Spatial Optimization of Wind Turbines

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ABSTRACT

The stochastic nature of wind resources renders turbine energy production highly unstable, reducing its market value and penetration. Prior research indicates that increasing the geographic distribution of wind turbines reduces overall production variability. The objective of this research was to develop a robust methodology utilizing Lagrange multipliers and a moving-window penalty method to determine the optimum spatial allocation of turbines to minimize power fluctuations while maximizing energy generation. The developed models are validated against a distributed generation network with uniform distribution of wind turbines, as well as previous studies on interconnected wind farms. The results indicate that the moving-window method is fairly effective at minimizing both short-term and long-term variability, as well as maximizing mean power output and baseload power supply.

KEYWORDS

distributed generation, lagrange multipliers, penalty method, power smoothing, variability reduction

INTRODUCTION

Wind energy is currently the fastest growing form of energy generation due to increasing cost-competitiveness, environmental awareness and political pressure (Kamau, 2009). Advances in energy production technology have reduced the average cost of wind energy from $0.38/KWh in 1982 to $0.04/KWh in 2001, leading the European Wind Energy Association (EWEA) to project that 12% of the world's power demand will be met by wind generation in 2020 (Kang, 2007). Meanwhile, parallel progress in the environmental arena regarding carbon emissions, energy security, and diminishing fossil fuel reserves have prompted social and economic support for wind energy development (Rowlands & Jernigan, 2008). However, despite decreasing production costs and enhanced levels of support across the socio-political spectrum, significant barriers to large-scale market penetration remain.

Whereas conventional power sources produce stable electricity supplies, wind energy exhibits greater volatility due to its stochastic nature (Milligan & Porter, 2005). This temporal variation in production supply leads to reduced market value because wind-produced energy cannot be reliably dispatched or forecasted, allowing power system operators less flexibility in responding to demand fluctuations (DeCarolis & Keith, 2005). As a result, wind energy critics contend that system operators must limit the level of wind energy penetration in the energy market to maintain overall grid stability (Rowlands & Jernigan, 2008). Currently, additional electrical reserves in the form of idling conventional power plants are kept on hand to ensure adequate supplies to meet peak demand (DeCarolis & Keith, 2006). However, such redundancy causes operating inefficiencies and additional overhead costs. Therefore, production smoothing is essential to reduce variability and facilitate large-scale wind integration into the existing grid system.

Wind resource management to smooth production can be accomplished via multiple methods. In recent years, extensive research has been conducted on various forms of energy storage to supplement power output during periods of low wind energy availability and divert excess capacity during periods of high availability (Ahmed, Miyatake, & Al-Othman, 2008). These techniques include the usage of batteries banks, ultracapacitors, super inductors, flywheels, and fuel cell systems (Khalid, 2010). Additional methods to stabilize real-time turbine output for fixed-speed wind generators include active pitch control, braking resistors, static synchronous compensators (STATCOM), and superconducting magnetic energy storage (SMES) systems (Ali & Wu, 2010). The primary disadvantage associated with these stabilization techniques is the prohibitive costs incurred in addition to the already high capital costs for wind turbines (Khalid, 2010).

An alternative to aggregate variability reduction that does not involve energy storage or active stabilization techniques is a distributed generation network. In a distributed generation system, wind developers diversify turbine locations to include multiple interconnected wind farms, thereby reducing the probability that wind speeds will drop simultaneously across all wind farm locations (Rowlands & Jernigan 2008). The effect of interconnected wind farms on output stability was first investigated by Kahn (1979), who analyzed reliability, availability, and effective load-carrying capacities for distributed networks of 2-13 sites in California (Archer and Jacobson, 2007). Kahn found that in general, as the number of sites increases, so does the output reliability. This finding is reinforced by Archer and Jacobson's (2007) study of the benefits of interconnecting wind farms for 19 sites in the Midwestern United States. Archer and Jacobson demonstrated that although most wind energy parameters, including intermittency, improved less than linearly with the number of interconnected sites, there was no saturation of benefits. These findings imply that the aggregate system reliability only improves with additional sites within a distributed network. However, there remains a gap in the literature with regards to the spatial optimization of wind turbines within a distributed network.

Although Cassola, Burlando, Antonelli, & Ratto (2008) developed a simplistic mathematical algorithm to optimize the allocation of wind turbines for minimal variability and maximum energy generation, the proposed methodology is limited by the scale and objectives of their study. For example, Cassola et al. (2008) utilize a brute force approach in which the proportion of wind turbines in each location is incrementally varied. This method is computationally expensive, offers limited resolution, and cannot be extended to large-scale applications involving tens or hundreds of sites. Additionally, the wind speed measurements used in the study are taken at 3 h and 24 h intervals, thereby neglecting short-term variations in wind energy supply. Given the lack of large-scale optimization techniques, wind developers typically optimize the layout of individual wind farms using heuristic methods such as genetic or evolutionary algorithms (Elkinton, 2005). A robust method of wind turbine spatial allocation is needed that is applicable to a wide range of geographical terrain and large datasets. To address this gap in systems-level optimization methodology, the objective of this study is to develop a general algorithm to determine the distribution of wind turbines over given regions that will minimize aggregate output variation and maximize energy generation.

METHODS

Study System

To determine the study system for the test sites, I used the National Renewable Energy Laboratory's (NREL) color-coded U.S. Western Wind Resources map to select the region with the highest prevalence of large wind speeds. The wind speed datasets are generated using Numerical Weather Prediction (NWP) techniques to recreate historical weather conditions in the western U.S. from 2004-2006 (NREL 2010). The criterion of large wind speeds reflects a wind developer's interest in maximizing power generation to enhance commercial viability. Using this guideline, I selected the mountainous region between Cheyenne and Laramie in southwestern Wyoming, which forms a natural corridor for year-round wind gusts averaging 10 m/s from the north. Although the relatively heterogeneous, complex terrain of sharp features and sparse vegetation may enhance modeling errors within the NREL Western Wind Resources dataset due to increased wind turbulence, such errors are inconsequential for the research of variability reduction. In addition, the increased terrain diversity acts to reduce statistical correlation between sites by introducing wind speed variability (Archer and Jacobson, 2007). Lastly, the presence of numerous coal-fired power plants in the region facilitates wind energy integration into existing grid lines.

Data Collection

To determine the individual locations within the study region for analysis, I randomly selected ten sites within the study region to simulate a distributed generation network. This procedure creates a scenario of high generation unpredictability to demonstrate proof of concept of overall power stabilization via the proposed methodology for spatial optimization. The average wind energy density for these sites ranged from 1626.8 W/m^2 to 689.9 W/m^2 (NREL 2010). These figures were obtained from simulated wind speed data corresponding to a height of 100m. The wind speed data used in this study, along with the rated power output (MW), SCORE-lite power output (MW), and corrected SCORE-lite power output (MW) for ten Vesta V-90 3MW commercial wind turbines, are available at ten minute intervals by NREL through the Western Wind and Solar Integration Study. The rated power output is the output determined by the manufacturer's power curve for various wind speeds (NREL 2010).

To reflect the stochastic nature of real wind turbines, the SCORE-lite power output exhibits greater short-term variability while the corrected SCORE-lite output accounts for wind turbine hysteresis at shut-down speeds of 25 m/s or greater (NREL 2010). For this study, I used the corrected SCORE-lite data corresponding to the two year interval from 2004-2005, representing approximately 105,270 data points per site.

Minimization of Power Variability while Maximizing Power Production

To minimize the overall power variability across all wind turbines while maximizing the energy generation, I minimized the ratio of the two quantities (Cassola et al., 2008)

$$\frac{\sum_{i=1}^{n}\left|P_{i,tot}-P_{i-1,tot}\right|}{\sum_{i=1}^{n}P_{i,tot}} \tag{1}$$

Here, the variable $P_{i,tot}$ represents the total power generation in MW across all ten sites at any time instant i, and can be calculated by

$$P_{i,tot} = N(k_1 P_{i,1} + k_2 P_{i,2} + \cdots + k_{10} P_{i,10}) \tag{2}$$

where N is the total number of wind turbines, k is the fraction of wind turbines at locations P_1, P_2, …, P_{10}, and n is the number of 10 minute time intervals from 2004-2005. In Eq. (1), the numerator represents the system variability, as calculated by the difference in total power production squared between adjacent time steps. This definition of variability is used to capture fluctuations in wind power output over short time intervals, which is of particular importance to grid system operators in supplying reliable power. Eq. 1 is minimized by decreasing the system variability or increasing the total energy generation. N is arbitrarily chosen to equal 1 to simplify calculations and calculate an overall proportional allocation.

Mathematical Formulation—Lagrange Multipliers

Given the size of each dataset and the number of sites within the study system, a brute force iterative approach towards minimizing Eq. 1 would be computationally-expensive and time-consuming. I attempted to minimize Eq. 1 using two methods—Lagrange multipliers to obtain a closed-form solution, and a moving-window optimization scheme to numerically approximate the desired k-values. The Lagrange multipliers method was used to determine the local minima of the function

$$\Lambda(x,y,\lambda) = f(x,y) + \lambda g(x,y) - c \tag{2}$$

where

$$f(x,y) = \frac{\sum_{i=1}^{n} |P_{i,tot} - P_{i-1,tot}|}{\sum_{i=1}^{n} P_{i,tot}} \tag{3}$$

$$g(x,y) = k_1 + k_2 + k_3 + k_4 + k_5 + k_6 + k_7 + k_8 + k_9 + k_{10} - 1 \tag{4}$$

The function $g(x,y)$expresses the constraint that the proportional allocation of wind turbines across the entire study system must add up to 1. Analysis of Eq. (2) yields the following simplified system of equations expressed for three study sites

$$\frac{\partial\Lambda}{\partial k_1} = \frac{(k_1y_1+k_2y_2+k_3y_3)x_1-(k_1x_1+k_2x_2+k_3x_3)y_1}{(k_1y_1+k_2y_2+k_3y_3)^2} + \lambda = 0 \tag{5}$$

$$\frac{\partial \Lambda}{\partial k_2} = \frac{(k_1 y_1 + k_2 y_2 + k_3 y_3)x_2 - (k_1 x_1 + k_2 x_2 + k_3 x_3) y_2}{(k_1 y_1 + k_2 y_2 + k_3 y_3)^2} + \lambda = 0 \tag{6}$$

$$\frac{\partial \Lambda}{\partial k_3} = \frac{(k_1 y_1 + k_2 y_2 + k_3 y_3)x_3 - (k_1 x_1 + k_2 x_2 + k_3 x_3) y_3}{(k_1 y_1 + k_2 y_2 + k_3 y_3)^2} + \lambda = 0 \tag{7}$$

$$\frac{\partial \Lambda}{\partial \lambda} = k_1 + k_2 + k_3 - 1 \tag{8}$$

$$x_1 = \sum_{i=1}^{n} (P_{i,1} - P_{i-1,1}) \qquad x_2 = \sum_{i=1}^{n} (P_{i,2} - P_{i-1,2}) \qquad x_3 = \sum_{i=1}^{n} (P_{i,3} - P_{i-1,3}) \tag{9}$$

$$y_1 = \sum_{i=1}^{n} P_{i,1} \qquad y_2 = \sum_{i=1}^{n} P_{i,2} \qquad y_3 = \sum_{i=1}^{n} P_{i,3} \tag{10}$$

where *n* represents the length of the time series from 2004-2005. The *x* and *y* values were determined by using the corrected SCORE-lite values from the Western Wind Resources dataset in MATLAB R2009b (NREL 2010).

Mathematical Formulation—Moving-Window Optimization

The moving-window optimization method was used to determine the optimal k-values that minimized Eq. 1 across multiple time steps. This was accomplished by defining a cost function to reformulate Eq.1 as a series of 10x10 systems of equations

$$C(a) = \frac{1}{2}\{a\}^T[K]\{a\} - \{a\}^T\{R\} - P^*\{a_i\} \tag{11}$$

$$[K]\{a\} = \{R\} \tag{12}$$

$$R_i = \frac{|P_{i,tot} - P_{i-1,tot}|}{P_{i,tot}} \tag{13}$$

$$K_{i,j} = \frac{|P_{i,j} - P_{i-1,j}|}{P_{i,j}} \tag{14}$$

where *a* represents the optimal solution of *k*-values, *R* represents the quantity to be minimized for the entire distributed network at each time step, and *K* is the ratio in Eq. 14 at sites *j* = 1, 2, … 10 for each time step. *K* is therefore a 10x10 matrix, while *a* and *R* are 10x1 vectors. It is necessary to obtain 10 time steps at a time to invert the *K*-matrix. Optimization using the moving-window technique yields a large array of *k*-values corresponding to the optimal solutions for each group of time steps. Because the physical distribution of wind turbines is stationary, the *k*-values are averaged across each test site. For both the Lagrange and moving-window methods, the derived *k*-values are normalized by their sum to satisfy the constraint expressed in Eq. 4.

$$\{k\} = \frac{\{k\}}{\sum_{j=1}^{10} k_j} \tag{15}$$

The term optimal solution is used when referring to Eq. 2 instead of exact solution because the exact solution contains negative entries, which have no physical correspondence to wind turbine placement. Therefore, the optimal solution is the set of positive *k*-values that comes closest to minimizing Eq. 1. Determination of this solution is achieved through the introduction of a penalty term in Eq. 11. The penalty factor P is set to be large enough such that it forces negative R values to approximate 0, yet not so large that it renders the *K*-matrix un-invertible. Eq. 12 is then solved iteratively in MATLAB R2009b until the solution converges within a specified tolerance of 0.001 (Appendix A).

Evaluation of Methodology Effectiveness

To determine the effectiveness of the developed variability reduction methods, I compared the average system power output and generation variability for the evenly distributed generation scenario, the optimized generation scenario, and the single site scenario using the Lagrange multiplier and moving-window optimization methods. The evenly distributed case represents the benchmark against which improvements are measured, without prior optimization. I selected the reference site as the location with the highest average power production. This comparison quantifies the tradeoff in total energy production using the power smoothing method versus the theoretical maximum energy generation over the time period 2004-2005 for real turbines in this system. The system variability is analyzed using the sum of the absolute differences between adjacent time steps, as well as the standard deviation

$$Var_{short} = \frac{\sum_{i=1}^{n}\left|P_{i,tot} - P_{i-1,tot}\right|}{\frac{1}{n}\sum_{i=1}^{n} P_{i,tot}} \tag{16}$$

$$Var_{long} = \frac{\sqrt{\frac{1}{n}\sum_{1}^{n}\left(P_{i,tot} - \frac{1}{n}\sum_{i=1}^{n} P_{i,tot}\right)^2}}{\frac{1}{n}\sum_{i=1}^{n} P_{i,tot}} \tag{17}$$

Eq. 16 represents variability resulting from short-term fluctuations in wind speed, while Eq. 17 represents overall long-term variability. Both metrics are ideally minimized for enhanced supply reliability. Eqs. 16 and 17 are both divided by the mean to reflect the range of fluctuations relative to the total plant output. Because wind energy integration requires high power output and generation stability, I analyzed the power output data for seasonal trends that may contribute to long-term variability. I then compared the baseload power generation corresponding to an 87.5% confidence interval for each distribution scenario. The baseload power represents the minimum power output that is available at least 87.5% of the time and was calculated using a Student's t-value of 1.1504. Finally, I compared the performance metrics corresponding to the calculated *k*-values with those of randomly-generated *k*-values to verify that the proposed methodologies produce true optima.

RESULTS

Mean Power Generation and Variability within Study Sites

Using the NREL Western Wind Resources map, I found that the area bounded by 40.83°N, 104.99°W and 42.16°N, 105.72°W experienced the highest average wind energy density within the study region. From this area, sites 13103, 13583, 14391, 15705, 16242, 18345, 18559, 19149, 19379, and 19501 were randomly selected to form the distributed generation network. Site 19379 exhibited the highest average power output at 14.1339 $\pm$ 11.5760 MW, while site 18559 exhibited the lowest average power output at 9.1869 $\pm$ 10.4860 MW (Table 1). The mean power output of the study sites was 12.1062 $\pm$ 11.2865 MW. The wind turbines for these sampling sites had an average capacity utilization of 41.61% for a mean wind velocity of 9.67 m/s.

Table 1. Individual and mean characteristics of the study sites.

k	Sites	Mean Power Output (MW)	Standard Deviation (MW)	Capacity Utilization (%)	Wind Speed (m/s)	Baseload Power (MW)
1	13103	12.6236	12.3050	44.2	10.8	-1.5320
2	13583	13.5037	11.5647	46.4	10.1	0.1997
3	14391	12.1247	11.0967	41.6	9.6	-0.6410
4	15705	13.4455	11.4845	46.3	10.2	0.2338
5	16242	11.9349	11.0468	41.5	9.3	-0.7733
6	18345	10.8990	10.6182	36.7	8.7	-1.3162
7	18559	9.1869	10.4860	32.0	8.3	-2.8762
8	19149	11.8958	11.8127	40.6	10.0	-1.6935
9	19379	14.1339	11.5760	48.0	10.7	0.8168
10	19501	11.3144	10.8742	38.8	9.0	-1.1952
Mean		12.1062	11.2865	41.61	9.67	-0.8777

Variability Minimization and Generation Maximization

Minimization of Eq.1 using the Lagrange multipliers and moving-window optimization techniques yielded the following non-uniformly distributed *k*-values. The majority of wind turbines using the Lagrange method are concentrated in study site 10 (19501) while sites 1, 3, 5, 7, and 9 (13103, 14391, 16242, 18559, and 19379) contain the least number of wind turbines (Table 2). The moving-window technique produces a much more even distribution of wind turbines, with the largest number of turbines concentrated in site 9 and the least number of turbines concentrated in site 7 (Table 2).

Table 2. Determined *k*-values for evenly-distributed, Lagrange, moving-window, and maximum output scenarios.

***k*-values**	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}
Even	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Lagrange	0	.1507	0	.2497	0	.0002	0	.1560	0	.4434
Moving	.0995	.0980	.1003	.1043	.0973	.0825	.0770	.1119	.1275	.1018
Max Out	0	0	0	0	0	0	0	0	1	0

Evaluation of Methodology Effectiveness

The even distribution of wind turbines within the distributed network is set as the benchmark for comparison in the absence of any optimization methodology. Utilizing the Lagrange distribution of wind turbines, the total mean power generation across the study sites becomes 11.7322 $\pm$ 9.5055 MW. The moving-window distribution produces a mean power generation of 11.7473 $\pm$ 9.1273 MW. The distribution that maximizes the total long-term power generation is concentrated in the single site with the highest average wind energy density. Using the *k*-values for maximum output yields a mean power generation of 14.2534 $\pm$ 11.7063 MW (Table 3).

Table 3. Mean power output, variability, and baseload power generation for evenly-distributed, Lagrange, moving-window and maximum output scenarios. Changes are referenced to the even distribution.

Distribution	**Mean Power Output (MW)**	**Var_{short}**	**Var_{long}**	**Baseload Power (MW)**
Even	12.1062 (+0%)	4764.5 (+0%)	0.7686 (+0%)	1.4025 (+0%)
Lagrange	12.2672 (+1.33%)	7239.6 (+51.95%)	0.7955 (+3.50%)	1.0407 (-25.80%)
Moving-Window	12.2237 (+0.97%)	4726.4 (-0.80%)	0.7658 (-0.36%)	1.4543 (+3.69%)
Maximum Output	14.1339 (+16.75%)	9809.7 (+105.89%)	0.8190 (+6.56%)	0.8168 (-41.76%)

Distributed generation of wind turbines according to the calculated *k*-values produces a smoother total power output profile than concentrated generation for maximum output (Fig. 1). The evenly-distributed, Lagrange, and moving-window scenarios produce 51.43%, 26.20%, and 51.82% reductions in short-term generation variability and 6.15%, 2.87%, and 6.50% reductions in long-term generation variability (Fig. 2). These variability reductions correspond to an average 13.69% decrease in mean total power output. All three distributed scenarios exhibit significantly higher baseload power production than the maximum output scenario (Table 1).

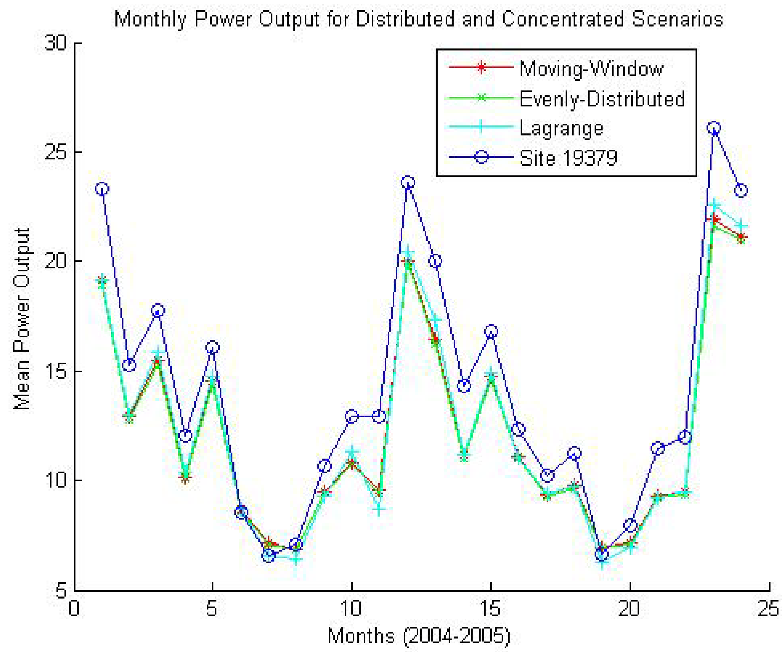

Figure 1. Distributed power output vs concentrated power output. Data is averaged across one month intervals, with month 0 corresponding to Jan. 1, 2004.

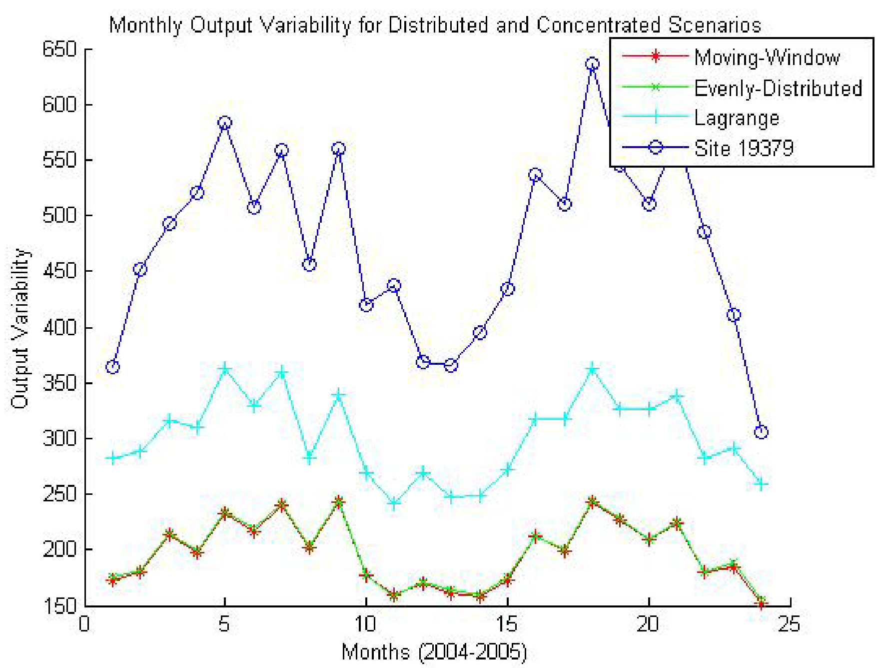

Figure 2. Distributed output variability vs concentrated output variability. Data is averaged across one month intervals.

The shapes of the graphs indicate strong seasonality trends for the magnitude and variability of wind energy density levels (Figs. 1 and 2). Power generation within the study system is maximized during the summer and minimized during the winter. System output variability exhibits an inverse relationship to the power generation, with variability maximized during summer and minimized during the winter (Figs. 1 and 2). The average power output at site 19379 is consistently higher than the other distributions, although it also demonstrates the highest variability. Although the Lagrange scenario roughly approximates the power output of the moving-window and evenly-distributed cases, it exhibits higher short-term and long-term variability (Figs. 1 and 2).

Compared to randomly selected combinations of *k*-values, the moving-window spatial optimization method is the only distribution that reduces the short-term and long-term variability while increasing the mean power output and baseload generation values referenced against the evenly-distributed case (Table 3). Additionally, the moving-window method displays the lowest short-term variability of the distribution permutations (Table 3). The average power output is not particularly sensitive to varying k-combinations, while the output variability is much more sensitive to differing k-values (Figs. 3 and 4).

Table 4. Randomly-generated combinations of k-values.

***k*-values**	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}
Rand1	.1705	.0851	.0528	.0882	.0211	.0288	.2058	.2089	.1257	.0131
Rand2	.0570	.0858	.1995	.0037	.0105	.0411	.1577	.1778	.1574	.1096
Rand3	.1215	.0658	.1654	.0420	.1525	.0408	.0818	.1389	.1733	.0180
Rand4	.1546	.1290	.0810	.0725	.0743	.0510	.0846	.0849	.1360	.1322
Rand5	.1024	.0602	.1290	.0847	.0557	.1492	.1392	.0874	.0989	.0933

Table 5. Mean power output, variability, and baseload power generation for random combinations of k-values. Changes are referenced to the even distribution.

Distribution	**Mean Power Output (MW)**	Var_{short}	Var_{long}	**Baseload Power (MW)**
Even	12.1062 (+0%)	4764.5 (+0%)	0.7686 (+0%)	1.4025 (+0%)
Moving	12.2237 (+0.97%)	4726.4 (-0.80%)	0.7658 (-0.36%)	1.4543 (+1.35%)
Rand1	11.9938 (-0.93%)	5449.3 (+14.37%)	0.7911 (+2.93%)	1.0782 (-23.12%)

Rand2	11.9475 (-1.31%)	5450.8 (+14.40)	0.7927 (+3.14%)	1.0521 (-24.98%)
Rand3	12.3139 (-1.72%)	5223.2 (+9.63%)	0.7695 (+0.12%)	1.4132 (+0.76%)
Rand4	12.2971 (+1.58%)	4890.5 (+2.64%)	0.7579 (-1.4%)	1.5759 (+12.36%)
Rand5	11.8713 (-1.94%)	4965.7 (+4.22%)	0.7820 (+1.75%)	1.1913 (-15.06%)

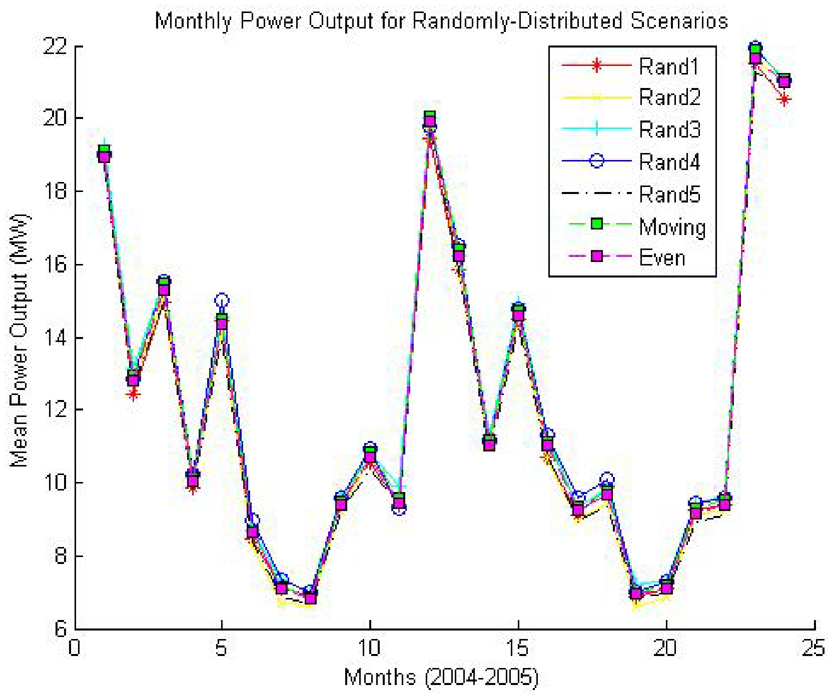

Figure 3. Power production profiles for random *k* distributions.

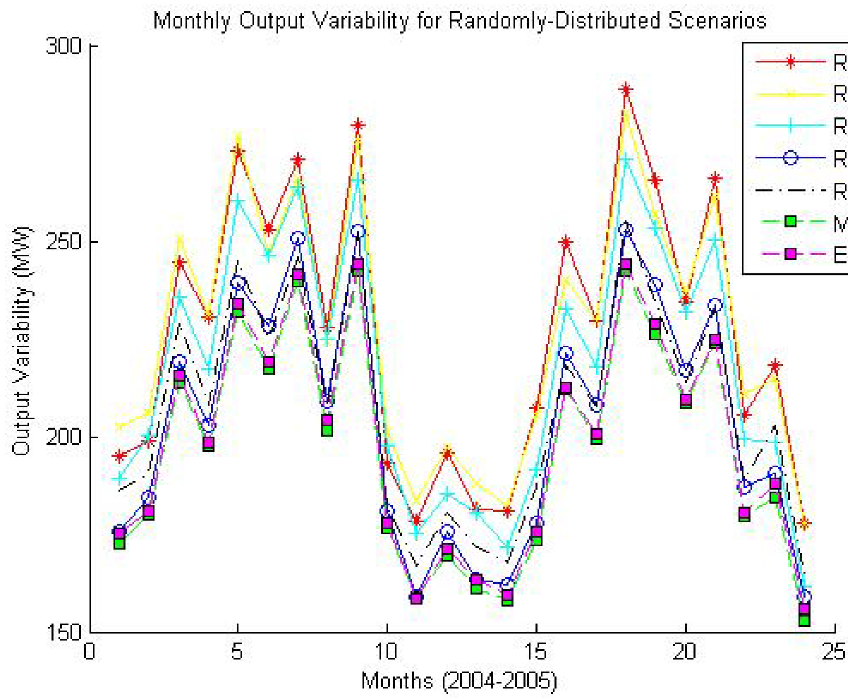

Figure 4. Output variability profiles for random *k* distributions.

DISCUSSION

Research on the spatial optimization of wind turbines is motivated by the recognition that wind energy intermittency represents a major barrier to large-scale integration into the existing electricity grid (Rowlands & Jernigan, 2008; DeCarolis & Keith, 2006). Such variations in wind energy generation can cause significant problems with frequency and voltage control, place large peak loads on transmission loads, and necessitate the usage of backup systems for smoothing periods of low energy production (Khalid, 2009). As a result of this production unreliability, grid operators typically pay less for wind power than energy from conventional sources (Hoste, Dvorak, & Jacobson, 2009). Although optimization techniques currently exist for situating wind turbines, they generally lack sufficient scale and focus on maximizing energy generation while minimizing installation costs (Mosetti, Poloni, & Diviacco, 1993). To address this knowledge gap, the objective of this study was to develop robust, mathematical algorithms for determining the optimal spatial configuration of wind turbines to minimize production variability while maximizing output. As such, the effectiveness of the algorithms has been evaluated by the resulting increase in mean power output and baseload power generation across the distributed network, as well the decrease in short-term and long-term variability.

Study System

The selected study sites displayed a large amount of long term, short term and spatial variability in average wind energy generation. For example, even the optimal moving-window *k*-values exhibited month-to-month fluctuations of up to 12.3822 MW, with average energy generation peaking at 21.9040 MW during the winter and dipping to 6.8662 MW during the summer (Figs. 2 and 4). Conversely, the short-term variability peaks during the summer months and decreases dips during the winter months. These trends suggest that winter months experience sustained periods of high wind speeds, while summer months experience sporadic low wind speeds. The magnitude and range of these fluctuations over time suggests an important consideration: the distributed generation alone approach cannot stabilize long-term power output within this study region and reflects a physical limitation of wind-derived energy. As such, the natural seasonal variability of wind resources may require additional power reserves to provide consistent baseload power for grid stability (Khalid, 2009; Milligan & Artig, 2008). These reserve requirements will likely be greatest during the summer months when the baseload power supply is the lowest and energy demand is the highest (Hoste et al., 2009). Potential avenues of further power smoothing include large-scale energy storage and individual turbine control (Archer & Jacobson, 2003; Ali & Wu, 2010)

Variability Minimization and Generation Maximization

Variability minimization using the Lagrange multipliers and moving-window optimization methods yielded a closed-form and numerical approximations to a set of non-uniformly distributed *k*-values. The non-uniform distribution of wind turbines agrees with the brute force computational model developed by Cassola et al. (2008) in which the k-values are incrementally varied across the test sites for each iteration. The distribution of values is an effective reality check on the functionality of the developed algorithms because the probability that the wind speed will drop simultaneously across multiple sites decreases with greater geographical variance, and non-uniform *k*-values are expected given the differing amounts of generation variability at each test location (Rowlands & Jernigan, 2008).

The degree of non-uniformity within the results is related to the variability of the wind energy density across the study sites—with perfectly uniform and static wind speeds, the allocation of wind turbines across the study sites would be the same. The variability of wind speeds is in turn related to the homogeneity of the terrain and spread of the test sites. With larger test sites and more diverse terrain, the statistical correlation between site pairs decreases nonlinearly (Archer & Jacobson, 2007; Kahn, 1979). The moving-window distribution of *k*-values also logically follows from the physical parameters in Table 1: the minimum *k*-value (k_7) corresponds to the site with the lowest mean power output (site 7), while the maximum *k*-value (k_9) corresponds to the site with the highest mean power output (site 9) (Tables 1 and 2).

In Table 2, the optimized sets of k-values contain positive entries that are close to zero. These values correspond to the negative entries in the solution when initially solving the system of equations. Application of the penalty method forces these entries to approximate zero within a specified tolerance. The observed reduction in short-term and long-term variability with increasing geographic diversity agrees with the body of literature on distributed generation (Milligan & Artig, 1998; Milligan & Porter, 2005). However, the marginal variability reductions diminish with increasing generation sites (Kahn, 1979; Katzenstein, 2010). As expected, the distribution that maximizes total power generation concentrates all wind turbines in the single location (site 9) with the highest average wind energy density (Cassola et al. 2008). It should be noted that site 9 represents a particularly attractive location for wind turbines as it exhibits the highest average power generation with relatively moderate output variability (Table 1).

Methodology Performance

Compared to the maximum output scenario, the moving-window optimization scheme yielded 51.82% and 6.50% reductions in short-term and long-term variability at the expense of a 13.52% reduction in average power output. The Lagrange distribution, however, only achieved 26.20% and 2.87% reductions in variability with a 13.21% decrease in average power output. The moving-window scenario is the only method that improved performance across all parameters compared to the evenly-distributed scenario, with increases of 0.97% and 3.69% in mean power output and baseload power production, along with decreases of 0.80% and 0.36% in short-term and long-term variability (Tables 2 and 4). Although the improvements are marginal, they are significant when considering the aggregate power output of large wind farms as well as the frequency and magnitude of wind speed fluctuations. Because each site exhibits varying generation profiles and output variability, the magnitude of potential improvements will vary depending upon the sites within the distributed network (Archer and Jacobson, 2007). These results indicate that the moving-window optimization scheme is an effective method of output maximization and variability reduction.

It is unsurprising that a perfectly uniform distribution of k-values should be close to the calculated optimum, given the stochastic nature of wind availability and the large sampling time frame. The effect of widespread turbine distribution is reflected in the relatively poor performance of the Lagrange-derived *k*-values, which concentrated wind turbines in four primary locations with an emphasis on site 10 (Table 2). The shortcomings of the Lagrange multipliers method may be attributable to the penalty method itself, as well as the formulation used in Eq. 3. This arises from the sensitivity of the convergence rate and solution of the original matrix equation to the penalty parameter (Babuska, 1973). Furthermore, the x and y values used in Eqns. 5-7 are calculated over two years as opposed to smaller time steps. This is problematic because data resolution is lost with the decreasing influence of individual fluctuations. A more robust method may be to solve for the optimal *k*-values across multiple time steps and average the resulting values over the sampling time frame.

Methodology Comparison

Through their brute force approach towards minimizing Eq. 1, Cassola et al. (2008) were able to achieve a 58% variability reduction at the expense of a 23% reduction in maximum power production, compared to the maximum output scenario. This variability, however, is defined as the sum of the absolute differences between adjacent time steps and does not differentiate between short-term and long-term variability. In addition, the authors were able to capture 16% of annual averaged wind power as the baseload for the electric grid (Cassola et al., 2008). In comparison, Archer & Jacobson (2007) were able to achieve baseloads of 14% and 23% of annual averaged wind power for 7 and 11 interconnected wind farms. These baseload values were calculated for 87.5% availability, which corresponds to the average amount of time from 2000-2004 that coal plants in the U.S. were free from scheduled or unscheduled maintenance (Giebel, 2007). The moving-window *k*-values determined in this study produced an 87.5% availability baseload power of 1.4543 MW, corresponding to approximately 11.90% of mean annual production.

Although the results appear similar, it is not possible to directly compare the methodology effectiveness because both methods utilize different sets of empirical data and may introduce additional error in the methodology analysis. A more effective comparison would be to evaluate the variability reduction achieved by both methods using the same data set. However, a brute force iterative method for 10 test sites is computationally-expensive because of the logarithmic scaling of the number of calculations required to assess all possible k-combinations in 0.01 increments (Cassola et al., 2008). Advantages of the moving-window method include greater scalability to accommodate more sites, as well as increased resolution of distribution proportions.

Direct comparison to the heuristic models used for individual farm layout would be uninformative given the differing optimization objectives and methods of implementation. Traditional genetic or evolutionary algorithms as applied to wind farms are based on the recognition that wind turbines can affect each other during operation through wake formations of disrupted air flow caused by individual turbines (Mosetti, Poloni, & Diviacco, 1993). These algorithms attempt to maximize wind farm efficiency by minimizing turbine interactions (Mosetti et al., 1993). Maximization of wind energy value is then achieved by combining turbine cost and wake models with an optimization scheme to determine the ideal layout (Elkinton, 2005).

The advantage of genetic algorithms is that they are more likely to identify global optima instead of local optima (Mosetti et al. 1993). However, the objective of these genetic algorithms is typically to situate wind turbines within a single wind farm for energy maximization, as opposed to minimizing the generation variability across multiple wind farms (Marmidis, Lazarou, & Pyrgioti 2008). As a result, the assumptions used in the creation of these models vary. For example, the genetic algorithm and Monte Carlo models assume a single wind direction with constant wind speed and intensity instead of utilizing actual wind data because the wind speed variability does not significantly affect the occurrence of turbine interactions (Marmidis et al., 2008). Given the increases in baseload power and mean power output, as well as the decreases in short and long-term variability, I conclude that the moving-window method is an effective and scalable means of maximizing system output while minimizing production variability.

Limitations

Successful application of the optimization procedure developed in this study is contingent upon access to extensive wind data. In this study, the wind simulation data has been generously supplied by the National Renewable Energy Laboratory as part of the Western Wind and Solar Integration Study (NREL, 2010). However, because the optimization methodology relies upon accurate power generation data, sources of error include the modeling inaccuracies using Numerical Weather Prediction techniques (NREL, 2010). Additionally, this methodology assumes that two years of training data is a sufficient indicator of regional wind behavior patterns, and may not account for long-term trends, which may be variable on a scale of years to decades depending on the regional climate trends of the proposed study area. Finally, the penalty method used to constrain the calculated distributions to positive values is highly sensitive to the choice of penalty parameter and may produce variable results depending upon the parameter used (Babuska, 1973). Further improvements to the study design include the use of more consistent methods of constrained nonlinear optimization, as well as a direct comparison with Cassola et al. (2008) and Archer & Jacobson (2007) to more rigorously analyze methodology effectiveness.

Further Research

Given the observed variability reduction and baseload power increases, future research include combining the moving-window method with wake and cost models to account for turbine-turbine interactions and reflect realistic operating constraints (Elkinton, 2005). Wake losses are important considerations for wind farm layout optimization because the linearly-expanding wake behind a turbine reduces the free stream wind speed within that region, decreasing turbine output (Kusiak, 2010). Further study is also needed to investigate the spatial distribution of wind speed over complex terrain (Uchida, 2008). This research has important applications for turbine layout optimization in regions where wind speed data are unavailable. Although the brute force iterative method cannot be scaled to large applications involving many test sites, a direct comparison of the brute force and Lagrange multipliers method utilizing identical turbine data may also provide additional information as to which method is preferable for small-scale applications.

Broader Implications and Conclusions

This study demonstrates two novel methods for determining the optimal spatial allocation of wind turbines for maximum energy production and minimum generation variability using Lagrange multipliers and a moving-window penalty method. The stochastic nature of wind energy limits its market penetration resulting from unreliable supply and subsequent grid instability. Therefore, minimizing generation variability is essential towards increasing the market value and integration of wind energy into the existing electrical infrastructure. Currently, no robust methodologies exist to optimize the layouts of large-scale, distributed generation networks given high-resolution wind speed data. This study demonstrates that the moving-window method is fairly effective at minimizing short-term and long-term variability, while maximizing power output and baseload supply. Implications of this research include enhanced wind energy market penetration stemming from increased supply reliability and reduced wind energy prices. The advancement of wind energy towards grid parity with coal will help enable the shift from a carbon-based economy.

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