{\mathbf{x}^*} (s - s^*) + \left[ \frac{\partial \mathbf{G}_i}{\partial q_{in}} \right]_{\mathbf{x}^*} (q_{in} - q_{in}^*),
\]

\[
+ \left[ \frac{\partial \mathbf{G}_i}{\partial T_g} \right]_{\mathbf{x}^*} (T_g - T_g^*) + \left[ \frac{\partial \mathbf{G}_i}{\partial \theta_m} \right]_{\mathbf{x}^*} (\theta_m - \theta_m^*),
\]

with the first term \( G_i(\mathbf{x}^*) \) equal to 0 by construct. When the differentiation in (17) is performed, a number of common terms appear with opposite sign in the \( T_g \) and \( \theta_m \) derivatives, due to the presence of the temperature difference \( T_g - \theta_m \) in the turbulent flux terms. Clearly, an equilibrium temperature difference exists; it is given the notation \( \Delta^* \), where \( \Delta = T_g - \theta_m \). Thus (17) may be rewritten to include the dependence of each steady-forcing component on the system’s departure from its equilibrium temperature difference, as well as on the departure from its equilibrium state in absolute terms:
The linear coefficients \(j\) storing terms (underlined) are the diagonal terms of the matrix and indicate the nondimensional disequilibrium in variable \(j\) (similar to \(Z\) scores in statistical analysis):

\[
\delta x_j = \frac{x_j - x_j^*}{\sigma_j}.
\]  

In the analysis the nondimensional disequilibria are referred to as \(\delta s\), \(\delta T^g\), and so forth. With this notation, (18) can be compactly written as

\[
G_j(x) = \sum_{i=1}^{5} \Lambda_{ji} \delta x_i = \Lambda_{1j} \delta s + \Lambda_{2j} \delta q_m + \Lambda_{3j} \delta T^g + \Lambda_{4j} \delta \theta_m + \Lambda_{5j} \Delta.
\]  

The linear coefficients \(\Lambda_{ji}\) are simply the derivatives of the steady-forcing function for the \(i\)th state variable with respect to the \(j\)th variable, evaluated at the equilibrium solution, and multiplied by the \(j\)th standard deviation. The sign and magnitude of these coefficients may be used to evaluate the strength of various physical restoring or disequilibrating factors.

3.2. Restoring and Coupling Terms

According to (20), each steady-forcing component \(G_j\) is approximated by a weighted sum of the four state disequilibria, where each weight \(\Lambda_{ji}\) is determined by the various physical processes through which a disequilibrium in variable \(j\) affects the evolution of variable \(i\). Let us first examine the terms where \(i = j\), that is, the effect of a variable’s disequilibrium upon its own evolution. For a self-restoring process, \(\Lambda_{ji}\) must be negative, consistent with the definition of a negative feedback as one that counters an anomaly. The terms \(\Lambda_{ji}\) indicate the tendency of the overall system to respond to an anomaly of one variable by restoring the equilibrium state of that same variable and therefore will be called “restoring” terms.

The terms \(\Lambda_{ji}, i \neq j\), express the effect of disequilibrium in variable \(j\) on the evolution of variable \(i\). If \(\Lambda_{ij}\) is positive, then an above-equilibrium \(\delta x_i\) will contribute to an increase in variable \(i\), regardless of whether \(\delta x_i\) is positive or negative. The terms \(\Lambda_{ij}, i \neq j\) are coupling terms; they signify the dependence of variables upon each other’s disequilibria.

Rewriting (20) in complete matrix form shows that the restoring terms (underlined) are the diagonal terms of the matrix that transforms the vector of nondimensional disequilibria to the approximate steady-forcing functions:

\[
\begin{bmatrix}
G_1 \\
G_2 \\
G_3 \\
G_4 \\
G_5
\end{bmatrix} = \begin{bmatrix}
\Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} \\
\Lambda_{21} & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} & \Lambda_{25} \\
\Lambda_{31} & \Lambda_{32} & \Lambda_{33} & \Lambda_{34} & \Lambda_{35} \\
\Lambda_{41} & \Lambda_{42} & \Lambda_{43} & \Lambda_{44} & \Lambda_{45} \\
\Lambda_{51} & \Lambda_{52} & \Lambda_{53} & \Lambda_{54} & \Lambda_{55}
\end{bmatrix} \begin{bmatrix}
\delta s \\
\delta q_m \\
\delta T^g \\
\delta \theta_m \\
\delta \Delta
\end{bmatrix}
\]  

(21)

After the expressions for \(\Lambda_{ji}\) in the analytic model of land-atmosphere interaction and coupled water and energy balance are derived, we may then evaluate their sign and magnitude to identify and quantify the feedback processes contributing to climatic and hydrologic variability (Figure 1).

3.3. Analytic Procedure

The selection of anomalies \(\delta s, \delta q_m, \ldots\), for evaluation of (20) is not arbitrary; not all combinations of the four states are physically probable. Thus the stochastic solution to (7) is an essential part of this analysis. Given a selected soil moisture anomaly, how will the corresponding physically realistic signs and magnitudes of the other state variables’ disequilibria contribute to recovery from or reinforcement of that anomaly? The analysis proceeds in three steps. First, the component parts of the weighting terms are decomposed and their physical meaning discussed (section 4). Then numerical values, based on the stochastic solution, are assigned to the terms (section 5). Finally, evaluation of (20) is carried out by taking the products of the weighting terms (from linear analysis) and physically realistic correlated state disequilibria (from the full nonlinear stochastic solution) (section 6).

The sign of each product cross term \(j \neq i\) in (20) depends on both the sign of the weighting term \(\Lambda_{ij}\) and the sign of the disequilibrium \(\delta x_j\). The determination of feedback effects, that is, whether variable \(j\) contributes to recovery from or reinforcement of anomalies in variable \(i\), requires the evaluation of the products \(\Lambda_{ij} \delta x_j\) and is reserved for section 6.

4. Decomposition of the Steady-Forcing Terms

The restoring and feedback factors quantified by the linear coefficients \(\Lambda_{ji}\) are physically based on the balance equations that describe the exchanges of mass and energy among the four state variables in two-way land-atmosphere interaction, given a steady wind speed. In this section the land-atmosphere model is linearized in terms of the coefficients \(\Lambda_{ij}\). In section 5 the coefficients \(\Lambda_{ji}\) and disequilibria \(\delta x_i\) are evaluated and ranked according to their magnitudes and signs.

4.1. Soil Moisture

The restoring term \(\Lambda_{11}\) represents the effect of soil moisture on the physical processes that reequilibrate it from a disequilibrated state. From (8) and the definition of \(\Lambda_{11}\),

\[
\Lambda_{11} = \frac{\partial G_1}{\partial s} \sigma_s
\]  

(a) \[= + (1 - b \left(\frac{M q_m U}{\rho_w} \right) \frac{d}{d s} (1 - \varphi) \sigma_{s},
\]

(b) \[+ (1 - b) \left(\frac{C E U_1 + C_2 \omega_i}{\rho_w} \right) \frac{q^* (T^g, p, q_m) - q_m}{\rho_w} \frac{d}{d s} [(1 - \varphi) \beta] \sigma_s,
\]

(c) \[= - \left(\frac{C E U_1 + C_2 \omega_i}{\rho_w} \right) \frac{q^* (T^g, p, q_m) - q_m}{\rho_w} \frac{d \beta}{d s} \sigma_s.
\]  

(22)

The subterms (a) and (b) represent the infiltration of precipitation. The factors \((1 - b)\) and the vapor gradient are positive at the equilibrium point. The subterm (a) is the infiltration of that precipitation derived from regional moisture advection.
The subterm (b) is the infiltration of precipitation derived from local evaporation. Because the infiltrated fraction of precipitation \((1 - \varphi)\) generally decreases with increasing soil saturation, the infiltration process due to large-scale advective forcing is a negative feedback mechanism for soil moisture anomalies (subterm (a) is always a negative contribution to \(\Lambda_{11}\)). The infiltration of locally derived (recycled) precipitation nonetheless may be either a positive or negative feedback, depending on the surface hydrologic partitioning. The runoff ratio \(\varphi\) and evaporation efficiency \(\beta\) are both increasing functions of soil saturation; the term \((d/ds)\{(1 - \varphi)\beta\}\) may take either sign depending on the partitioning of hydrologic fluxes at the surface. The locally recycled precipitation may thus constitute a positive feedback on soil moisture anomalies. The final subterm (c) represents the evaporative loss of soil moisture in the water balance equation. Dry soils reduce evaporation efficiency, so that further drying is slowed; moist soils are subject to enhanced evaporation if the energy is available. \(\Lambda_{11}\) will usually be negative overall; however, the indeterminate sign of (b) suggests that soil moisture may not always be self-restoring (particularly for dry states).

The term \(\Lambda_{12}\) quantifies the coupling of soil moisture steady forcing to specific humidity disequilibrium:

\[
\Lambda_{12} = \frac{\partial G_1}{\partial q_m} \sigma_q = \left[ 1 - \left(1 - \beta \right) \left( 1 - \varphi \right) \right] \frac{\rho}{\rho_w} \left( C_s \bar{U} + C_w \bar{w}_b \right) \sigma_q \tag{23}
\]

This term is necessarily positive. A moist disequilibrium in air reduces the humidity gradient in the surface sublayer and thus suppresses net evaporation. In this expression, net evaporation equals total evaporation minus recycled precipitation.) Conversely, a dry-air humidity disequilibrium enhances net evaporation. Depending on the correlation of soil moisture and air specific humidity anomalies (that is, what sign \(\delta x\) takes in (8)), this physical interaction may constitute either positive or negative feedback.

The coupling of soil moisture steady forcing to disequilibrium in soil temperature is given by \(\Lambda_{13}\):

\[
\Lambda_{13} = \left( \frac{\partial G_1}{\partial T_g - \bar{\varphi}} \right) \sigma_T = \left[ 1 - \left(1 - \beta \right) \left( 1 - \varphi \right) \right] \frac{\rho}{\rho_w} \frac{dq^*}{dT_g} \left( C_s \bar{U} + C_w \bar{w}_b \right) \sigma_T \tag{24}
\]

As discussed above, the term \(\Lambda_{13}\) does not include all the constituents of \(\partial G_1/\partial T_g\); the items that arise in common with \((- \partial G_1/\partial \theta_m)\) are indicated by \(\bar{\varphi}\) and appear in \(\Lambda_{15}\), multiplying the temperature difference, \(\Delta\). A very important property to note is that \(\Lambda_{13}\) is necessarily negative; \(1 - (1 - \beta)(1 - \varphi)\) is always positive, as is the temperature dependence of ground surface saturation specific humidity \((dq^*/dT_g)\) in (24). Positive \(T_g\) anomalies thus exert a drying influence in the steady part of soil moisture evolution. (Because higher temperatures are typically associated with dry soils, this weighting term will tend to reinforce soil moisture anomalies, as discussed in later sections.)

A disequilibrium in absolute air slab potential temperature drives soil moisture steady forcing through the term \(\Lambda_{14}\):

\[
\Lambda_{14} = \left( \frac{\partial G_1}{\partial q_m} + \bar{\varphi} \right) \sigma_q = \left[ 1 - \left(1 - \beta \right) \left( 1 - \varphi \right) \right] \frac{\rho}{\rho_w} C_s \sqrt{\frac{T_g - \theta_m}{\theta_m}} \frac{1}{\theta_m} \beta \sigma_q \tag{25}
\]

This term arises from the presence of \(\theta_m\) in the denominator of the buoyancy velocity scale, \(w_B\). In \(w_B\), the expression \(g(T_g - \theta_m)/\theta_m\) represents the buoyancy acceleration that a parcel of air at the surface would experience if lifted adiabatically into the mixed layer. The square root of this acceleration multiplying a distance (here, the mixed-layer thickness \(h\)) gives a velocity scale for the buoyant thermals, which is related to the surface turbulent fluxes. For a given temperature difference \((T_g - \theta_m)\), buoyant acceleration decreases as the coupled absolute temperatures increase. Therefore in (25) an increase in absolute \(\theta_m\), independent of a change in \(\Delta\), suppresses evaporation and provides a moistening influence.

The effect of land-atmosphere thermal coupling on soil moisture evolution is given by \(\Lambda_{15}\):

\[
\Lambda_{15} = \bar{\varphi} \sigma_{\Delta} = -\left[ 1 - \left(1 - \beta \right) \left( 1 - \varphi \right) \right] \frac{\rho}{\rho_w} \left[ q^* \left( T_g \right) - q_m \right]
\]

\[
\cdot C_s \sqrt{\frac{T_g - \theta_m}{\theta_m}} \frac{1}{\theta_m} \beta \sigma_T \tag{26}
\]

This necessarily negative term arises from the presence of \(\Delta = T_g - \theta_m\) in the numerator of the buoyancy velocity scale. As the land-air temperature difference increases, thermal buoyancy and consequently evaporation are enhanced.

### 4.2. Soil Temperature

The weighting terms \(\Lambda_{3j}\) in the approximation to \(G_3\) are estimates of the contribution of disequilibrium in each variable \(j\) to the state-dependent deterministic evolution of soil temperature in the absence of fluctuations in wind speed.

The effect of soil moisture disequilibrium is through \(\Lambda_{13}\):

\[
\Lambda_{13} = \frac{\partial G_3}{\partial \delta s} \sigma_s \tag{a}
\]

\[
= -\frac{d\alpha}{ds} \frac{1}{C_s \bar{U}} \sigma_s
\]

\[
- \lambda(C_s \bar{U} + C_w \bar{w}_b) \frac{dq^* (T_g p_s) - q_m}{d\delta s} \frac{1}{C_s \bar{U}} \sigma_s \tag{27}
\]

Here, subterm (a) is the effect of soil moisture–controlled albedo \((\alpha(s))\) on \(dT_g\). Moist soil (high \(\delta s\)) means a less reflective surface; \(d\alpha/d\delta s\) is negative; and thus (a) is positive (in the long run when \(RS > 0\). That is, the contribution of anomalously moist soil through this term is a warming influence. Although albedo may depend only weakly on soil moisture, this term may be large, due to the dominance of solar radiation in the energy balance. Subterm (b) reflects soil moisture control of evaporation. Evaporation efficiency \((\beta(s))\) increases with moister soil \((d\beta/d\delta s\) is positive), so that (b) is typically negative. Thus through this term, moister soil contributes a cooling influence. In (27) the \(s\) dependence of the soil heat capacity \(C_s(s)\) does not appear; this is because that term of the \(\partial G_3/\delta s\) evaluates to zero at \(x^*\).
Disequilibrium in air slab specific humidity \( q_m \) drives soil temperature evolution through \( \Lambda _{32} \):

\[
\Phi _{32} = \frac{\partial G_3}{\partial q_m} \sigma_q
\]

\[
(a) \quad = (1 - \alpha )RS_y \frac{\partial Y_c}{\partial q_m} \frac{1}{CZ_i} \sigma_q
\]

\[
(b) \quad + (1 - e_{\text{col}}) \frac{\partial RL_{\text{ad}}}{\partial q_m} \frac{1}{CZ_i} \sigma_q
\]

\[
(c) \quad - RL_{\text{ad}} \frac{\partial}{\partial q_m} \frac{1}{CZ_i} \sigma_q
\]

\[
(d) \quad + \left( \sigma X_{\text{ad}} \frac{\partial}{\partial q_m} e^2 C_{\text{ad}} \right) \frac{1}{CZ_i} \sigma_q
\]

\[
(e) \quad + \left( RL_{\text{ad}} \frac{\partial}{\partial q_m} X_c \right) \frac{1}{CZ_i} \sigma_q
\]

\[
(f) \quad + \lambda \beta (C_1 U_x + C_2 w_y) \rho \frac{1}{CZ_i} \sigma_q
\]  

(28)

In subterm (a), \( Y_c \) signifies the effect of clouds in blocking solar radiation. The value \( \partial Y_c/\partial q_m \) is negative (more water vapor, more cloud, less solar radiation); therefore a positive air humidity disequilibrium has a cooling effect on soil temperature if this factor is considered alone. Subterm (b) represents the enhancement of above-slab downwelling longwave due to both vapor and cloud; \( RL_{\text{ad}} \) increases with increased humidity, a warming effect; however, this effect is largely cancelled by (c). Subterms (d) and (e) reflect the effect of \( q_m \) on the downward longwave radiation from the mixed-layer to the soil through the clear-sky radiation \( e^2 \) and the cloud correction \( X_c \) that multiplies it. Both derivatives are positive. Finally, an anomalously moist air slab has a warming influence on the soil by suppressing the latent heat flux from the soil slab, so that (f) is a positive influence.

The soil temperature state influences its own steady-forcing evolution through the restoring term \( \Lambda _{33} \):

\[
\Lambda _{33} = \left( \frac{\partial G_3}{\partial T_g} - \bar{\varepsilon} \right) \sigma_T
\]

\[
(a) \quad = -4 \sigma T_g^3 \frac{1}{CZ_i} \sigma_T
\]

\[
(b) \quad - \lambda \beta (C_1 U_x + C_2 w_y) \frac{dq^*}{dT_g} \rho \frac{1}{CZ_i} \sigma_T
\]  

(29)

These terms are both necessarily negative, as expected for a self-restoring state. Term (a) reflects the influence of longwave back radiation, and (b), the dependence of potential evaporation on temperature-dependent saturation specific humidity \( dq^*/dT_g > 0 \). The important roles of soil temperature in the sensible heat flux occur due to the gradient between \( T_g \) and \( \theta_m \), not due to absolute \( T_g \); therefore these terms (indicated by \( \bar{\varepsilon} \)) appear in \( \Lambda _{33} \) below.

Absolute mixed-layer potential temperature affects \( T_g \) through \( \Lambda _{34} \):

\[
\Lambda _{34} = \left( \frac{\partial G_3}{\partial T_g} + \bar{\varepsilon} \right) \sigma_T
\]

\[
(a) \quad = (1 - \alpha )RS_y \frac{\partial Y_c}{\partial \theta_m} \frac{1}{CZ_i} \sigma_T
\]

\[
(b) \quad + \left( \frac{\partial}{\partial \theta_m} RL_{\text{ad}} X_c + RL_{\text{ad}} \frac{\partial X_c}{\partial \theta_m} \right) \frac{1}{CZ_i} \sigma_T
\]

\[
(c) \quad + \left( \frac{\partial}{\partial \theta_m} RL_{\text{ad}} X_c + RL_{\text{ad}} \frac{\partial X_c}{\partial \theta_m} \right) \frac{1}{CZ_i} \sigma_T
\]

\[
(d) \quad + \lambda \beta C_1 \rho (q^* - q_m) \sqrt{\frac{gh}{2}} \frac{T_g - \theta_m}{\theta_m} \frac{1}{CZ_i} \sigma_T
\]

\[
(e) \quad + C_2 \rho C_3 \left( T_g - \theta_m \right) \sqrt{\frac{gh}{2}} \frac{T_g - \theta_m}{\theta_m} \frac{1}{CZ_i} \sigma_T
\]  

(30)

All of these terms are positive, so that a positive potential air temperature disequilibrium contributes a warming influence on the soil temperature through all the mechanisms that depend on absolute \( \theta_m \). Term (a) reflects the role of warmer air in decreasing relative humidity (for fixed \( q_m \)) so that with fewer clouds, the shortwave radiation is increased. Both the overlying atmosphere and slab downwelling longwave fluxes respond to the \( \theta_m \) dependence and to the longwave cloud correction \( X_c \) through terms (b) and (c). An elevated \( \theta_m \) in the buoyancy velocity denominator suppresses the turbulent fluxes, contributing an additional warming influence (d).

The term \( \Lambda _{35} \) gives the effect on soil temperature of a disequilibrium in land-atmosphere thermal coupling:

\[
\Lambda _{35} = \bar{\varepsilon} \sigma_T
\]

\[
(a) \quad = -\rho C_p (C_1 U_x + C_2 w_y) \frac{1}{CZ_i} \sigma_T
\]

\[
(b) \quad - \rho C_p C_3 \left( T_g - \theta_m \right) \sqrt{\frac{gh}{2}} \frac{T_g - \theta_m}{\theta_m} \frac{1}{CZ_i} \sigma_T
\]

\[
(c) \quad - \lambda \beta C_1 \rho C_3 (q^* - q_m) \sqrt{\frac{gh}{2}} \frac{T_g - \theta_m}{\theta_m} \frac{1}{CZ_i} \sigma_T
\]  

(31)

All terms are negative. A large temperature gradient in the surface sublayer (high \( \delta \Delta \)) enhances the turbulent fluxes through (a) the gradient dependence in sensible heat and ((b) and (c)) the transfer coefficient in both the latent and sensible heat fluxes, through the buoyancy velocity. The result is that a positive disequilibrium in \( \Delta \) leads to cooling of the soil layer, and a negative disequilibrium to warming, regardless of the sign of the \( T_g \) disequilibrium.

5. Evaluation of the Term-Wise Decomposition

In this section the component subterms described in section 4 for the mean forcing of soil moisture and temperature states are evaluated. The results are presented in Tables 4 and 5. In accordance with Equation (17), all functions included in the derivatives are evaluated at the equilibrium solution, \( x^* \). This stage of the analysis quantifies the relative strength of the coexisting positive and negative feedbacks in the evolution of moisture and temperature anomalies, for a specified set of deterministic and stochastic forcings.

In Tables 4 and 5 a positive entry in the column labeled "scaled value" means that a positive disequilibrium in variable
j contributes to an increase in variable i through the itemized physical mechanism. A negative value indicates that variable i will decrease due to a positive disequilibrium in variable j.

Only the precipitation recycling mechanism could oppose the self-restoring soil moisture effect in the steady part of soil moisture evolution (see the discussion of (22)). In this case (Table 4) the positive recycling feedback is insufficient to outweigh the strong negative (restoring) feedback mechanisms of s-dependent infiltration and evaporation efficiency. Thus all the \( L_{13} \) terms in the water balance are negative feedback factors. The other four states influence soil moisture deterministically through one physical process each; the temperature dependence of surface saturation specific humidity dominates (\( L_{13} = -0.48 \text{ mm day}^{-1} \)). \( L_{13} \) is important because it contributes to strong climatic feedbacks and intensification of moisture anomalies. From (24) it is evident that a key factor is the response of the saturation specific humidity to temperature deviations (the thermodynamic Clausius-Clapeyron relationship). The direction and strength of the feedback depends on the sign and magnitude of the temperature anomaly.

Disequilibrium of the land-air temperature difference is influential (\( L_{15} = -0.22 \text{ mm day}^{-1} \)) through the dependence of buoyancy velocity (and thus the turbulent surface fluxes) upon that difference. Figure 2 is a summary plot of the terms in Table 4 in the context of the linkages identified in Figure 1. Clearly, the surface water and energy balances are in direct competition for restoring soil moisture and temperature anomalies. Through the saturation specific humidity in the latent heat flux dependence, restoring factors in the water balance are strongly reduced by the tendency for the energy balance to use latent heat flux to restore from a temperature disequilibrium. The temperature disequilibrium in the surface air layer also serves as a positive feedback in the coupled system.

The situation is more complicated for soil temperature evolution (Table 5), where each state contributes through at least two physical mechanisms. Between the two influences of soil moisture in \( L_{31} \), soil moisture control of evaporation efficiency dominates. Of the several influences of atmospheric humidity, the relatively large and negative shortwave cloud contribution is essentially cancelled by several positive terms in \( L_{32} \): downwelling longwave radiation, and vapor deficit. Vapor-enhanced downwelling longwave radiation from above is counteracted by vapor-enhanced reabsorption in the mixed layer; in fact, because of the greater vapor concentration at lower levels, the soil receives less of this radiation overall.

The soil temperature state is strongly self-restoring. Interestingly, this restoration owes more to the temperature dependence of saturation specific humidity than to longwave cooling. The restoring contribution of the ground longwave flux is about half that of the term containing \( \sigma T_g \).

In \( L_{34} \) the decrease in cloudiness due to increased air temperature creates an effect on soil temperature evolution that equals the effect of net downwelling longwave subterms (b) and (c) of (17). The contribution of absolute mixed-layer temperature through the buoyancy velocity is small.

A land-air temperature difference greater than the equilibrium value has a strong cooling influence, \(-1.87^* \text{ day}^{-1}\) through the sensible heat flux (subterms (a) and (b) of (18)) and \(-1.05^* \text{ day}^{-1}\) through latent heat flux (subterm (c) of (18)). The presence of \( \Delta \) in buoyancy velocity contributes \(-1.4^* \text{ day}^{-1}\) (subterms (b) and (c) of (18)), about half of the total \( \Delta \) contribution, demonstrating the importance of free convection to turbulent transfer. Figure 3 provides a summary of the estimates in Table 5 of the detailed factors in the physical linkages identified in Figure 1. There is a strong negative feedback and restoring force associated with anomalies in ground temperature. These factors are mostly associated with energy balance (thermal back radiation and turbulent heat flux). Figure 3 also shows that soil moisture (and the surface water mass balance) as well as the soil temperature link to downwelling thermal radiation in a coupled land-atmosphere

### Table 4. Decomposition of Terms in \( G_1 \) (Soil Moisture Steady Forcing)

<table>
<thead>
<tr>
<th>Term</th>
<th>Multiplies</th>
<th>Subterm</th>
<th>Scaled Value, mm day(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{11} )</td>
<td>( \delta s )</td>
<td>infiltration*</td>
<td>-0.25</td>
</tr>
<tr>
<td>( L_{11} )</td>
<td></td>
<td>recycled precipitation†</td>
<td>-0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>evaporation efficiency‡</td>
<td>-0.36</td>
</tr>
<tr>
<td></td>
<td></td>
<td>sum</td>
<td>-0.64</td>
</tr>
<tr>
<td>( L_{12} )</td>
<td>( \delta q_m )</td>
<td>vapor gradient in potential evaporation</td>
<td>0.08</td>
</tr>
<tr>
<td>( L_{13} )</td>
<td>( \delta T_g )</td>
<td>saturation specific humidity</td>
<td>-0.48</td>
</tr>
<tr>
<td>( L_{14} )</td>
<td>( \delta \theta_m )</td>
<td>buoyancy velocity in potential evaporation</td>
<td>0.00</td>
</tr>
<tr>
<td>( L_{15} )</td>
<td>( \delta \Delta )</td>
<td>buoyancy velocity in potential evaporation</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

*Advection precipitation.
†Including evaporation efficiency and infiltration.
‡Total evaporation loss.

### Table 5. Decomposition of Terms in \( G_3 \) (Soil Temperature Steady Forcing)

<table>
<thead>
<tr>
<th>Term Multiplies</th>
<th>Subterm</th>
<th>Scaled Value, deg day(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{31} )</td>
<td>( \delta s )</td>
<td>albedo</td>
</tr>
<tr>
<td></td>
<td></td>
<td>evaporation efficiency</td>
</tr>
<tr>
<td></td>
<td></td>
<td>sum</td>
</tr>
<tr>
<td>( L_{32} )</td>
<td>( \delta q_m )</td>
<td>SW cloud correction</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LW from above mixed layer*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>column absorption of same</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LW from mixed layer†</td>
</tr>
<tr>
<td></td>
<td></td>
<td>LW from mixed layer‡</td>
</tr>
<tr>
<td></td>
<td></td>
<td>vapor deficit in ( E_p )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>sum</td>
</tr>
<tr>
<td>( L_{33} )</td>
<td>( \delta T_g )</td>
<td>LW from soil saturation specific humidity in ( E_p )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>sum</td>
</tr>
<tr>
<td>( L_{34} )</td>
<td></td>
<td>SW cloud correction</td>
</tr>
<tr>
<td></td>
<td></td>
<td>above-mixed-layer downwelling LW*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>downwelling LW from slab †</td>
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<tr>
<td></td>
<td></td>
<td>buoyancy velocity in ( H )</td>
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<tr>
<td></td>
<td></td>
<td>buoyancy velocity in ( E_p )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>sum</td>
</tr>
<tr>
<td>( L_{35} )</td>
<td>( \delta \Delta )</td>
<td>gradient in sensible heat flux</td>
</tr>
<tr>
<td></td>
<td></td>
<td>buoyancy velocity in ( H )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>buoyancy velocity in ( E_p )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>sum</td>
</tr>
</tbody>
</table>

*Clear sky and cloud.
†Clear sky.
‡Cloud correction.
Figure 2. Detailed and quantified version of Figure 1 for influences on soil moisture, \( s \). Relative magnitudes are noted based on Table 4; signs are reversed with respect to Table 4 where appropriate to account for negative correlation between state variables. Negative values indicate restoring forces or negative feedbacks; positive values represent anomaly-reinforcing factors. Units in millimeters per day.

6. Analysis of Moisture Anomalies

In this section the normalized tendencies to restore or reinforce anomalies in soil moisture and temperature are evaluated for both dry and moist anomalies. A relevant question in the study of land-atmosphere interaction and the local stability of water and energy balance is whether recoveries from dry and moist anomalies are significantly different. For example, does recovery from hot/dry conditions take longer than recovery from cool/moist conditions? Are the strengths of the restoring forces and feedback mechanisms different for each anomaly? Focusing on the land surface, a hydrologic drought condition (dry anomaly) is defined as when the soil moisture state \( s \) is at or below the fifth percentile on its probability distribution. The moist anomaly is defined as when the soil moisture is at or above the ninety-fifth percentile (as by Brubaker and Entekhabi [1996]). The probability distribution is defined by the integration of (7). Significant covariance between the model states results from routing external white noise forcing through the physical linkages in the balance model [Entekhabi and Brubaker, 1995]. Thus the conditions of the three remaining state variables must be weighted according to the joint probability distribution for the system when considering any measure (such as \( \Lambda_{ij} \)) evaluated at the dry or moist soil moisture anomaly states. For the sake of length, only the overall contributions from the \( \Lambda_{ij} \) (and not the component-by-component contributions as in section 5) are presented.

The approximate \( G_1 \) and \( G_3 \) functions, according to (18), are tabulated in Tables 6 and 7. Table 6, which presents the analysis of the steady part of soil moisture evolution, \( G_1 \), will be discussed in detail; the format of Table 7 is identical. The expected value of each disequilibrium conditional on the selected soil moisture anomaly,  

\[
E[\delta_t | s_0] = \frac{E[(x_j - x_0^*) | s_0] s_0}{\sigma}
\]

is computed from the stationary conditional probability density functions and nondimensionalized by the respective stationary standard deviation. The product \( \Lambda_{ij} E[\delta_t] \) is given for each of the four state disequilibria and \( \Delta \). As a product of terms a large entry can result from either a large \( \Lambda_{ij} \) or a large \( \delta_t \), or both. The terms are summed to give the result in the line “Linearized \( G_i \).” To judge the adequacy of the truncated Taylor series approximation, the actual expected value of \( G_i \) conditional on the selected anomalies, “\( E[\text{Nonlinear } G_i] \),” gives the value as computed from the nonlinear stochastic solution.

The dry anomaly represents a soil moisture disequilibrium of \(-1.15 \sigma_s\). The restoring term multiplying \( \delta_s \) is large and negative so that the product of a negative restoring \( \Lambda_{11} \) and negative \( \delta_s \) gives a positive (moistening) contribution to the steady-forcing evolution of \( s \) \((G_1)\). Thus the soil water balance which constitutes \( \Lambda_{11} E[\delta_t] \) is a self-restoring system, with built-in negative feedbacks. However, dry soil is associated with high soil temperature, with an expected \( T_g \) disequilibrium of \( 1.09 \sigma_T \). The coupling term \( \Lambda_{13} \) multiplying \( E[\delta_T] \) is large and negative; when multiplying the positive (warm) \( \delta T_g \), it gives a negative (drying) contribution to \( G_3 \). The coupled energy balance for the land surface contributing to the terms in \( \Lambda_{13} E[\delta_T] \) thus serves as a positive feedback on soil moisture anomalies. Table 6 shows that in this case, the evolution of soil moisture anomalies, the soil water balance is strongly drawn back toward equilibrium, but the energy balance coupled to it provides for an almost equal strength reinforcement of the dry anomaly. Under the moist anomaly state in Table 6, the water and energy balances also work in opposing directions, but with slightly weaker contributions because \( s \) and \( T_g \) are less disequilibrated than in the dry anomaly \((\delta s = 1.05 \text{ and } \delta T_g = -1.02)\).

For the dry anomaly in Table 6, a moist (positive) specific humidity anomaly \((\delta q_m)\) multiplies a positive \( \Lambda_{12} \), a moisten-

Figure 3. Detailed and quantified version of Figure 1 for influences on soil temperature, \( T_g \). Relative magnitudes are noted based on Table 5; signs are reversed with respect to Table 5 where appropriate to account for negative correlation between state variables. Negative values indicate restoring forces or negative feedbacks; positive values represent anomaly-reinforcing factors. Units in degrees per day.
ing effect due to the decreased vapor gradient in potential evaporation. A positive temperature difference disequilibrium, $E[\delta T]$, multiplying a negative $\Lambda_s$ contributes further drying, due to the enhanced turbulent fluxes. As a result of these competing influences, mostly that of soil temperature, the soil moisture’s self-restoring tendency $0.74$ mm day$^{-1}$ is reduced to less than one third of its strength. The exact reverse occurs for the moist anomaly, where a cool disequilibrium in $T_g$ reduces the evaporative demand, contributing to moistening and opposing the restoring term’s drying effect.

Because the dry anomaly is slightly more disequilibrated than the moist anomaly, the magnitude of the self-restoring negative feedback is larger on the dry side. However, the ground temperature is also more disequilibrated for the dry anomaly than for the moist one, contributing a stronger positive feedback on the dry side. In this particular case the net result is a slightly stronger restoring force for the dry anomaly.

All else being equal, a much larger temperature disequilibrium ($\delta T_g$) corresponding to the dry anomaly would strengthen the positive feedback, $\Lambda_s E[\delta T_g]$, perhaps even causing further drying before recovery to normal.

For the steady part of soil temperature evolution ($G_3$) from anomalously dry and moist states (Table 7), the contribution of a strong $T_g$ restoring term, $\Lambda_{33}$, is counteracted by both the $s$ and $\theta_m$ contributions. In the warm/dry case, reduced soil moisture inhibits cooling by suppressing evaporation. It thus contributes a positive feedback on the soil temperature anomaly evolution. Increased mixed-layer air temperature ($\theta_m$ disequilibrium of $1.25\sigma_m$) in the dry soil case also suppresses cooling by reducing the sensible heat flux in addition to warming the soil by longwave radiation. The various products $\Lambda_j E[\delta_j]$ sum to give nearly equal contributions to $G_3$ at the two anomalies, although considered separately, each variable (except $\Delta$) contributes a stronger positive or negative feedback on the dry side, due to the variables’ greater disequilibrium when the soil moisture is anomalously dry.

For the small perturbations from equilibrium analyzed here, the linearized $G_1$ and $G_3$ functions are not very different from the expected values of the nonlinear functions. The nonlinearities are more apparent in the temperature than in the soil moisture steady-forcing functions. For temperature (Table 7) the actual nonlinear function shows greater asymmetry of response between the dry and moist anomalies than does the linearized approximation. For soil moisture (Table 6) the reverse is true. This difference is attributable to the nonlinearity of $g^*(T_g)$, in contrast with the linear form of $\beta(s)$ (see Table 1) in this version of the model.

### Table 6. Terms in $G_1$ (Soil Moisture Steady Forcing)

<table>
<thead>
<tr>
<th>$j$</th>
<th>Variable</th>
<th>Dry Anomaly, $\Lambda_j E[\delta_j]$, mm day$^{-1}$</th>
<th>Moist Anomaly, $\Lambda_j E[\delta_j]$, mm day$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$s$</td>
<td>0.74</td>
<td>-0.67</td>
</tr>
<tr>
<td>2</td>
<td>$q_m$</td>
<td>0.12</td>
<td>-0.10</td>
</tr>
<tr>
<td>3</td>
<td>$T_g$</td>
<td>-0.58</td>
<td>0.54</td>
</tr>
<tr>
<td>4</td>
<td>$\theta_m$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>5</td>
<td>$\Delta$</td>
<td>-0.07</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>linearized $G_1$</td>
<td>0.21</td>
<td>-0.15</td>
</tr>
<tr>
<td></td>
<td>$E[\text{nonlinear } G_1]$</td>
<td>0.19</td>
<td>-0.16</td>
</tr>
</tbody>
</table>

### Table 7. Terms in $G_3$ (Soil Temperature Steady Forcing)

<table>
<thead>
<tr>
<th>$j$</th>
<th>Variable</th>
<th>Dry Anomaly, $\Lambda_j E[\delta_j]$, deg day$^{-1}$</th>
<th>Moist Anomaly, $\Lambda_j E[\delta_j]$, deg day$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$s$</td>
<td>1.02</td>
<td>-0.93</td>
</tr>
<tr>
<td>2</td>
<td>$q_m$</td>
<td>-0.09</td>
<td>0.07</td>
</tr>
<tr>
<td>3</td>
<td>$T_g$</td>
<td>-4.13</td>
<td>3.90</td>
</tr>
<tr>
<td>4</td>
<td>$\theta_m$</td>
<td>1.12</td>
<td>-1.00</td>
</tr>
<tr>
<td>5</td>
<td>$\Delta$</td>
<td>-0.75</td>
<td>0.81</td>
</tr>
<tr>
<td></td>
<td>linearized $G_3$</td>
<td>-2.85</td>
<td>2.87</td>
</tr>
<tr>
<td></td>
<td>$E[\text{nonlinear } G_3]$</td>
<td>-2.64</td>
<td>2.32</td>
</tr>
</tbody>
</table>

### 7. Analysis of Perturbation-Induced Forcing

Similar analysis of the perturbation-forcing function $g(x)$ reveals the effect of coupled wind speed fluctuations and state-variable disequilibrium. This analysis quantifies the relative susceptibility of soil moisture and soil temperature anomalies to fluctuations in wind speed, which affects advection and turbulent fluxes. The wind speed influence is mediated by the soil and air temperature and moisture states, through the fluxes of water and energy that link them.

The perturbation-forcing functions ($g_j$) multiply the random noise in (7) and thus allow the system state at the time of the perturbation to control the perturbation’s effect on each state.

The components of the perturbation-forcing function are expanded in Taylor series about the same $x^*$ as in (17), with the result,

$$g_j(x) = g_j(x^*) + \lambda_{i1}\delta s + \lambda_{i2}\delta q_m + \lambda_{i3}\delta T_g + \lambda_{i4}\delta \theta_m + \lambda_{i5}\delta \Delta.$$  

(32)

In (32), $g_j(x^*)$ must be included because it is not necessarily equal to 0. In fact, for a stochastic solution to exist, it is necessary that at least one $g_j$ be nonzero at $x^*$. If all the $g_j$ were zero at the steady-equilibrium, then noise could not perturb the system from that equilibrium, and once in the equilibrium state the system would no longer evolve either randomly or deterministically. Each $\lambda_{ij}$ in (32) represents the susceptibility of variable $j$ to random perturbations in wind speed through the effect of a disequilibrium in variable $i$. (Again, it is emphasized that the expressions for $\Lambda_j$, like the $\Lambda_j$ in section 3.1, are derived from an analytic model of land-atmosphere interaction, which represents the coupled water and energy balance between the soil and the near-surface atmosphere.)

In interpreting the numerical results for the perturbation-forcing terms, it is important to keep in mind that the perturbations themselves are zero mean and can be either positive or negative with equal probability. This wind anomaly multiplies either a positive or negative disequilibrium in each state (32), through the approximate $g_j$ functions. If $g_j$ (the soil moisture perturbation-forcing function) is positive, for example, then a positive (above-average) wind speed fluctuation contributes a moistening effect to whatever steady wind forcing is occurring at that time, and a negative (below-average) wind speed perturbation contributes a drying effect to the instantaneous steady forcing.
7.1. Soil Moisture

The effect of soil moisture on its own susceptibility to random wind perturbations $\lambda_{11}$ is made up of three contributions:

$$\lambda_{11} = \frac{\partial g_1}{\partial s} \sigma_s$$

(a) $$= + (1 - b) \frac{Mq_w \sigma_u}{\rho_w} d \frac{d}{ds} (1 - \varphi) \sigma_s$$

(b) $$+ (1 - b) \frac{\rho}{\rho_w} (q^* - q_0) C_1 \frac{d}{ds} [(1 - \varphi) \beta] \sigma_u \sigma_s$$

(c) $$- \frac{\rho}{\rho_w} (q^* - q_0) C_1 \frac{d \beta}{ds} \sigma_s$$

The infiltrated fraction of precipitation $(1 - \varphi)$ generally decreases with increasing soil saturation, so that the $s$-dependent infiltration of anomalous advection, subterm (a), is a negative factor multiplying the random noise. As discussed in the modulations of evaporation loss due to noise, subterm (c), is negative; that is, an above-average wind speed would tend to dry the soil, and this drying effect increases with increased saturation, through $\beta$ being an increasing function of $s$.

The magnitude of noise-induced soil moisture perturbations is affected by a disequilibrium in air specific humidity $q_m$ through the direct influence of humidity on potential evaporation,

$$\lambda_{12} = \frac{\partial g_1}{\partial q_m} \sigma_q$$

$$= [1 - (1 - b)(1 - \varphi)] \frac{\rho}{\rho_w} C_1 \sigma_u \beta \sigma_q$$

and by a soil temperature disequilibrium through the temperature dependence of saturation specific humidity,

$$\lambda_{13} = \frac{\partial g_1}{\partial T_s} \sigma_T$$

$$= -[1 - (1 - b)(1 - \varphi)] \frac{\rho}{\rho_w} C_1 \sigma_u \beta \frac{dq^*}{dT_s} \sigma_T$$

The $g(x)$ function does not include buoyancy velocity $w_B$, which has no noise component in this model. Because $w_B$ is the only way that air slab potential temperature affects the evolution of soil moisture, $s$ perturbation forcing is not affected directly by disequilibrium in $\theta_m$ and $\Delta$:

$$\lambda_{14} = 0$$

$$\lambda_{15} = 0$$

The terms in (32) are quantified in Table 8. The influence that a unit variation in wind speed will have on soil moisture when all states start from equilibrium, $g_1(x^*)$, is positive, indicating that above-average wind speed has a moistening influence; that is, the anomalous importation of moisture outweighs the anomalous evaporation at the equilibrium point in state-space. If an anomalously strong wind encounters a wet soil (positive $\delta s$), decreases in percolation rates and evaporation efficiency combine to reduce the equilibrium. If the strong wind encounters a dry soil, then its moistening effect is also decreased due to elevated potential evaporation through $q^*(T_s)$. The effect of anomalously moist air is small and of the opposite sign: suppressed evaporation would allow the wind anomaly to produce slightly greater soil moistening than at equilibrium (Table 8).

7.2. Soil Temperature

The soil moisture dependence of evaporation efficiency and soil heat capacity affect the susceptibility of the temperature state to wind variations:

$$\lambda_{32} = \frac{\partial g_3}{\partial q_m} \sigma_q$$

(a) $$= - \lambda C_1 \sigma_u \rho [q^*(T_v, p_v) - q_m] \frac{d}{ds} \beta \frac{1}{C_s Z_s} \sigma_q$$

(b) $$- [C_1 \sigma_u \rho C_p (T_{v0} - \theta_m) + \lambda \beta C_1 \sigma u \rho [q^*(T_v, p_v) - q_m]] \frac{d C_s}{ds} \frac{1}{C_s Z_s} \sigma_q$$

Because $d \beta / ds$ and $d C_s / ds$ are both positive, subterms (a) and (b) are both negative.

The mixed layer specific humidity moderates the effect of noise on soil temperature through the vapor gradient,

$$\lambda_{33} = \frac{\partial g_3}{\partial T_s} \sigma_T$$

$$= \lambda \beta C_1 \sigma_u \rho \frac{1}{C_s Z_s} \sigma_T$$

The absolute temperature state affects its own sensitivity to wind variability through surface saturation specific humidity in potential evaporation:

$$\lambda_{33} = \left( \frac{\partial g_3}{\partial T_s} \right) \sigma_T$$

$$= - \lambda \beta C_1 \sigma_u \rho \frac{d}{dT_s} q^* \sigma_T$$

Again, because buoyancy velocity does not appear in the perturbation-forcing terms, $T_s$ perturbation forcing is insensitive to absolute mixed-layer potential temperature.
8. Conclusions and Discussion

State-dependent responses in isolation and the steady part of system evolution, this is due largely to the asymmetry of the correlated state disequilibria that correspond to these percentiles of the soil moisture distribution in the stochastic solution.

Table 9. Decomposition of Terms in \( g_s \) (Soil Temperature Perturbation Forcing)

<table>
<thead>
<tr>
<th>Term Multiplies</th>
<th>Subterm</th>
<th>Scaled Value, deg day(^{-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_{33} )</td>
<td>( \delta \theta_m ) vapor deficit in potential evaporation</td>
<td>0.10</td>
</tr>
<tr>
<td>( \lambda_{33} )</td>
<td>( \delta T_s ) saturation specific humidity in sum</td>
<td>0.00</td>
</tr>
<tr>
<td>( \lambda_{33} )</td>
<td>( \delta \theta_m ) (does not appear)</td>
<td>-0.31</td>
</tr>
<tr>
<td>( \lambda_{33} )</td>
<td>( \delta \Delta ) gradient in sensible heat flux</td>
<td>-0.31</td>
</tr>
</tbody>
</table>

\[
\lambda_{34} = \left( \frac{dg_s}{d\theta_m} + \bar{\omega} \right) \sigma_\theta
\]

However, perturbation forcing of \( T_s \) is dependent on the temperature difference, \( \Delta \), through the sensible heat flux:

\[
\lambda_{35} = \bar{\omega} \sigma_\Delta = -\rho C_p \sigma_\Delta \frac{1}{C_{1r} \sigma_g} \sigma_s
\]

For soil temperature (Table 9), \( g_s(x^*) \) is strongly negative, meaning that at equilibrium an anomalously strong wind cools the soil. If the soil is moist, then this cooling is enhanced due to increased evaporation efficiency \( (d\beta/ds) \) but suppressed by greater soil heat capacity. If the anomalously high wind encounters warm soil and/or a strong temperature gradient, wind-induced cooling is enhanced with respect to the equilibrium state’s response. As in the soil moisture case, a disequilibrium in air humidity has a slight counteracting effect by suppressing the perturbation-induced component of evaporation.

Both soil temperature and soil moisture tend to be more susceptible to perturbations in wind speed at the 5% (dry) soil moisture anomaly than at the 95% (moist) anomaly. As in the steady part of system evolution, this is due largely to the asymmetry of the correlated state disequilibria that correspond to these percentiles of the soil moisture distribution in the stochastic solution.

8. Conclusions and Discussion

A basic linearization technique has been combined with a model of two-way land-atmosphere interaction incorporating coupled water and energy balances to analyze how and why feedback mechanisms arise in the linked moisture and temperature states at the land surface and in the lower atmosphere. The findings include the observation that anomalous moisture and temperature states affect each other so as to create mutual positive feedbacks. The contribution of this analysis is to identify and quantify the pathways of those feedbacks. The stochastic solution gives a physically consistent probability density function, incorporating realistic covariances among the states by means of functions that describe, respectively, the system’s state-dependent response to insolation and the steady part of wind speed forcing and its simultaneous state-dependent susceptibility to random perturbations.

The quantitative results presented here are specific to a solution of this particular model, given a particular parameter set. They are illustrative, not universal. The competition between the water mass and energy budgets to set the evaporation rate is a physical reality. This analysis demonstrates that this competition is significant even for small perturbations from an equilibrium climatic state.

Decomposition of the steady- and perturbation-forcing functions into the dependence of each component physical process upon each model state shows the following:

1. Soil moisture control of infiltration and of evaporation efficiency are self-restoring forces of comparable strength for the soil moisture state.
2. The temperature dependence of surface saturation specific humidity is a major factor in reinforcing soil moisture anomalies.
3. The individually strong, multiple (mostly radiative) effects of atmospheric humidity on ground temperature take opposite signs and cancel one another.
4. Soil moisture control of evaporation efficiency is the major mechanism by which the moisture state tends to reinforce temperature anomalies.
5. The temperature dependence of surface saturation specific humidity is a major self-restoring factor for the temperature state, and it exceeds the thermal radiation factor.
6. The buoyancy velocity is a significant recovery factor for temperature anomalies because it affects both the soil temperature and its coupling to air temperature. Soil temperature is positively correlated with temperature gradient anomalies; when the soil is anomalously warm, a strong gradient that enhances cooling also tends to be present.
7. Both the soil moisture and the soil temperature state are more susceptible to variations in wind speed (noise) when the system is dry than when it is moist.

It is well known that the soil moisture and soil temperature states are negatively correlated (cool/moist or warm/dry). These soil states communicate their covariability partially through local-scale interaction with the near-surface atmosphere. Because of the negative correlation between the states, the physical mechanisms that serve as restoring forces for each state individually (soil moisture control of evaporation and temperature dependence of saturation specific humidity) act as anomaly-enhancing positive feedback mechanisms for the other state. These feedback mechanisms are not apparent if the hydrologic variable soil moisture is considered alone. The coupled energy balance and energy states of the soil and near-surface atmosphere must be taken into account when seeking to understand and predict the persistence of hydrologic anomalies.

Notation

\( A_{\text{top}} \) empirical coefficient in expression for sensible heat entrainment at slab top [dimensionless].

\( a \) scaled amount of (mainly water vapor) mass in air column between \( p_v \) and \( p_v \) [cm].

\( b \) partitioning parameter, fraction of incoming water vapor that moistens the air slab; the remainder precipitates [dimensionless].

\( C_1, C_2 \) empirical constants in turbulent transfer coefficient [dimensionless].
\( C_{pa} \) dry air specific heat at constant pressure [J kg\(^{-1}\) deg\(^{-1}\)].
\( C_s \) soil layer bulk heat capacity [J m\(^{-3}\) deg\(^{-1}\)].
\( dw_i \) differential of Wiener process (white noise) [s\(^{1/2}\)].
\( E \) evaporation from the soil into the air slab [kg m\(^{-2}\) s\(^{-1}\)].
\( G \) steady-forcing function.
\( g \) perturbation-forcing function.
\( g \) acceleration of gravity [m s\(^{-2}\)].
\( H \) turbulent flux of sensible heat from soil to air slab [W m\(^{-2}\)].
\( H_{in,out} \) lateral advection of sensible heat into and out of the region [W m\(^{-2}\)].
\( h \) height of mixed layer [m].
\( L \) length scale of region [m].
\( M \) mixed-layer air mass per unit width [kg m\(^{-1}\)].
\( N_c \) cloud fraction [dimensionless].
\( n \) soil porosity [dimensionless].
\( P_h \) atmospheric pressure at air slab top [mbar].
\( p_s \) atmospheric pressure at surface (defined as 1000 mbar).
\( P_{ref} \) reference pressure in \( \theta_m \) [mbar].
\( Q_{in,out} \) lateral advection of water vapor into and out of the region [kg m\(^{-2}\) s\(^{-1}\)].
\( q^* \) saturation specific humidity [(g H\(_2\)O) (kg air\(^{-1}\)].
\( q_{in} \) effective specific humidity of incoming air [(g H\(_2\)O) (kg air\(^{-1}\)].
\( q_m \) model mixed-layer specific humidity [(g H\(_2\)O) (kg air\(^{-1}\)].
\( R_d \) gas constant for dry air [J kg\(^{-1}\) deg\(^{-1}\)].
\( R_v \) gas constant for water vapor [J kg\(^{-1}\) deg\(^{-1}\)].
\( RL_{od} \) downwelling thermal (longwave) radiation flux density at top of mixed layer [W m\(^{-2}\)].
\( RL_{uod} \) upwelling thermal (longwave) radiation flux density from the soil surface [W m\(^{-2}\)].
\( RL_{od} \) downwelling thermal (longwave) radiation flux density from the mixed layer [W m\(^{-2}\)].
\( RL_{sod} \) clear-sky downwelling thermal (longwave) radiation flux density from the mixed layer [W m\(^{-2}\)].
\( RL_{us} \) upwelling thermal (longwave) radiation flux density from the mixed layer [W m\(^{-2}\)].
\( RL_{suc} \) clear-sky upwelling thermal (longwave) radiation flux density from the mixed layer [W m\(^{-2}\)].
\( RS \) solar (shortwave) radiation flux density at soil surface [W m\(^{-2}\)].
\( RS_e \) solar (shortwave) radiation flux density at top of atmosphere [W m\(^{-2}\)].
\( r \) exponent in runoff parameterization [dimensionless].
\( s \) model relative soil saturation (soil moisture) [dimensionless].
\( T_g \) model soil layer temperature [deg].
\( U_z \) mixed-layer wind speed [m s\(^{-1}\)].
\( U \) mean mixed-layer wind speed [m s\(^{-1}\)].
\( w_R \) buoyancy velocity scale [m s\(^{-1}\)].
\( X_c \) correction for cloud in longwave radiation [dimensionless].
\( x^* \) model equilibrium solution with daily-averaged solar forcing.
\( Y_c \) correction for cloud in shortwave radiation [dimensionless].
\( Z_h \) hydrologically active soil depth [m].
\( Z_t \) thermally active soil depth [m].
\( \alpha \) soil-surface albedo [dimensionless].
\( \beta \) evaporation efficiency [dimensionless].
\( \Delta \) surface sublayer temperature difference (\( T_g - \theta_m \)) [deg].
\( \delta x \) nondimensional disequilibrium in variable \( x \).
\( \varepsilon \) effective emissivities of the mixed layer for upwelling and downwelling longwave radiation [dimensionless].
\( \varepsilon_{od} \) bulk longwave emissivity/absorptivity of the mixed layer [dimensionless].
\( \varepsilon \) coefficient in runoff parameterization [dimensionless].
\( \theta \) potential temperature [deg].
\( \theta_m \) model air slab potential temperature [deg].
\( \Lambda_{ij} \) linear coefficient for effect of state variable \( j \) on variable \( i \) in steady forcing.
\( \lambda_{ij} \) linear coefficient for effect of state variable \( j \) on variable \( i \) in perturbation forcing.
\( \lambda \) latent heat of vaporization of water [J kg\(^{-1}\)].
\( \rho \) air density [kg m\(^{-3}\)].
\( \rho_w \) density of liquid water [kg m\(^{-3}\)].
\( \sigma \) Stefan-Boltzmann constant [W m\(^{-2}\) deg\(^{-4}\)].
\( \sigma_x \) stationary standard deviation of variable \( x \).
\( \sigma_a \) standard deviation of mixed-layer wind speed [m s\(^{-1}\)].
\( \varphi \) runoff ratio (runoff/precipitation) [dimensionless].

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