Snowpack in California's Sierra Nevada: Modeling and Prediction of Water Supply

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ABSTRACT

Maritime snow climates like that of the California Sierra Nevada are especially susceptible to global warming and the threat of declining seasonal snowpack. California's fresh water supply depends upon seasonal snowpack as a source of summertime water storage, and future water resource management necessitates the long-term prediction of snowpack. Current models are not accurate enough to confidently plan for possible future drought conditions. I collected snowpack and climate data from the past thirty years and created a model of seasonal snowpack for the Stanislaus River watershed. This allowed me to predict seasonal SWE for the Stanislaus River watershed and assess the feasibility of applying such a model to data-poor regions in more remote areas of the Sierra Nevada. Within the Stanislaus watershed the model had a total average error of 8.9%, and it predicted maximum seasonal SWE within 20% of measured values in 78% of all cases. It was most accurate for low to mid-elevation (<9000ft) western slopes, and least accurate for high elevation eastern slopes. When tested as a range-wide predictive model, it predicted maximum seasonal SWE within 30% of measured values in 62% of all cases. It was most accurate for west-facing drainage basins in the central Sierra, and least accurate for the far southern and eastern Sierra. The model was highly correlated with runoff measurements in central Sierran drainages, proving its usability as a water resource management tool. The long-term scope and snowpack specificity of this model fills the previous gap in predictive capacity between short-term mass balance snow accumulation/ablation models and long-term climate models.

KEYWORDS

El Nino Southern Oscillation (ENSO), Pacific Decadal Oscillation (PDO), Pacific-North American Pattern (PNA), Snow-Water Equivalence (SWE), Stanislaus

INTRODUCTION

Global warming raises concerns about the future security of fresh water supplies. Regions that depend almost entirely on seasonal snowmelt as a source of fresh water will be affected most heavily. However, warming on a global scale does not affect all snowpack regimes equally. Maritime snow climates like those seen in the Cascades and European Alps show a large reduction in seasonal snowpack in response to warming (Serreze et al. 1999), while continental snowpacks located farther inland have more variable and inconclusive responses (Harpold et al. 2012). Recent modeling of European alpine snow-cover temperature sensitivity in such maritime ranges predicted that, for every 1°C of warming, there would be a maximum reduction in snow cover similar to a 700m decrease in elevation (Hantel and Hirtl-Wielke 2007). This indicates a conspicuous capacity for reduction in maritime alpine snowpack with continued warming.

California's Sierra Nevada is one such maritime mountain range, and while its seasonal snowpack has predictably decreased over the past half-century (Knowles et al. 2006), many of the finer details and mechanisms behind this change are still unclear. One such mystery relates to altitude buffering, whereby increased altitude disproportionately lessens the effects of warming. Warming actually resulted in *increased* snow water equivalent (SWE) — a measure of the snowpack's water content — at higher elevations in the Sierra over the past 30 years (Howat and Tulaczyk 2005). In the Tuolumne and Merced drainage basins, which drain the Sierra Nevada near Yosemite, such buffering was also seen for snow-cover in addition to SWE (Rice et al. 2011). Currently, no theories have attempted to describe the reasons for this buffering effect. Nevertheless, even though high elevations from 2100-3000m contribute 40-60% of annual snowmelt, the greater susceptibility of lower elevations to climate change will still likely cause an overall reduction in seasonal runoff (Rice et al. 2011).

Runoff measurements corroborate the earlier prediction of a general reduction in snow-derived water, but also indicate the seasonality of future water shortages. Headwater catchments in the Kings River drainage basin — another west-facing basin slightly farther south than the Tuolumne or Merced — have indicated that peak discharge lags peak SWE by 60 days at high elevations, but only by 20-30 days at lower elevation (Hunsaker et al. 2012). This indicates that climate warming would result in *earlier* runoff. In addition, an increase in the winter rain:snow precipitation ratio at lower elevations would directly increase early-season runoff as well as lengthen the growing season, which in turn would result in a lower runoff:precipitation ratio (Hunsaker et al. 2012). All of this adds up to less water availability during summer, the peak water-use season.

A long-term decline in snowpack presents a pressing problem for California and necessitates more well informed management practices. Many highly populated areas, such as southern and central California, depend almost entirely on seasonal snowmelt for fresh water. In addition, water demand will continue to rise with growing human population, especially in large cities like Los Angeles and San Francisco (National Science and Technology Council 2008). To effectively prepare for the future, water managers need tools to predict future water supply (Forest Service 2008). However, current models of snowpack are too non-specific or lack the requisite long-term predictive capabilities. For example, the aforementioned model created by Hantel and Hirtl-Wielke using data from the Austrian Alps only predicts snow cover, not SWE (2007). Snow cover is not effective for predicting overall seasonal runoff volume, not to mention that such a model may not be accurate when applied to a different mountain range. In the United States, the premiere and ubiquitous snowpack model — the government-funded Snow Data Assimilation System, or SNODAS — similarly proves unfit for future projection of Sierra Nevada snowpack. SNODAS incorporates thousands of data points from multiple platforms including remote sensing into an advanced energy-and-mass-balanced snow model called SNTHERM (USACE 2011), making it highly accurate, but it only creates predictions of snowpack variables one day in advance (Barrett 2003). This limited prediction range curtails the model's usefulness from a water resource management perspective. A new region-specific long-term model would prove invaluable for building a conservation framework for a single watershed or the entire range (Forest Service 2008).

I collected snowpack and climate data from the past thirty years and created a seasonal snowpack model for the Stanislaus River watershed. I selected this watershed because of its data richness and site distribution representing a range of physiographic variables. This model allows us to (1) describe current and long-term trends in snowpack for the Stanislaus River watershed, (2) predict seasonal SWE for the Stanislaus River watershed, and (3) assess the feasibility of applying such a model to data-poor regions in more remote areas of the Sierra Nevada.

METHODS

Raw data collection

To account for all the potential factors affecting seasonal SWE, I assembled data from different locations representing a wide range of physiographic conditions. Within the Stanislaus watershed, I gathered daily SWE readings from all fifteen California Department of Water Resources snow courses and two NOAA Snow Telemetry (SNOTEL) sensors, also noting the elevation and geographic coordinates of each data collection site. I linearly interpolated single-day misreadings if there were relatively few during the snow year (<20), and eliminated all data from the snow year if there were larger data gaps.

To obtain a numerical approximation of global climate I recorded the Multivariate ENSO Index (MEI), the Pacific Decadal Oscillation (PDO) index, and the Pacific North American Pattern (PNA) index at a monthly resolution. MEI values approximate the condition of the El Niño Southern Oscillation (ENSO) by accounting for sea-level pressure, zonal and meridional components of surface wind, sea surface temperature, surface air temperature, and total cloudiness fraction of the sky. PDO index values approximate the condition of the more northerly Pacific Decadal Oscillation by similar means. PNA index values approximate the location/shape of the Pacific and Polar jet streams over North America.

Data transformation - creating model inputs

To make my raw data useful as inputs for my model I used the geographic coordinates of data collection sites to determine slope, aspect, and tree cover. I first plotted all 17 sites on Google satellite imagery to screen for accuracy. I rectified coordinates in cases when a sensor apparatus was clearly visible on satellite imagery and the location conflicted with the given coordinates. To determine slope and aspect of each site I plotted the rectified site coordinates in ArcGIS on a USGS Lidar[1]-derived terrain model, extrapolated the tangent plane at that point, and mathematically determined slope and aspect in degrees. For tree cover I created a

[1] Lidar, short for "Light Radar," uses a laser to record distances, which can be translated into very accurate surface and terrain models.

categorical variable, classifying site locations by full tree cover, partial cover (including shrubs and less-protective foliage), or no cover. I determined this using 1m resolution National Agriculture Imagery Program orthophotos.

Model characterization

I created my model in two stages because I did not have enough processing power to analyze all my variables at once. For Stage I, I used data from only one collection site at a time and produced a unique SWE-predictive equation for each site. I repeated the Stage I modeling process 17 times, once for each data collection site. For Stage II, I incorporated all of the site-specific equations produced in Stage I along with the physiographic variables of each site to create a single SWE-predictive equation for the entire watershed.

Stage I

Inputs: *daily SWE readings, date, MEI, PDO, & PNA index calculations,*

Output: *Gaussian-based site-specific function predicting seasonal SWE*

I used a fourth-order skewed gaussian function as the basis for my model, fitting it to SWE data with date as the main independent variable.[2] I allowed MEI, PDO, and PNA indices to affect both the skewness and amplitude of the function, reflecting the effect of global climate on the seasonality and volume of precipitation. I used a Gauss-Newton algorithm to run a non-linear regression against SWE readings using the total residuals over the 30 year dataset as a measure of fit.

Stage II

Inputs: *Stage I derived parameters, elevation, aspect, slope, tree cover*

Output: *Watershed-specific function predicting seasonal SWE*

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[2] See Appendix A for full description of gaussian-based function and modeling process.

Stage II built on Stage I by using the numerical constants derived from Stage I functions as regression data. I allowed elevation, aspect, slope, and tree cover to affect the function's amplitude based on these variables' effects on overall SWE and effective seasonal duration. I treated the parameters derived from Stage I as constants, letting each function become a solvable expression with an arbitrary numerical answer. I then ran a linear regression on these arbitrary values using residuals as a measure of fit. I also ran individual regressions for each Stage II variable to determine correlational strength. Ultimately, Stage II produced a single watershed-wide function with date, MEI, PDO, PNA, elevation, aspect, slope, and tree cover as independent variables, and SWE as the dependent variable.

RESULTS

Model inputs

NRCS and CA DWR data provided reliable daily SWE readings with relatively few data gaps. 89% of all snow year data did not contain data gaps. The abundance of interpolatable data gaps such as single-day misreadings, malfunctions, or flagged data points increased with the age of the data, with almost no data gaps present in data from the past 10 years. In almost all cases, snow years that were eliminated due to large data gaps contained greater than 80% missing or flagged data points. 47% of eliminated yearly data did not contain any SWE readings at all.

My data sites represented a wide range of physiographic variables, including slopes from 0° to 17°, aspects encompassing every major direction, and elevations from 6500ft to 9300ft (Figure 1). Tree cover was relatively evenly distributed, with each of the three categories containing at least 24% of all data points. No site variables were significant outliers. Rectification of site coordinates proved necessary in only 34% of all cases, always on the basis of positive visual site identification.

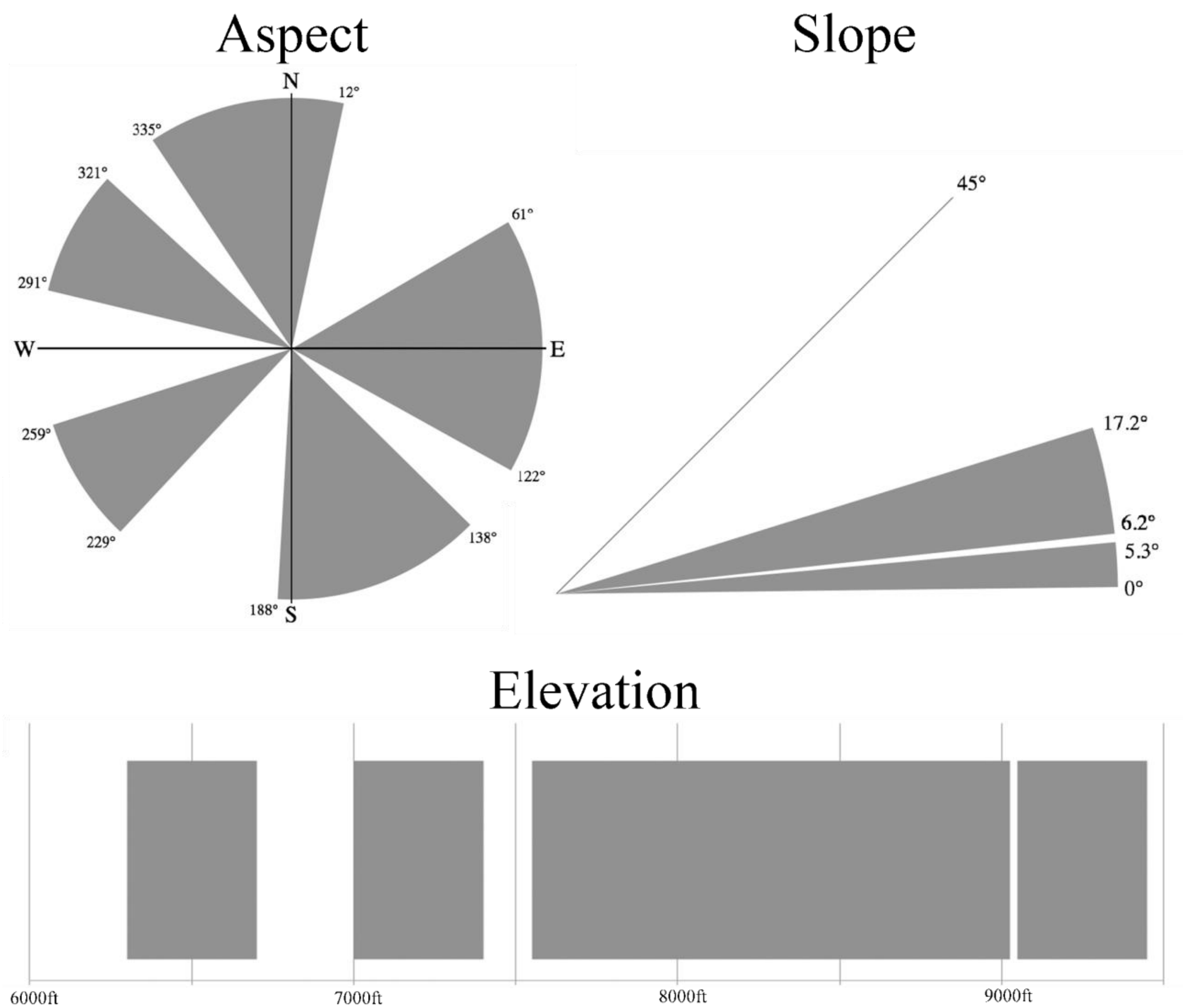

Figure 1. Range of data site physiographic variables. Grey areas depict values that are well represented by the dataset, as determined by absolute error in calculation. In terms of relative error, these ranges are: ±0.5% for slope, ±4.0% for aspect, and ±2.5% for elevation.

Stage I results

My Stage I models fit my data with a high degree of accuracy, with average error ranging from only 4.4% to 9.8%. My input variables tended to fit yearly ideal parameters with a varying degree of accuracy, however final Stage I parameters resulted in a low residual-derived average error (shown for a single site in Table 1 and Figure 2). Correlation analysis combined with a sense of how the input variables *should* affect snowfall lead to my final Stage I equations. Epsilon was correlated with alpha, likely because both terms describe the seasonality of maximum SWE in different ways. Omega correlated strongly with R, which made sense because both variables are essentially scale terms for the amplitude of the SWE curve. Because the overall amplitude should be a product of the *average* global climate status for that year, I defined R in terms of the integral of the seasonal MEI, PDO, and PNA equations. This ended up correlating well with ideal R and omega values. Likewise, alpha should be a product of the overall *trend* in the ENSO status, so I defined it in terms of a variable M, the slope of a linear least-squares regression of the seasonal MEI, PDO, and PNA values. This correlated well with ideal alpha and epsilon values.

Table 1. Stage I parameters and average error at Sonora Pass site.

	P_a	P_b	P_g	P_h	P_i	P_j	P_k	P_l	P_m
R	-	-	0.0203777 39	0.170	0.029	388.3	-	-	-
Ω	0.0015	3.44	-	-	-	-	-	-	-
E	-133.8	121.8	-	-	-	-	-	-	-
A	-	-	0.648	0.139	-0.108	0.057	-0.691	-0.132	0.099
Average Error	-8.26%								

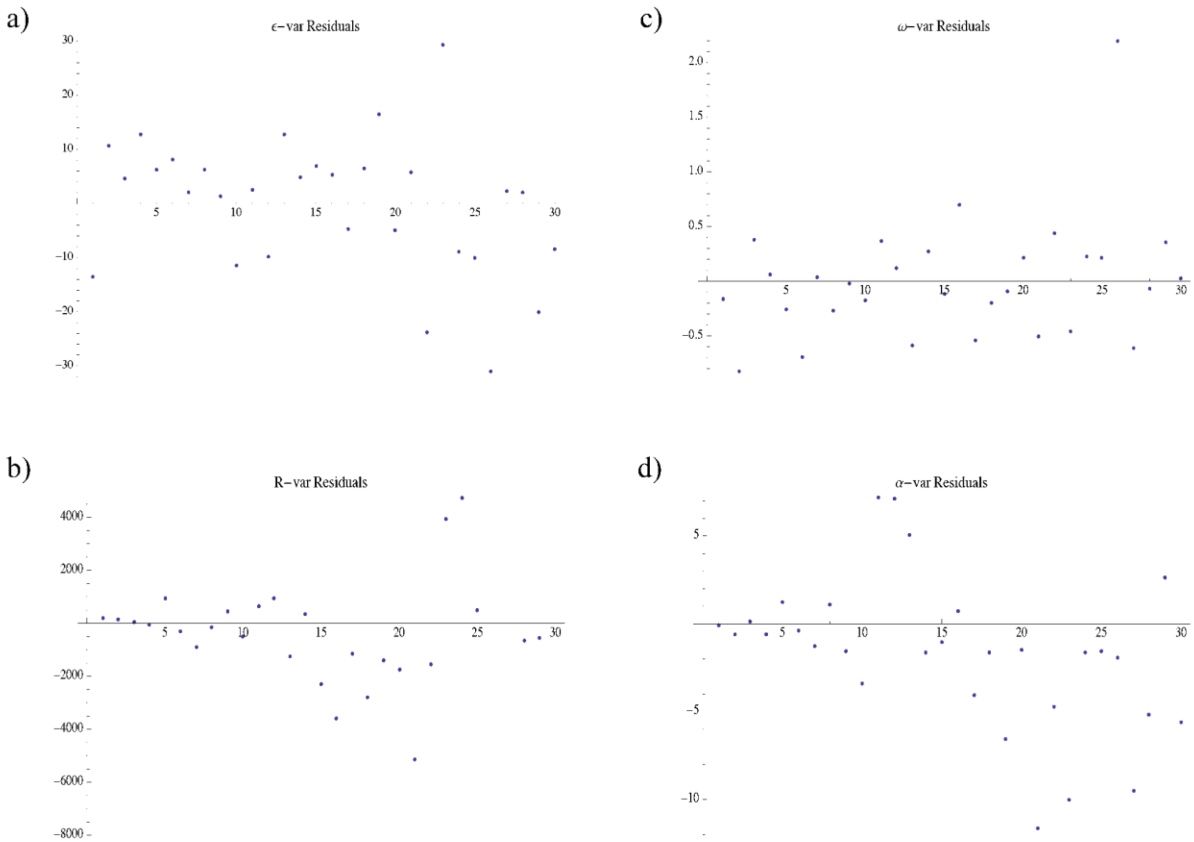

Figure 2. Residuals when fitting ideal parameters to input variables for the Sonora Pass site.

Stage II results

My Stage II model fit my data with a relatively high degree of accuracy, with average error ranging from 1.3% to 38.1% when compared with historical data for all data sites. I found the strongest correlations between R_g and aspect, R_h and aspect, R_i and slope, R_j and aspect, Ω_a and elevation, Ω_b and elevation, E_a and aspect, E_b and aspect, A_g and aspect, A_h and aspect, A_i and aspect, A_j and aspect, A_k and aspect, A_l and aspect, and A_m and aspect (Table 2).

Table 2. R^2 values for linear fitting of Stage I parameters to physiographic variables. R^2 values describe the amount of variation in a given Stage I parameter that is described by the site physiographic variables.

	P_a	P_b	P_g	P_h	P_i	P_j	P_k	P_l	P_m
R	-	-	0.595	0.703	0.813	0.758	-	-	-
Ω	0.579	0.849	-	-	-	-	-	-	-
E	0.704	0.839	-	-	-	-	-	-	-
A	-	-	0.568	0.703	0.904	0.602	0.577	0.696	0.884

Model Testing - Stanislaus Watershed

The Stage II model predicts SWE within the model development area (Stanislaus Watershed) with an average of 91.1% accuracy. When tested against the 2014 snow year data, the model over-predicts maximum SWE for the Sonora Pass data site by 2.40 inches with an error of 18.2% (Figure 3). When compared against NRCS snow course data that was not part of the dataset used to derive the model, the model still tends to over-predict total SWE.

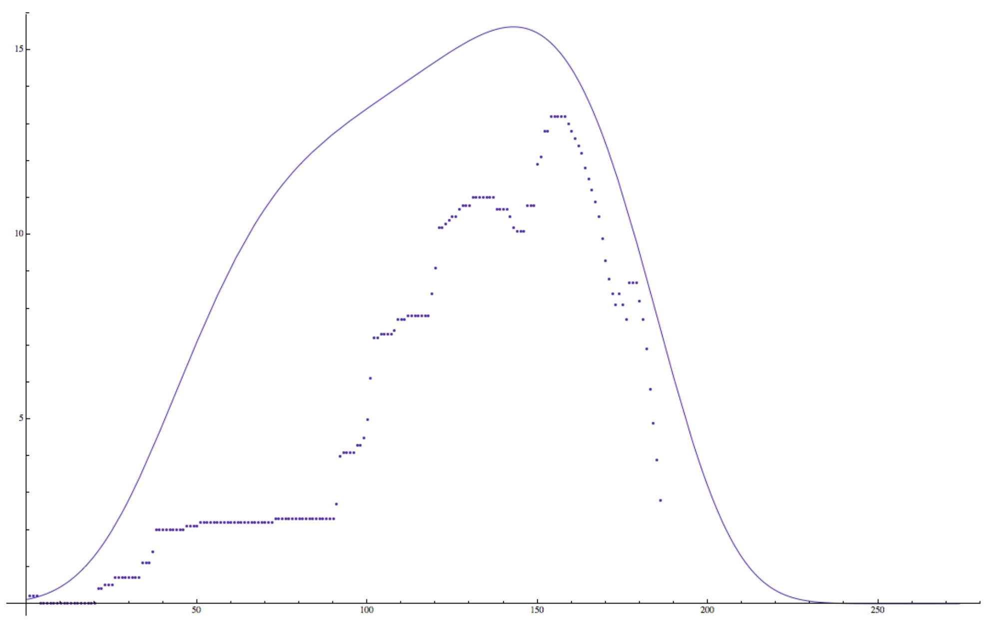

Figure 3. Final model vs. 2014 data for the Sonora Pass site. The seasonality of maximum SWE is accurate, but the model over-predicts total SWE.

Model testing - Northern Sierra & East-side

The Stage II model predicts SWE outside of the model development area (in data-poor regions) within 30% of maximum SWE in 62% of all tested cases. When compared with data from the Mokelumne drainage basin - north of the Stanislaus, but still on the western slope of the Sierra Nevada - the model predicted seasonal SWE with an average of 13% error. Average error increased to 41% when compared to the Truckee drainage basin farther to the north. When compared with data from the West Walker River and Virginia Creek drainage basins - on the eastern slope of the Sierra Nevada - the model predicted seasonal SWE with an average of 532% error. The model tended to radically over-predict SWE on the eastern slope, where increasing site distance from the Sierra crest corresponded with increased error. Even in west-facing drainage basins (Mokelumne), or areas near the Sierra crest (Truckee), prediction error generally increased when compared to eastern data sites within the region.

An error map for the entire Sierra Nevada shows that the model works best in west-facing drainages close to the Stanislaus basin. Increased latitudinal distance from the basin of origin results in more limited predictive capacity. In addition, most areas east of the Sierra crest are unsuitable for application of the model due to extremely high over-prediction error (Figure 4).

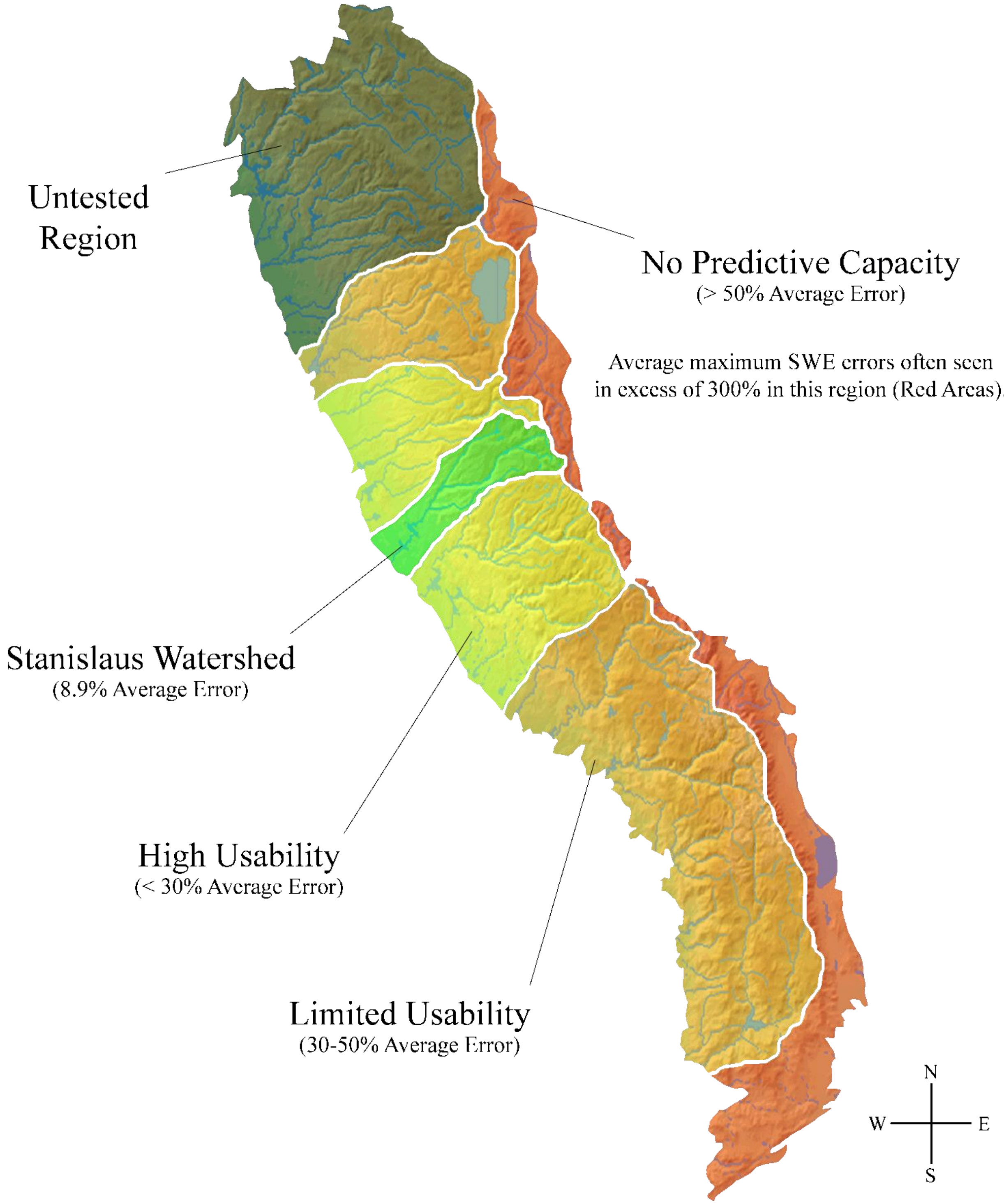

Figure 4. Map of model usability coded by average error.

DISCUSSION

My model provides a generalized, long-term prediction of Sierra Nevada snowpack not seen in previous models. It is applicable to a broad spectrum of physiographic regimes within the Sierra Nevada, although some areas are not within standards of reasonable usability due to variation in broad-scale physiographic effects like rain shadow. The model shows similarities with some other models, but no others predict SWE on a yearly scale with so few input variables.

Model applicability

This model can be highly accurate and useful in certain situations, namely west-facing slopes with elevations under 9000ft. A lower prediction accuracy is seen on higher elevation northeast-facing slopes, perhaps because of a lack of higher elevation sites in the initial dataset. The range-wide spatial accuracy seen in the error map (Figure 4) suggests that the model may only be applicable to western slopes in the central Sierra Nevada. However, a variation of this model could prove useful in other areas with maritime snow climates.

While the model is highly accurate at predicting seasonal SWE in the Stanislaus watershed, it is much less accurate and unsuitable for use in the Eastern Sierra. The orange and red areas seen on the error map (Figure 4) indicate that the model will be of decreasing usability in those areas. When compared to runoff, the model maintains its real-world accuracy in the Stanislaus, Kings, and American River drainage basins while predictably becoming less reliable in the east-side drainage basins. Realistically, the model becomes less useful to water managers in the far southern drainage basins, and an average of 2 miles east of the Sierra crest.[3]

Comparison with other models

The model seems to fit as expected within the scope of other studies, squarely between mass-balance and climate models in terms of accuracy and prediction range. When compared to mass-balance models like SNOWPACK or the 2012 Langlois et al. model, my model has a much

[3] Figure given is an average distance range-wide. Accuracy can become prohibitively poor in <1mi in some high-relief southeastern basins.

lower daily level of accuracy (Lehning 2002, Bellaire 2011, Barrett 2003). However, my model also had a much longer effective prediction range. The Langlois, SNOWPACK, and SNODAS models need to be seeded with climate input data at every storm event, and their yearly predictions depend on the accuracy of such input data. They can be very accurate for estimation of ablation and snowpack characteristics, such as those important for avalanche forecasting (Lehning 2002, Bellaire 2011), allowing them to more consistently out-predict my model in weekly or daily predictions. This is to be expected, as the predictions made by such models are not on the same temporal scale as my model.

A more relevant comparison may be found in longer-term climate models like MM4-based climate models, which predicts winter storms at a much broader temporal scale similar to my model. However, these models are often based on large-scale grids because they necessarily must estimate global climate in order to predict winter storms. In comparison my model has a much better spatial resolution (any point can be selected for prediction without new inputs or model re-derivation), but its accuracy can be more variable (Dickinson et al. 1989). Because the prediction range of MM4-based models can be selected at-will (e.g. 1-day, 1-week, 1-month prediction), short-term predictions can closely approximate actual precipitation in storm events (Dickinson et al. 1989, Giorgi et al. 1993). My model does not have the ability to predict such short-term individual storm events. From a yearly perspective, my model is more accurate than MM4-based models, while from a sub-monthly perspective my model is less accurate (Dickinson et al. 1989, Giorgi et al. 1993).

In comparison to climate models that are designed to predict on a longer temporal scale (yearly or greater, similar to my model), my model had a higher level of accuracy in yearly overall SWE prediction. Models like the Coupled Model Intercomparison Project series (CMIP1-5) predict yearly SWE or precipitation at a much lower resolution and accuracy, but for a longer prediction range (Waliser et al. 2011). Such extreme long-term predictions were not tested with my model due to difficulty in obtaining global climate variable predictions more than a year in advance.

Perhaps most relevant to the model's real-world usability, SWE predictions correlated well with runoff from the Merced, Carson, and American River drainage basins (Hunsaker et al. 2012, Dettinger et al. 2004). Areas of low model accuracy were less accurately correlated with runoff, while areas of high model accuracy were predictably more accurately correlated with runoff. The model was most highly correlated with actual runoff measurements in the Stanislaus watershed, and least highly correlated with runoff in the eastern Sierra, similar to SWE prediction accuracy. This error schema is consistent with range-wide patterns of snow accumulation, where high error areas correspond closely with the largest changes in climate patterns (Aguado 1990). Because the range-wide model is accurate in predicting actual runoff, it will be useful in predicting future water resource availability.

Limitations and future directions

The input dataset for this model only spanned 30 years and it may not encompass longer-term climatic cycles and trends. This suggests the possibility of decreased accuracy with longer-timespan predictions and/or increasing local variance from a historically normal climate regime. Specifically, temperature may need to be more heavily weighted in the future as snowfall becomes more and more dependent upon the SWE:PRE ratio (Harpold et al. 2012), and altitude buffering effects may increase or decrease unpredictably with continued warming (Howat and Tulaczyk 2005).

While this model is accurate in predicting seasonal SWE historically and for the upcoming snow year, it would benefit from integration with an advanced climate model and a mass-balance snowpack model. The addition of more accurate inputs could yield better short-term accuracy and lead to a longer effective prediction range (> yearly). However, one must be wary of adding too much complexity, as it may come at the cost of prediction range. The addition of more variables necessarily adds more opportunities to introduce error, which can propagate exponentially in the long-term. SNODAS exemplifies this quandary, whereby a high temporal resolution and complex calculations limit the prediction range to a matter of days (Barrett, 2003). To maintain model usability there needs to be a balance between temporal resolution and prediction range.

Broader implications

Water resource managers like the California Department of Water Resources (DWR) can use this model to accurately predict seasonal water storage in snowpack, allowing for earlier anticipation of drought or high-flow conditions. Current California drought predictions stem from low-accuracy climate models, which are essentially flawed in their water storage prediction capacity. California DWR does not consider such models "suitable for decision-making." (CA DWR, 2013) Because most of California's summertime water supply is stored in snowpack in the Sierra Nevada and other mountain ranges, a SWE-specific predictive model will be much more useful for drought prediction. With the addition of more variables or the integration of a climate model as described in the *future directions* section, my model is well prepared to provide long-term SWE predictions that would be suitable for decision-making by the DWR.

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APPENDIX A: Modeling Details

General Model

I started with a skew-normal probability density function (PDF) as the basis for my model.

Skew-normal PDF: $P(x)=\frac{2}{\omega}\phi\left(\frac{x-\varepsilon}{\omega}\right)\Phi\left(\alpha\left(\frac{x-\varepsilon}{\omega}\right)\right)$

where $\phi(x)$ is the normal PDF: $\phi(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$

$\Phi(x)$ is the normal CDF: $\Phi(x)=\int_{-\infty}^{t}\phi(t)\,dt$

α represents skewness, ε represents location, and ω represents scale.

Modifications from General Form

I modified the normal PDF and skew-normal PDF to meet the demands of snowpack modeling.

The normal PDF became $\phi(x)=\frac{1}{2\pi}e^{\left(\frac{-x}{20}\right)^4}$ and the skew-normal PDF became

$f(x)=\frac{r}{\omega}\phi\left(\frac{x-\varepsilon}{\omega}\right)\Phi\left(\alpha\left(\frac{x-\varepsilon}{\omega}\right)\right)$ with *r* an arbitrary variable allowing for easy scaling to seasonal SWE values.

Regression and Model Creation

I used a Gauss-Newton algorithm to fit my modified function f(x) to seasonal SWE values where x ranged from 0 (Nov 1) to 274 (Jul 31), with leap day data removed. This yielded the ideal parameters (r, ε, ω, and α) for each snow year. For example, 2009 data from the Gianelli Meadow site produced the following curve and parameters:

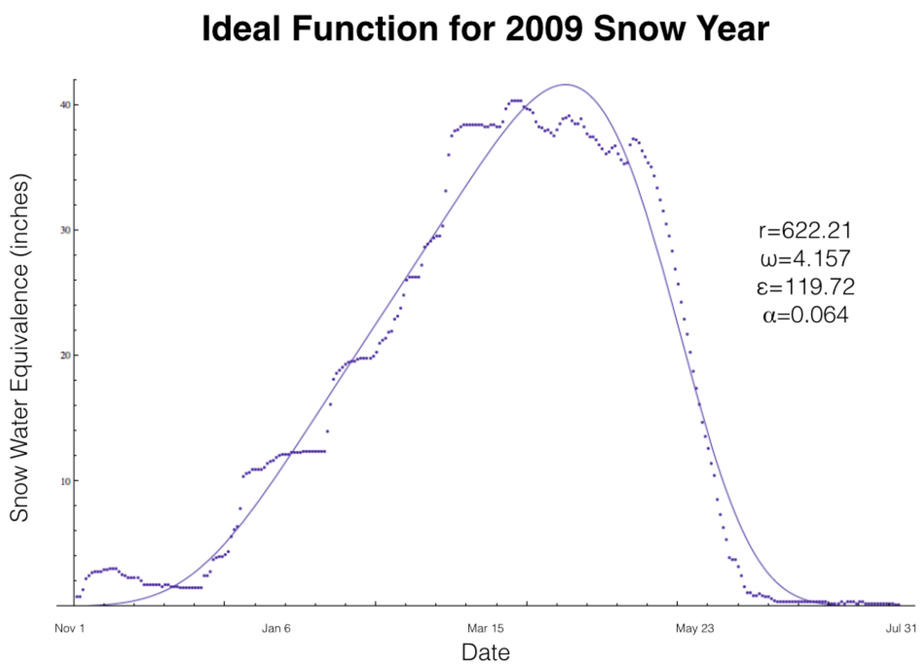

Figure A1. Ideal function and parameters for Gianelli Meadow data site during 2009 snow year.

I then defined global climate index data as a series of piecewise functions linking data points with straight lines (Figure A2).

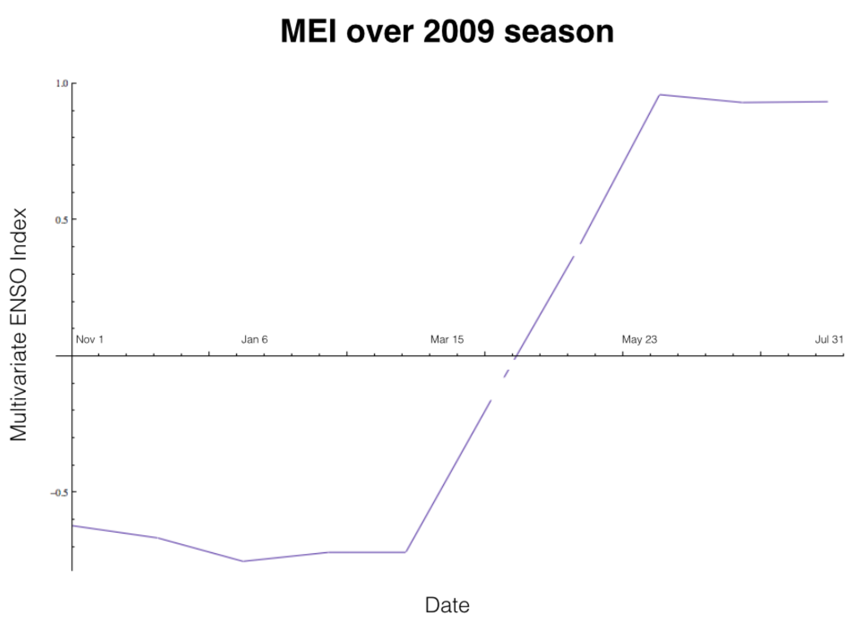

Figure A2. Piecewise-defined MEI function for 2009 snow year

This allowed me to determine the relationship between the ideal parameters and the functions defined by the global climate index variables for the same time period. I defined a new function $F(x)=\frac{R}{\Omega}\phi\left(\frac{x-\mathrm{E}}{\Omega}\right)\Phi\left(\mathrm{A}\left(\frac{x-\mathrm{E}}{\Omega}\right)\right)$ to represent my completed Stage I model and defined variables as follows:

Table A1. Stage I variables.

Variable	Representing	Defined in terms of	Rationale
Ω	Scale	Integral of piecewise functions	Average state of global climate determines whether it is high/low snow year
E	Location	A	Location and skewness are both measures of the seasonality of maximum SWE
A	Skewness	**m**, slope of linear least-squares regression for piecewise functions	Represents the overall trend in global climate over the snow year, skewing the curve positively/negatively
R	Scale	Ω	Both R and Ω represent scale, they are linearly related.

Stage II Modeling

After solving for the parameters of the functions that best fit the Stage I variables, I then fit those parameters to the site physiographic variables for elevation (H), slope (S), aspect (θ), and tree cover (TC) by re-running the Gauss-Newton algorithm. This produced one equation for the entire watershed with inputs MEI, PDO, PNA, H, S, θ, and TC to define the yearly equation for any given site, which then produces continuous estimates of SWE at any time, x, throughout the year. An example is shown for 2013 snow year at the Gianelli Meadow data site (Figure A3).

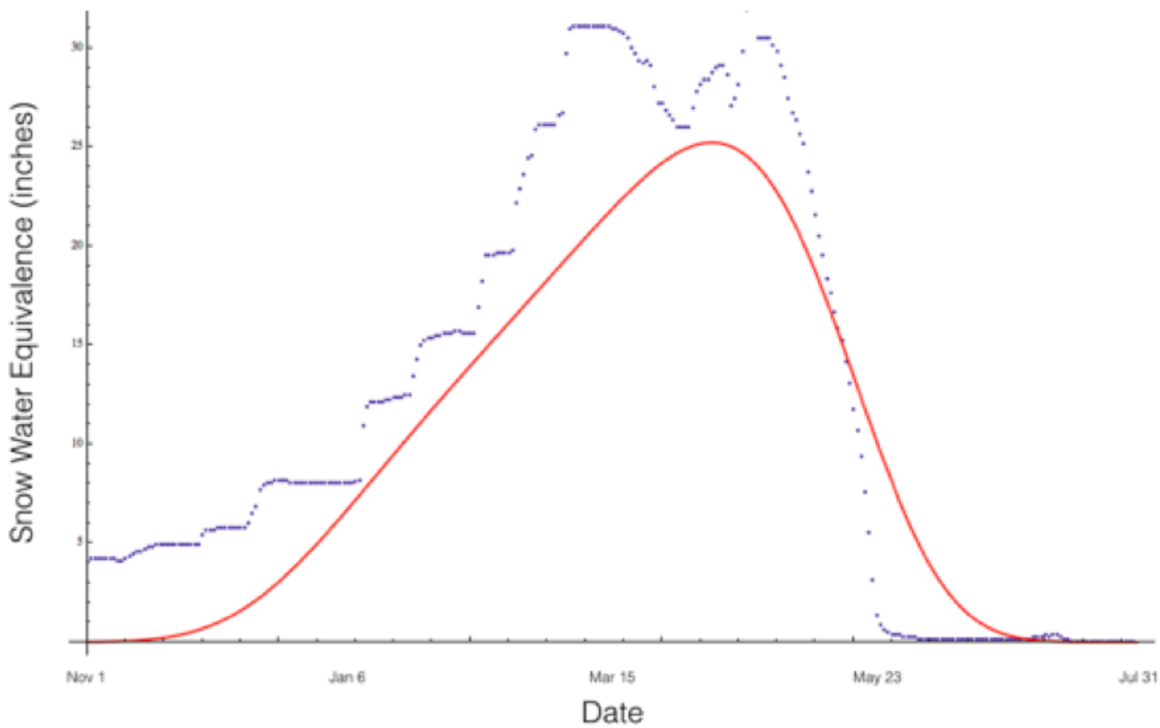

Figure A3. Stage II prediction for 2013 snow year at the Gianelli Meadow site. The predicted SWE curve can be seen in red, while the actual measured data is plotted as a series of blue points. Seasonality of maximum SWE is accurately predicted, while there is a 19.35% error between predicted maximum SWE and actual maximum SWE, with actual SWE being under-predicted by the model.