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The Decline of Cartesianism in Mechanics: The Leibnizian-Cartesian Debates

By Carolyn Iltis *

I. INTRODUCTION

THE DECLINE OF THE CARTESIAN WORLDVIEW in the early decades of the eighteenth century has been described from several vantage points.[1] As a metaphysical system it reflected the failure of the ontology of substance philosophy.[2] The categories substance and modification were too limited in scope; the essences extension and thought so different in kind as to forbid causal interaction. As a methodological system it failed because certain knowledge of the temporal phenomenal world could not be deduced from its logical axioms.[3] As a planetary explanation its vortical aethereal motions were shown by Newton to be inconsistent with Kepler's laws, while the gradual demise of these aethereal hypotheses has been recently documented by E. J. Aiton.[4]

My purpose in this paper is to indicate the inadequacies of the Cartesian worldview in handling problems in terrestrial mechanics. I shall show that although Cartesian presuppositions were used in mechanical problems, the mathematical results supported Leibnizian or Newtonian conclusions. Cartesianism was unable to maintain

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[1] Cartesianism and its influence on the natural philosophy of the seventeenth and eighteenth centuries have been described in many works. E.g., see Paul Mouy, *Le développement de la physique cartésienne, 1646–1712* (Paris: Vrin, 1934) and "*L'influence de Descartes sur le développement de la physique*," *Scientia*, 1930, *48*:227–236; A. R. Hall, "Cartesian Dynamics," *Archive for History of Exact Sciences*, 1961, *1*:172–178; Thomas L. Hankins, "The Influence of Malebranche on the Science of Mechanics During the Eighteenth Century," *Journal of the History of Ideas*, 1967, *28*:193–210; Leonard M. Marsak, "Cartesianism in Fontenelle and French Science. 1686–1752," *Isis*, 1959, *50*:51–60; Michael A. Hoskin, "'Mining All Within.' Clarke's Notes to Rohault's 'Traité de physique,'" *The Thomist*, 1962, *24*:353–363; George Sarton (on Rohault's *Traité de physique*), *Isis*, 1948, *38*:141; Pierre Costabel, "Essai critique sur quelques concepts de la mécanique cartésienne," *Archives Internationales d'Histoire de Sciences*, 1967, *20*:235–252; Richard J. Blackwell, "Descartes' Laws of Motion," *Isis*, 1966, *57*:220–234; Gerd Buchdahl, "The Relevance of Descartes' Philosophy for Modern Philosophy of Science," *British Journal for the History of Science*, 1963, *1*: 227–249, esp. p. 236.

[2] Richard A. Watson, *The Downfall of Cartesianism, 1673–1712* (The Hague: Martinus Nijhoff, 1966).

[3] Norman Kemp Smith, *Studies in the Cartesian Philosophy* (London: Macmillan, 1902).

[4] E. J. Aiton, *The Vortex Theory of Planetary Motions* (New York: American Elsevier, 1972). Alexandre Koyré, "Newton and Descartes," in *Newtonian Studies* (Chicago: University of Chicago Press, 1965).

its integrity as a system of nature because its scientific metaphysics could not produce a unique Cartesian mechanics. Descartes' followers in the early eighteenth century used his aether theories to support Leibnizian or Newtonian concepts of force. Thus the mechanical conclusions eventually undermined the metaphysical hypotheses.

I shall deal with the Leibnizian-Cartesian debate as it was played out in the Paris Academy of Sciences during the 1720s. In an earlier article I discussed Leibniz' initial disagreements with Cartesians on the subject of *vis viva*.[5] The metaphysical, physical, and socio-psychological aspects of the Newtonian-Leibnizian arguments in the 1720s are also described elsewhere.[6] Aspects of a somewhat later phase of the *vis viva* controversy have been investigated by Thomas L. Hankins, L. L. Laudan, and myself.[7]

The intellectual background for the Cartesian worldview in mechanics was provided by René Descartes and his followers, two of the most important being Nicholas Malebranche and Jacques Rohault.[8] Several aspects of Descartes' philosophy of nature were used by Cartesians in the 1720s in attacking mechanical problems. His theory of matter stated that the world was composed of three elements formed from primitive matter.[9] The first element, which formed the material of the sun and stars, consisted of very subtle minute matter capable of moving at enormous speeds and of filling in the small spaces surrounding the other two denser forms of matter. The second element consisted of spherical particles formed from rotation of the original primitive matter. These spheres, the main constituent of celestial matter, moved in large vortices at high speeds and could transmit pressure instantaneously. The third element was coarser and slower and composed the earth and planets.

The properties and action of the spherical aether particles were used by Cartesians in explaining the action of mechanical bodies. As the aether swirled in a centrifugal vortical motion around the earth it caused continual impulses on terrestrial objects.[10] Since the subtle matter moved faster than terrestrial matter and exerted a force toward the center, bodies above the earth would fall toward the earth's center.

A second important aspect of Descartes' worldview was his belief in God's initial and continued action in the universe. God initially created matter in motion and set the vortices rotating.[11] His continued re-creation of the world from instant to instant guaranteed his presence in the creation. In the hands of the Cartesian Malebranche this concern for God's continual action became an occasionalist philosophy in which God acted at the moment of each collision to determine its outcome. The small aetherial spheres of subtle matter become little vortices rotating like miniature whirlpools and occasionally rupturing during violent activities.[12]

[5] Carolyn Iltis, "Leibniz and the *Vis Viva* Controversy," *Isis*, 1971, *62*:21–35.

[6] Carolyn Iltis, "The Leibnizian-Newtonian Debates," *Br. J. Hist. Sci.* (in press).

[7] Thomas L. Hankins, "Eighteenth-Century Attempts to Resolve the *Vis viva* Controversy," *Isis*, 1965, *56*:281–297; L. L. Laudan, "The *Vis viva* Controversy, a Post-Mortem," *Isis*, 1968, *59*:131–143; Carolyn Iltis, "D'Alembert and the *Vis Viva* Controversy," *Studies in History and Philosophy of Science*, 1970, *1*:135–144.

[8] Nicholas Malebranche, *Oeuvres complètes*, ed. André Robinet (Paris:Vrin, 1962–1967). Jacques Rohault, *Traité de physique* (Paris, 1671; 3rd ed., 1675). *J. Rohaulte physica*, trans. with notes by S. Clarke (London, 1697).

[9] Aiton, *Vortex Theory*, pp. 43–47. René Descartes, *Principia philosophiae*, in *Oeuvres de Descartes*, ed. Charles Adam and Paul Tannery, 13 vols. (Paris:Cerf, 1897–1913), Vol. VIII, Pt. 3, Prin. 49.

[10] Aiton, *Vortex Theory*, pp. 55–58.

[11] Descartes, *Principia*, Pt. 2, Prin. 37.

[12] Hankins, "Malebranche," *loc. cit.*, pp. 198–199; Aiton, *Vortex Theory*, p. 71.

A third relevant Cartesian principle was the doctrine of matter in motion measured by the amount of matter m and its speed v. The quantity of motion $m|v|$ was conserved in collisions between bodies.[13] It was soon discovered that Descartes' rules of collision deduced logically from the laws of his mechanics were inconsistent with experience; accordingly, early adherents to Descartes' physical system of matter in motion did not claim allegiance to his seven rules of collision. Abbé Catelan, Denis Papin, Antoine Arnauld, and Malebranche brought forward other reasons for supporting the concept quantity of motion.[14]

I shall show that in the debates on the communication of motion sponsored by the Academy of Sciences in the 1720s French Cartesians used various presuppositions of the Cartesian worldview in order to argue that mv is the proper measure of the force of a body in motion. Thus Jean-Pierre de Crousaz appealed to Descartes' and Malebranche's concept of God's volition and the necessity of his active power in sustaining the natural world. Pierre Mazière used the "little vortices" of Malebranche as a basis for analyzing the action of elastic bodies. The Cartesian Jean Bernoulli, who supported the Leibnizian position in dynamics, used Descartes' conception of air and aetherial matter in supporting his concept of elastic bodies. Abbé Charles-Etienne Camus related the force of rising and falling bodies to Bernoulli's Leibnizian analysis of expanding springs. Jacques Eugène de Louville utilized the Cartesian impulsive aether in deriving a concept of impulsive force. Jean Jacques de Mairan tried to reduce cases of acceleration to uniform motion, thereby eliminating the concept of force, a viewpoint held by Malebranche. To the extent to which these men were successful in giving a mathematical description of mechanical problems, they supported mechanical points made by Leibniz or Newton. While the scientific metaphysics of Descartes stimulated analysis of the physical world, it could not support a mechanics built around the concept of quantity of motion.

II. CARTESIANISM AND THE ACADEMY OF SCIENCES

In the Cartesian-Leibnizian debates of the 1720s it was the Paris Academy of Sciences which provided a sounding board for the three prevailing systems of natural philosophy. In sponsoring contests on various controversial matters in the sciences the Academy was pursuing its self-image as a neutral arbitrator. It conceived of itself not as the proponent of any particular worldview such as Cartesianism or Newtonianism but as an official bureau for the publication of controversial and often contradictory opinions.[15] It officially reserved judgment until it considered sufficient evidence had been gathered on a particular issue. To take a position on any one of the numerous uncertain controversial arguments of the early eighteenth century would

[13] Descartes, *Principia*, Pt. 2., Prin. 37.

[14] Iltis, "Leibniz and the *Vis Viva* Controversy." Abbé Catelan, "Courte Remarque de M. l'Abbé D. C. où l'on montre à M. G. G. Leïbnits le paralogisme contenu dans l'objection précédente," *Nouvelles de la République des Lettres*, Sept. 1686, *8*:1000–1005. Denis Papin, "De gravitatis causa et proprietatibus observationes," *Acta Eruditorum*, Apr. 1689, pp. 183–188. Antoine Arnauld, "Correspondence with Arnauld," in G. W. von Leibniz, *Discourse on Metaphysics, Correspondence with Arnauld, and Monadology*, trans. George R. Montgomery (La Salle, Ill.: Open Court, 1957), pp. 142–149. Nicholas Malebranche, "Extrait d'une lettre du P. M. à M. l'Abbé D.C.," *Nouv. Répub. Lett.*, 1687, *9*:448–450.

[15] See Roger Hahn, *The Anatomy of a Scientific Institution: The Paris Academy of Sciences, 1666–1803* (Berkeley: University of California Press, 1971), pp. 31–32.

have inhibited the progress of science. "In the Academy no system should dominate to the exclusion of others," said Bernard de Fontenelle.[16] In sponsoring contests on new and debatable viewpoints it functioned admirably as an integrative device forcing the opposing parties to ponder and evaluate the validity of their own and their adversary's fundamental presuppositions.

Yet ultimately the Academy displayed its Cartesian bias. Of three contests it sponsored on the question of motion and its transfer, the winners were Cartesians or supported the Cartesian measure *mv*. The winner of the 1720 contest on the nature and communication of motion was the Cartesian philosopher Crousaz. The essay of the Englishman Bishop Berkeley, published in 1721 as *De motu*, was rejected.[17] The 1724 contest on the laws of motion of hard bodies was won by the Newtonian Colin Maclaurin,[18] whose conclusions supported the measure of force *mv*. Jean Bernoulli's Leibnizian analysis was disqualified from this contest because it rejected the concept of a hard body on logical grounds; his paper took only honorable mention in the 1726 contest on elastic bodies which was won by the Cartesian Mazière.

Bernoulli's paper, published in the prize volume, inspired the reactions of the Cartesians Camus, Louville, and Marain to his *vis viva* (mv^2) argument based on the expansion of elastic springs. I shall first discuss the papers on collision mechanics and then those on elastic springs, showing how metaphysical presuppositions were related to mechanical results.

III. THE OCCASIONALISM OF CROUSAZ

The Cartesian measure of force *mv* was supported by the Swiss professor of philosophy and mathematics at Lausanne, Jean-Pierre de Crousaz. A vociferous opponent of Leibniz' philosophy, Crousaz had rebuked Leibniz for errors in his attacks on Descartes. In his paper "Discourse on the Principle, Nature and Communication of Motion," which won the prize of the Academy in 1720, he examined the nature and origin of motion.[19]

Crousaz followed Descartes in maintaining that extension was a substance. Since matter was indifferent to motion or rest, it could not change its state by any force of internal origin. Only an external cause could change the state of motion or rest of a particular body.[20] The first motion in the universe derived from the power and will of God. Crousaz' conception of God was that of an eternal intelligence who could produce motion voluntarily, continually, and with infinite ease. In the continual creation of new motion the collisions of bodies were merely occasions.[21] For Crousaz, as for Malebranche, matter was moved only by the continual action of the Creator. Occasional causes are natural causes created anew and of which the collision of bodies is simply the occasion.[22]

[16] Quoted by Hahn, *ibid.*, p. 31.

[17] George Berkeley, "De motu," in *Berkeley's Philosophical Writings*, ed. David M. Armstrong (London: Collier-Macmillan, 1965).

[18] Colin Maclaurin, "Démonstration des loix du choc des corps," *Recueil des pièces qui a remporté les prix de l'Académie Royale des Sciences* (Paris, 1724), Vol. I, pp. 1–24, separate pagination.

[19] Jean-Pierre de Crousaz, "Discours sur le principe, la nature et la communication du mouvement," *Recueil des pièces qui ont remporté les deux prix de l'Académie Royale des Sciences* (Paris, 1720), pp. 1–67.

[20] *Ibid.*, pp. 16–17.

[21] *Ibid.*, p. 56.

[22] *Ibid.* On Malebranche's occasionalism see Beatrice K. Rome, *The Philosophy of Malebranche* (Chicago: University of Chicago Press, 1963), pp. 226–228.

The laws of motion were expressions of God's volition which enabled bodies to act and to move. The initial motion was imparted to a universe perfectly at rest by the wish of the supreme intelligence.[23] In particular collisions motion was merely the state of a body traversing a particular space. Quantity of motion was the product of the weight (*pesanteur*) and the velocity (*vitesse*); the force of motion was identical with this quantity of motion.[24] Like other Cartesians, Crousaz accepted time as essential to the measure of motion. One must compare those distances traversed in equal times when comparing velocities of colliding bodies.[25] Crousaz thus defined force as the measure mv, a kinematic rather than dynamic analysis of nature.

IV. MAZIÈRE AND THE "LITTLE VORTICES"

Whereas Crousaz' analysis of collisions derived from a voluntarist notion of God and the re-creation of motion from instant to instant, Pierre Mazière presented a mechanical analysis of collision based on the vortex theory of Descartes and Malebranche. The winner of the Academy's 1726 contest on elastic impact, Mazière discussed "The Laws of Impact of Perfectly or Imperfectly Elastic Bodies, Deduced from a Probable Explanation of the Physical Cause of Elasticity."[26] His essay, based on Cartesian presuppositions, won the contest over Jean Bernoulli's Leibnizian analysis.

Mazière divided his memoir into two parts, the first containing a probable explanation of the physical cause of elasticity and the second deriving the laws of the collisions of elastic and semi-elastic bodies exemplified in problems. He attempted to explain the cause of the elastic virtue (*vertu elastique*) of bodies by use of the Cartesian-Malebranchean vortex theory. Soft bodies remain at rest after collision, as do perfectly hard inflexible bodies, since no new cause of motion occurs. Elastic bodies rebound because their elastic parts are pressed together during the moment of compression and restored during the moment of restitution by the action of tiny aetherial vortices—*petit tourbillons*. His explanation was sought in physical terms rather than as a resort to the immediacy of God's action on matter. This physical cause could not be found in matter itself, since rest is an essential property of matter. Nor could solid parts of matter cause motion, since they could not of themselves return to their original position during the restitutive phase of the collision.[27]

The physical cause of elasticity was hypothesized to be the corpuscles of a subtle aether penetrating all bodies and having an infinite force given to it by the Creator. It could cause perfect rebounding by the restitution of the primitive forces of bodies after collision.[28] As a perfectly elastic fluid, the aether could transmit the vibrations received by a solid body, causing its parts to change and re-establish their original positions in the smallest instants of times. It flowed through all bodies with extreme facilitity, leaving no void in the immense spaces that it occupied. It was so subtle that one corpuscle of air could contain a million corpuscles of subtle matter. The aetherial

[23] Crousaz, "Discours," p. 20.
[24] *Ibid.*, p. 27.
[25] *Ibid.*, pp. 48, 50.
[26] Pierre Mazière, "Les loix du choc des corps a ressort parfait ou imparfait, déduites d'une explication probable de la cause physique du ressort," *Recueil des pièces* (Paris, 1727), Vol. I, pp. 1–108, separate pagination.
[27] *Ibid.*, pp. 6–8.
[28] *Ibid.*, pp. 8–12.

corpuscles, although ordinarily spherical, could divide themselves into smaller parts in the manner of mercury particles when a change occurred in a gross body. The particles could incorporate themselves with other corpuscles or change their figures to ellipses.[29]

Mazière's subtle aetherial matter was conceived to be infinitely compressible and responded to the force compressing it with an equal power. It was composed of an infinity of vortices (*tourbillons*) whirling about their centers with extreme rapidity, counterbalancing each other as did the large vortices of Descartes. Both the large Cartesian and the small Malebranchean vortices counterbalanced each other by their centrifugal forces. This dynamic balance prevented them from moving away. Their centrifugal forces were inversely proportional to their diameters, increasing as their diameters decreased. The centrifugal force of the infinitely small vortices was therefore infinitely greater with respect to the infinitely large vortices. And thus, concluded Mazière, the centrifugal force of the smallest vortices was infinite. It was this property which was the physical cause of elasticity.[30]

Using the theory of *petit tourbillons* Mazière defined an elastic body as one filled with an infinity of pores through which the subtle aether moves in circular motions. Each pore contained one or more little vortices, giving, by their centrifugal force, stability to solid bodies. The smaller the vortices the greater the elasticity of the body, because a greater centrifugal force repelled the external forces tending to separate the parts of the bodies.

Suppose, argued Mazière, that two bodies hit each other directly with equal and opposite forces. In successive instants of time they use their primitive forces to mutually compress one another. The aetherial matter never resists motion and partially leaves the pores in the direction toward which it is pushed. Motion is communicated successively through the first pores to the others: the pores are flattened, assuming elliptical configurations, and continue to be flattened up to the precise instant that the bodies have exhausted all their primitive force in mutual compression.

The centrifugal forces of the vortices outside the two bodies remain the same as before the collision when the exterior and interior vortices were in equilibrium. But the centrifugal forces of the vortices remaining inside are augmented because their diameters are diminished. At the end of compression the interior vortices have increased their centrifugal forces, while those outside have not. The exterior vortices do not have centrifugal forces sufficient to stop the tendency of the interior vortices to enlarge the pores, and they continue to enlarge the pores until the point where the compressed parts have been re-established. Thus bodies having perfect elasticity expand with velocities equal to those with which they were compressed, as a result of the infinite force of the little vortices.[31]

> If in a collision the force with which the elasticity is restored in two bodies is to that with which they were compressed e.g. 15 to 16, then in all other collisions of these two bodies, or of two others of the same nature, these two forces will always be as 15 to 16. That is why if one knows the elastic constant, r, and the force lost or gained by one of the two bodies during compression, one can obtain that force lost or gained in restoration, by multiplying the force lost during compression by the elastic constant r. This constant

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[29] *Ibid.*, pp. 13–15.
[30] *Ibid.*, pp. 16–21.
[31] *Ibid.*, pp. 22–25.

is equal to unity when the elasticity is perfect and less than unity when the elasticity is imperfect.[32]

Mazière then proceeded to derive a general solution for elastic and semi-elastic collisions. His general formula for the laws of colliding bodies in terms of the velocity after collision was

$$a' = a - B\left[\frac{(r+1)(a-b)}{A+B}\right] \text{ and } b' = b + A\left[\frac{(r+1)(a-b)}{A+B}\right]$$

where A and B are the masses of bodies A and B, a and b their velocities before collision, a' and b' the velocities after collision, and r the elastic constant. The velocity of a body after collision has two parts. The first is the primitive velocity a, which is always positive, or the primitive velocity b, which is positive when the movements are in the same direction and negative when they are in opposite directions. The second part is the total velocity that each body gains or loses by the compression and restoration in the two moments of collision. That of the attacking body is always negative; that of the body hit is positive.[33]

When bodies have perfect elasticity the elastic constant is equal to unity, so that $r + 1 = 2$; thus 2 will appear in the general formula. When the bodies are not elastic and are supposedly perfectly soft, the elastic constant will be 0; hence $r + 1 = 1$. When the elastic constant is equal to the ratio of the mass of the attacking body to the body hit, one has $r = A/B$ and consequently $r + 1 = (A + B)/B$.

Mazière showed that special cases of collision substituted in the general equation checked with the previously known facts. When the elasticity is perfect and the masses A and B are equal, the general equations come down to $a' = b$, $b' = a$. Thus the bodies exchange their velocities. For the case of body B at rest before the collision one has for perfect elasticity

$$a' = \frac{Aa - Ba}{A+B} \text{ and } b' = \frac{2Aa}{A+B}$$

For bodies without elasticity:

$$a' = b' = \frac{Aa}{A+B}$$

showing that the bodies stick together and proceed with a common velocity.

When the body hit is at rest and the velocity of the attacking body is equal to the sum of the masses $a = A + B$, the formula becomes

$$a' = A - rB \text{ and } b' = (r+1)A$$

from which is deduced

$$r + 1 = \frac{b'}{A}$$

This gives an easy method of determining in experiments the value of $r + 1$ and consequently the proper value given to two bodies in a given experiment, or to two other bodies of the same nature. In the case for perfect elasticity, $a' = A - B$, $b' = 2A$; for no elasticity $a' = A$, $b' = A$. For the case of body B at rest and infinitely greater than body A attacking, one supposes $A = 0$. One obtains $a' = -ra$. This is the case

[32] *Ibid.*, pp. 28–29.

[33] *Ibid.*, pp. 33–34.

of direct reflection, body *A* rebounding with its primitive velocity when elasticity is imperfect and with a velocity equal to its primitive velocity when the elasticity is perfect.[34]

Mazière thus presented a general solution for inelastic and elastic collisions in terms of force defined as *mv*. His explanation of the nature of matter, upon which the solution was based, was cast in terms of the Cartesian vortex theory and was supported by the concept of centrifugal force. His equations were consistent with the Newtonian laws of impact and momentum conservation which took the sign of the velocity into consideration. In this way a Cartesian view of matter supported a physics which became known as Newtonian mechanics. The vortex theory of matter along with the vortex theory of planetary motions was soon to die out as a philosophy of nature.

V. JEAN BERNOULLI'S ELASTIC MATTER

Jean Bernoulli, whose analysis of planetary motions was based on the Cartesian vortex theory, supported the Leibnizian measure of force in dynamics, mv^2. His "Discourse on the laws of the communication of motion," disqualified from the Academy's 1724 contest on hard bodies, was awarded honorable mention in the debate on elastic bodies of 1726.[35] His analysis of the elasticity of matter which stemmed from both Cartesian and Leibnizian presuppositions supported a Leibnizian physics.

Bernoulli followed Leibniz in his definitions of *vis viva*, or *force vive*, mv^2, and *vis mortua*, or *force morte*, *mdv*, as well as in the assumption that hard atoms could not exist in nature. He argued, as had Leibniz, that since hard-body collisions would violate the law of continuity, only perfectly elastic bodies are theoretically possible. Since every act occurs by infinitely small degrees and "nature does not operate through leaps," motion cannot pass suddenly into rest, or rest into motion as would be necessary in the collision of two hard bodies.[36] Hard bodies being inflexible and unbreakable, they would not rebound after colliding; their speed would drop to zero without going through intermediate steps. If this were true, there would be no reason why nature would choose one state of motion or rest in preference to another. Having no liaison between the two states, rest to motion or motion to rest, no reason would determine the production of one over the other.

Bernoulli rejected hardness taken in the common sense of perfectly solid atoms. Such atoms were only imaginary corpuscles existing in the minds of their champions.[37] Instead he argued that hardness existed only in the sense that bodies are like heavy "balloons filled with compressed air." The greater the pressure, the harder the surface, but likewise the more perfect the body's elasticity. If the density of the air in the balloon is increased to an immense degree of resistance such that an extremely powerful force is necessary to compress it, the balloon, although elastic, will differ in no essential aspect from a hard body.

Bernoulli's concept of matter as essentially elastic stemmed from Decartes' analysis

[34] *Ibid.*, pp. 35–38.

[35] Jean Bernoulli, "Discours sur les loix de la communication du mouvement," *Recueil des pièces* (Paris, 1727), Vol. II, pp. 1–108, separate pagination.

[36] *Ibid.*, p. 5.

[37] *Ibid.*, pp. 6–9. See also Wilson L. Scott, *The Conflict between Atomism and Conservation Theory 1644 to 1860* (London: MacDonald; New York: Elsevier, 1970), pp. 23–24.

of air and aether and Robert Hooke's elaboration of the theory. In the *Principia philosophiae* Descartes had set forth the notion that air particles were surrounded by a spherical space within which they moved, being confined to this space by the impacts of the surrounding aether:

1) Air particles are of the third element, of all shapes, and are put into motion by the aether.
2) Each particle retains for itself a "little spherical space" in which it moves and from which it keeps other particles.
3) This space is larger when the air is heated and the particles move faster, so that the air then expands.
4) When this motion is forcibly compressed, the particles try to regain their previous space, and hence exert increased pressure.[38]

Descartes did not assign elasticity to the air particles and held that they moved at random in the spherical space to which they were confined by the impulses of the aether. In expanding on Descartes' ideas Hooke had suggested that vibrating matter conveys pulses to neighboring particles through the subtle aether surrounding and penetrating them.[39] Similar particles will vibrate together in harmony, whereas dissimilar ones will make differing vibrations and repercussions. Particles of matter have varieties of substance, figure, and bulk and are agitated by pulses or vibrations uniting or loosening the cohesions between them.[40] Bernoulli did not mention Hooke's law for springing bodies, but this as well as the theory of an elastic vibrating matter underlay his own work on elastic springs.

Bernoulli imagined an infinite number of small spheres full of extremely condensed air in a common envelope. Each portion of this mass can be as small as desired and is enclosed in its own envelope. The small spheres represented elementary molecules, and the envelopes took the place of the Cartesian ambient fluid, or aether, which by its own activity pressed and compressed the entire mass and each particular part. If an immensely large degree of elasticity is given to the air contained in these balloons, their entire mass cannot be sensibly compressed by a

> ... finite force as large as can be supposed: A body will conform to our idea of hardness when its sensible parts change their situation only with difficulty.... Elasticity is perfect when all the parts return to their original state; it is imperfect when some of the parts do not return.[41]

Bernoulli used the term "stiffness" (*roideur*) to mean perfect elasticity, whether infinite or finite. Infinitely stiff bodies do not exist in nature, but the term "hard body" referred to an actually infinite stiffness or perfect elasticity.[42] To this concept of elasticity in matter Bernoulli applied Leibniz' concept of *vis viva*, and he developed a mathematical proof of the *vis viva* principle based on the expansion of elastic springs.

Bernoulli was in close correspondence with Leibniz over the development of the ideas of the calculus and was one of those whom Leibniz consulted concerning the best mathematical notation by which the concepts might be expressed.[43] Although Leibniz

[38] Descartes, *Principia*, Pt. IV, Prin. 45–47, quoted in Mary Hesse, "Hooke's Vibration Theory and the Isochrony of Springs," *Isis*, 1966, *57*:433–441, on p. 436.

[39] Hesse, "Hooke's Vibration Theory," p. 434.

[40] Robert Hooke, *Micrographia* (London, 1665; New York: Dover, 1961), pp. 15, 16.

[41] Bernoulli, "Discours," p. 9.

[42] *Ibid.*

[43] Carl B. Boyer, *The History of the Calculus and its Conceptual Development* (New York: Dover, 1949), p. 205.

did not make extensive use of his notation in his popular writings on living force, Bernoulli did apply this notation to the problem of expanding springs. He employed $\int p\,dx$ for the integral of the pressure of the spring over a distance increment and expressed his results in the form of an equation: $\frac{1}{2}\,vv = \int p\,dx$.[44]

Bernoulli's concept of "force" was similar in its essential details to that of Leibniz. *Force vive*, or living force, is that which resides in a body when it is in uniform motion. *Force morte*, or dead force, is "that which a body not in motion receives, when solicited or pressed toward motion," or which moves it more or less fast when the body is already in motion.[45]

If an obstacle prevents local motion from occurring in a body, the body has dead force. The force of gravity is an example. A body placed on a horizontal table makes a continual effort to descend: because of gravity, at each instant an infinitely small degree of velocity is created and immediately absorbed by the resistance of the obstacle. "These small degrees of velocity perish on creation and are reborn in perishing."[46]

The nature of living force is totally different. Time is needed to produce living force and likewise to destroy it. Living force is produced successively in a body as the pressure applied to the body little by little produces increments of local motion. Motion which is acquired in increments becomes finite, eventually remaining uniform when the cause which produced it ceases to act on the body. Thus living force is equivalent to that part of the cause which is consumed in producing it.[47] The living force of a body produced by the dilation of some elastic body or elastic spring is capable of compressing it again to its original state. The efficient cause and its effect are equal.

Although Bernoulli considered himself the foremost champion of "living forces" since Leibniz' death, he did not find it necessary also to accept the role that "force" played in Leibniz' metaphysical system of monads. Leibniz postulated an inner "force" of matter whose action caused it to move unless hindered by some obstacle. Bernoulli simply accepted this inherent force in the concept of *vis mortua* as pressure and *vis viva* as the force produced when the obstacle is removed.

Because he believed that Leibniz' demonstration of 1686 was by itself unconvincing, he developed an argument for *vis viva* derived from elastic springs (see Fig. 1). He showed that the pressure of the expanding spring as it moved through a certain distance would give an mv^2 to a body accelerated by it. He argued that a compressed spring has a certain pressure or dead force.[48] This force turns out to be equivalent to the impressed force of Newtonian mechanics. Bernoulli compared springs having equal elastic constants but composed of an unequal number of units expanding against "equal" bodies. As the spring expands, the dead force is converted to living force mv^2, imparted to a body set in motion by the spring's expansion. As the moving body accelerates, the increment of velocity dv in an increment of time dt depends on the pressure or force of the spring: $dv = p\,dt$. The velocity at any instant is $v = dx/dt$; hence the increment of time dt is dx/v. By substitution Bernoulli arrived at the well-known result $v\,dv = p\,dx$, the integral of which he wrote as $\frac{1}{2}\,v^2 = \int p\,dx$.[49]

[44] Bernoulli, "Discours," p. 44.
[45] *Ibid.*, p. 19.
[46] *Ibid.*, p. 32.
[47] *Ibid.*, p. 33.
[48] *Ibid.*, pp. 39–40.
[49] *Ibid.*, pp. 41–45.

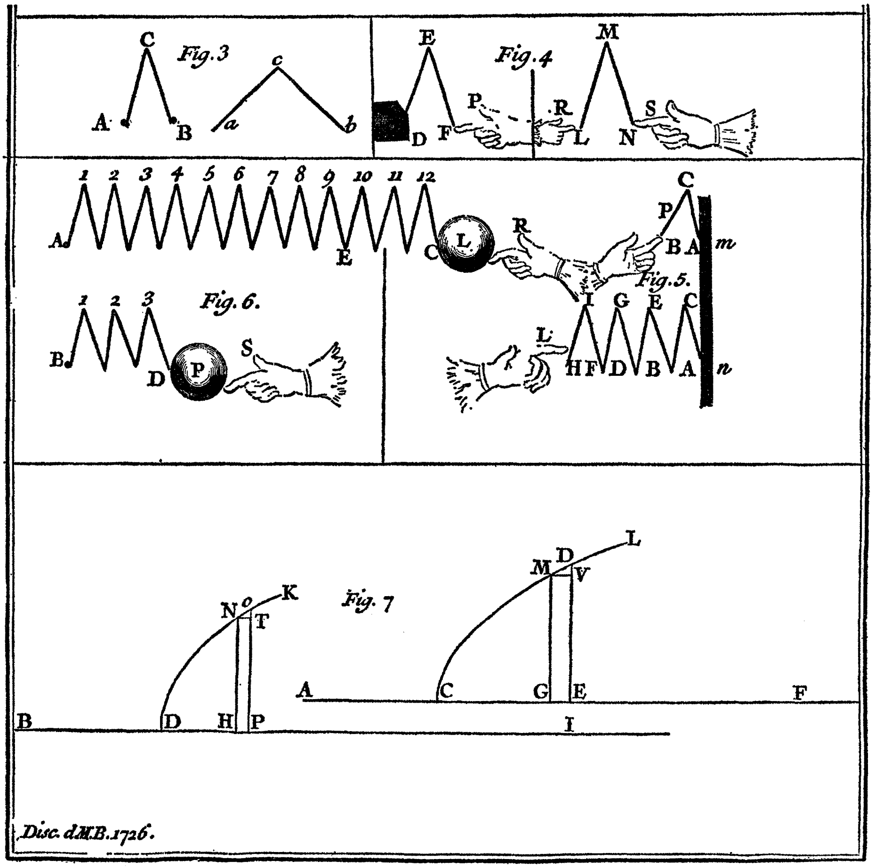

Figure 1. Jean Bernoulli, **Recueil des pièces qui ont remporté les deux prix de l'Académie Royale des Sciences** *(Paris, 1727),* **plate 1.**

By use of proportions Bernoulli compared the *vires vivae*, mv^2, imparted to accelerated bodies with the pressures exerted by compressed springs of different numbers of elastic units.

Bernoulli's Leibnizian mechanics was thus derived from a philosophy of nature based on both Cartesian and Leibnizian presuppositions. Although he did not accept Leibniz' "monadology," he used the Leibnizian concepts of force and conservation along with a Cartesian aether theory to establish a fundamental Leibnizian point, the elasticity of matter.

VI. CAMUS' LEIBNIZIAN MECHANICS

Bernoulli's "Discourse" inspired a series of essays in the Memoirs of the Academy of Sciences examining his opinion concerning elastic springs. The writers included the French scientists Abbé Charles-Étienne Camus (1728), Jacques Eugène de Louville (1729), and Jean Jacques de Mairan (1728). Camus extended the Leibnizian analysis of the mechanics of springs.

Leibniz' 1686 demonstration of the *vis viva* principle had been given in terms of bodies of different weights falling from different heights. An attempt to further relate the free-fall problem to that of compressed springs was made in Camus' 1728 article "On Accelerated Motion due to Springs and the Forces Residing in Moving Bodies."[50] (See Fig. 2.) In discussing the *vis viva* of rising or falling bodies in relation to the force of compression or expansion of springs, Camus expanded on Bernoulli's argument. He showed that the forces producing these accelerated motions were proportional to the masses of bodies accelerated by gravity or pushed by compressed springs, and the squares of their velocities. Although stimulated by Bernoulli's physical ideas, Camus fell back on the older methods of geometry and proportions, choosing not to employ the newer concepts of the calculus.

Camus defined an elastic spring as "a body which after having been compressed re-establishes itself nearly or exactly in the same state as before compression." A spring is perfectly elastic if "in re-establishing its state before compression, it returns to the distorting body the same velocity lost by that body."[51] A spring is imperfectly elastic if it does not return all the velocity to the compressing body. Springs with similar elasticity (*ressorts semblables*) are those whose resistances or stiffnesses (*roideur*) are always similar for equal apertures.

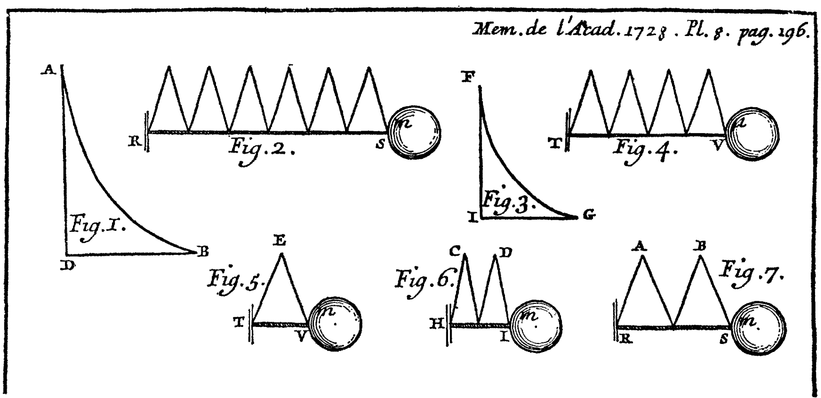

Figure 2. Camus,* Mémoires de l'Académie Royale des Sciences, *1728, p. 196.

In Camus' discussion of the varying resistances of these expanding springs he was essentially using a concept of the potential energy of a spring, $Fs = \frac{1}{2}ks^2$. He argued that if two springs A and B are such that the resistance or stiffness of spring A when it is compressed is to the resistance or stiffness of spring B when it is compressed as the resistance or stiffness of spring A when it is open or held at an aperture of 15° is to the resistance or force of spring B when it is also open or retained at an aperture of 15°, then springs A and B are similar.[52] In modern terms these two springs would have equal ratios of potential energy when compressed and when partially expanded.

[50] Charles-Étienne Camus, "Du mouvement accéléré par des ressorts et les forces qui résident dans les corps en mouvement," *Mémoires de l'Académie Royale des Sciences*, 1728, 159–196.

[51] *Ibid.*, p. 159.

[52] *Ibid.*

Camus then derived the laws of motion pertaining to the compression and expansion of these similar springs in contact with other bodies. He compared the resistance of a compressed spring to the resistance which gravity presents to a body ascending geometrical curves representing the variation in the "force" (i.e., potential energy) over the distance in compressing a spring. He argued that since the distribution of resistance along the curves representing the ascent and the compression of the spring are the same, a body m which rises along the curve will with the same velocity and in the same time compress the spring. A second body, u, compressing a similar spring (i.e., having the same coefficient of elasticity) will meet resistances in the same ratio as body m meets in compressing the first spring.[53]

The laws of accelerated motion for bodies descending along the curves will be the same as those for bodies pushed by expanding springs. Camus derived four relations for expanding springs in the form of proportions:[54]

	Camus' notation	*Modern equivalents*
A.	$f\,t\,t\,\mu\,\epsilon = \phi\,\theta\,\theta\,m\,e$	$Ft^2 = ms$
B.	$f\,e\,\mu\,v\,v = \phi\,\epsilon\,m\,u\,u$	$Fs = mv^2$
C.	$f\,t\,\mu\,v = \phi\,\theta\,m\,u$	$Ft = mv$
D.	$\epsilon\,u\,t = e\,v\,\theta$	$s = vt$

where f, ϕ, and F are force; e, ϵ, and s space; t and θ time; m and μ mass; and v and u velocity. Although Camus obtained proportions for both the *vis viva* and momentum relationships, he did not discuss the relation of momentum to compressed springs. Because he stated his relationships as proportions and did not use infinitesimals, the modern equivalents lack numerical constants and are valid relationships only for the initial and final values of the motions. Thus in the Cartesian Leibnizian debates within the Academy the Frenchman Camus supported a Leibnizian analysis of mechanics, helping to disseminate the *vis viva* viewpoint within the Cartesian stronghold.

VII. LOUVILLE'S IMPULSIVE AETHER

The French Cartesian Jacques Eugène de Louville attacked the problem of the momentum imparted to moving bodies by Bernoulli's elastic springs. In the *vis viva* controversy both the Cartesians and the Newtonians employed the measure mv for force, correcting Descartes' original estimate $m|v|$ to consider the sign of the velocity. It is well known that Newton wrote his second law as "the change of motion is proportional to the motive force impressed," meaning by motive force the impetus $F = mv$.[55] The contact forces which produce these changes in motion in elastic impacts are essentially instantaneous. For the action of continuous noncontact forces such as those found in gravity, magnetism, and centripetal force, Newton indicated that the accelerative quantity of the centripetal force was to be employed. The accelerative quantity is proportional to the velocity which it generates in a given time.[56] In free-fall problems, the Newtonians employed as a measure of the continuous constant gravitational force the proportion $F \propto mv/t$, that is, the impulse Ft.

[53] *Ibid.*, pp. 159–162.
[54] *Ibid.*, p. 169.
[55] I. Bernard Cohen, "Newton's Second Law and the Concept of Force in the *Principia*," *Texas Quarterly*, 1967, *10*:127–157.
[56] *Ibid.*, pp. 130–131.

In the expansion of elastic springs, however, the force is continual and nonconstant over a finite time interval. Louville's problem was to apply the Newtonian-Cartesian concept *mv* to the variable force of an expanding spring. His 1729 essay, "On the Theory of Varying Motions, that is, Continually Accelerated or Retarded, With a Method of Estimating the Force of Bodies in Motion," defined concepts of force which he called instantaneous, actual, and virtual.[57] At the root of his argument lay the concept that force was fundamentally impulsive and that forces were derived from the impulsions of a subtle fluid, a Cartesian aether pervading all space. The impulses of the aether determined acceleration in free fall, as well as in the expansion of elastic bodies. The impulses of expanding elastic bodies were like the impulses of expanding springs. In discussing the action of Bernoulli's springs he arrived at results equivalent to establishing the impulse of expanding springs and the momentum they impart to moving bodies. However, in so doing he considered himself to have refuted the validity of the *vis viva* concept.

The relationship of the velocity to the "force" and mass is expressed as $f = mv$. The definition of force as *mv* has been disputed by clever geometers, said Louville, but he hoped in this memoir to establish it so firmly that no doubt would remain.[58] The equation $f = mv$ shows that when the velocities of two bodies are in a ratio reciprocal to their masses these bodies have equal forces. In the case of gravitation the contact forces of the aether acted as impulses of short duration on a body. Each impulse impressed a small velocity and a small force in each time increment. In the intervals of time between impulses there would be no increase in force or in velocity. Nevertheless the space traversed was continually augmenting. It was because of the impulsive nature of the force causing the body's motion that this motion should be measured by the velocity rather than the space traversed.[59]

In material bodies the force of elasticity comes from the movement of this same subtle fluid. It does not act with its entire mass (*masse*) but only with sufficient and repeated impulses as will halt the motion of another body. Elastic bodies are similar in action to elastic springs. Louville criticized Bernoulli's elastic spring demonstration on the basis of his own concept of "force" as fundamentally impulsive. It is not sufficient to know the magnitude of the dead force or pressure; the number of impulsions in a given time interval must also be known. In all three cases, gravitation, elastic bodies, and elastic springs, the effect produced by the impulses of the aether or the spring will be proportional to the number and magnitude of the impulses in each time interval.[60]

Louville defined *force instantanée* as being equivalent to Bernoulli's *force morte*, or pressure. In describing the force of bodies in motion, however, one must use the concept of *force actuelle*, which is *mv* rather than *force vive*, mv^2.[61] *Force actuelle* resembles our concept of impulse and is equal to *mv*. The purpose of his essay, he said, was to clarify the meaning of *force actuelle*. The force of each impulsion (F_i) communicated only in an instant is "instantaneous force."[62] These are equivalent to

[57] Jacques Eugène de Louville, "Sur la théorie des mouvements variés, c'est à dire, qui sont continuellement accélérés ou continuellement retardés; avec la manière d'estimer la force des corps en mouvement," *Mém. Acad. Sci., Paris*, 1729, 154–184.

[58] *Ibid.*, pp. 154–156.

[59] *Ibid.*, p. 156.

[60] *Ibid.*, pp. 170–171.

[61] *Ibid.*, p.170.

[62] *Ibid.*, pp. 167, 176.

Leibniz' dead forces, or pressures, and to Newton's impressed forces. "Actual force" is the product of the force of each impulsion (F_i) by the number (sum) of impulsions (nF_i) the moving body receives in equal times.

In this essay Louville was struggling to define the impulse of a force which varies with time for the case of expanding springs. These are nonconstant forces; for a compressed spring the force starts at a maximum and decreases as the acceleration of the body starts at zero and increases to a maximum. The total number or sum of the elements of the instantaneous forces in equal units of time in modern notation is $\Sigma_{i=1}^{n} = F_i \Delta t$, or the integral of the impulses $\int_{t_1}^{t_2} F\,dt$. Louville used infinitesimals in parts of his essay but not to express his concept of "force." The total "force" which an expanding spring communicates to a body is Louville's *force actuelle*, equivalent to Ft and mv. Louville considered this to be the correct measure of what Leibniz called living force. Actually he was defining a different concept: the impulse of the force which is equal to the momentum.

Louville also defined a third meaning for force. "Virtual force" (*force virtuelle*) is the force or potential energy of a compressed spring.[63] It pertains chiefly to the accelerations of similar elastic springs, composed of a different number of parts. Each of these units can produce an acceleration during its total expansion. Those springs composed of a great number of parts equally compressed will follow the moving body which it accelerates over a longer time and path than a spring of a lesser number of parts.

Louville's analysis of the *vis viva* controversy was based on a particular concept of force: the nature of force was basically impulse. Using this concept he succeeded in showing that moving bodies acquire a momentum (mv) from the expansion of the compressed springs, but he did not successfully demonstrate that they do not also acquire *vis viva*. This was because he confused Leibniz' and Bernoulli's living force mv^2 with the concept he had defined—impulse—which is equivalent to the change in a body's momentum. Although he employed ideas which were later rendered exact by the calculus, he did not employ its mathematical notation. It remained for Leonhard Euler to express the differential form of Newton's second law in the notation of the calculus.[64]

VIII. MAIRAN AND THE ELIMINATION OF FORCE

The third paper in the series of Academy responses to Bernoulli's essay was that of the Cartesian Jean Jacques de Mairan, secretary of the Academy. His 1728 "Dissertation on the Estimation and Measure of the Moving Forces of Bodies" depended on a concept of "force" defined as the uniform motion of matter.[65] It was primarily an attempt to reduce cases of accelerated and retarded motion, where the *vis viva* principle appeared to hold, to cases of uniform motion where quantity of motion mv is valid. In Mairan's analysis "force" produces uniform motion rather than acceleration.

Mairan's approach is consistent with another of Descartes' principles: the world is to be described by matter in uniform motion. It also shows the influence of Male-

[63] *Ibid.*, pp. 172–173, 177.

[64] See C. Truesdell, "Reactions of Late Baroque Mechanics to Success, Conjecture, Error, and Failure in Newton's *Principia*," *Texas Q.*, 1967, *10*:238–258, p. 247.

[65] Jean Jacques de Mairan, "Dissertation sur l'estimation et la mesure des forces motrices des corps," *Mém. Acad. Sci., Paris*, 1728, 1–49.

branche's rejection of the concept force.[66] At the basis of both Newtonian and Leibnizian mechanics lay the concept of force; to demonstrate that accelerated motions could be described without the concept of force would strike a blow at these systems of natural philosophy competing with Cartesianism.

Mairan's paper was later hailed by the Academy as having settled the issue, perhaps because he was the Academy's secretary. It was this paper, however, which touched off a renewed debate in the 1740s when it was attacked by Madame du Châtelet and as a consequence reprinted.

Mairan presupposed that in most phenomena nature behaved in a perfectly uniform manner with regard to the "forces" of moving bodies.[67] He argued for the importance of using uniform motion in measuring "force." If a "force" does not impede movement, it will produce it. Movement can be uniform or nonuniform, which in turn can be accelerated or retarded. In uniform motion the effect is that of equal spaces traversed in equal times; uniform motion or velocity itself is the space divided by the time. Quantity of motion is measured by the mass times its velocity; that is, by uniform motion. If two bodies A and B of the same mass move uniformly with the same "force" and with the same velocity, but one moves for 1 hour and the other for 2 hours, they have two different quantities of motion, in the ratio of 1 to 2. Those bodies whose movement is not uniform do not represent nature as it is.

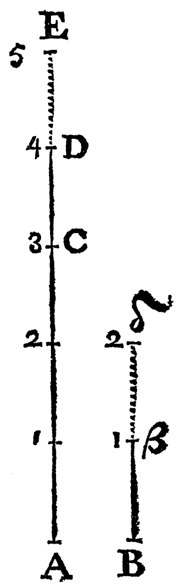

Figure 3. Mairan, **Mémoires de l'Académie Royale des Sciences,** *1728, p. 30.*

Collisions between bodies which produce changes in the world of Descartes' mechanical philosophy do not alter the uniformity of the body's motion, since the collision is essentially instantaneous. Although the magnitude and direction of the motion is altered, the progress remains uniform. Descartes had not dealt specifically with the problem of accelerated and retarded motion which Newton soon after explained by the action of a force. But it was in just such cases—for example, free fall and elastic springs—that *vis viva* proponents had advanced their most successful arguments. Mairan tried to return to the Cartesian description of matter in uniform motion by reducing accelerated and retarded motion to uniform motion. And whereas Newton had shown that a change in motion requires a force $F=\Delta mv$, Mairan defined force as mv, the uniform motion of a body.

For the case of free fall Mairan reduced accelerated motion to uniform motion by the following technique (see Fig. 3). Imagine the ascent of two equal bodies to be possible with uniform motion. Let B travel 2 *toises* in the first instant; A, having double the velocity, will then travel 4 in the first instant.[68] However, under motion retarded by gravity, B, having a velocity of 1, will rise only 1 *toise* by Galileo's relation between average and

[66] Hankins, "Influence of Malebranche," pp. 205–207.

[67] Mairan, "Dissertation," sec. 3.

[68] *Ibid.*, sec. 39.

accelerated motion. The distance *not* traveled, which would be under uniform motion, is $2-1 = 1$. Body A, having a velocity of 2, will rise in retarded motion to a total height of 4, 3 units of which will be traversed in the first instant and 1 in the second instant. Therefore, for body A the distance *not* traveled in the first instant is the uniform motion of A minus its retarded motion, or $4 - 3 = 1$. In the second instant of retarded motion A rises 1 unit, but it would have traveled 2 under uniform motion. The distance *not* traveled in the second instant is again 1.[69] The *spaces not traversed* in each instant represent the force lost or consumed in each instant or the effort of the contrary force which destroys or consumes it. But the sum of all the lost forces or of the contrary forces is equal to the total force, or mv, of the body.[70]

For the case of springs Mairan showed how retarded motion could be reduced to uniform motion and the space traversed in this uniform motion used to measure "force" (see Fig. 4). He conceived his springs to be little elastic bands:

> For example let there be impulsions, obstacles, or any resistances whatever repeated and placed on the path AF of moving body A. [These can be] for example particles of matter 1, 2, 3, 4, etc., or elastic strips (*lames de ressort*) to be displaced, knocked down lifted, or bent.[71]

These elastic bands were similar to the impulsions of gravity in that they offered resistance to an object moving against them.[72]

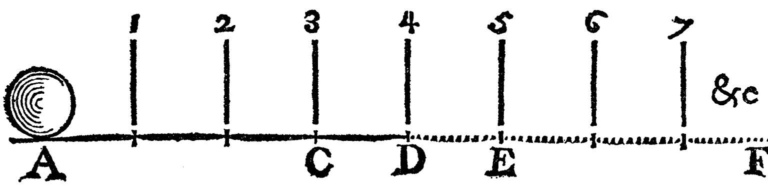

Figure 4. Mairan,* Mémoires de l'Académie Royale des Sciences, *1728, p. 31.

Mairan argued that the momentum of a moving body could be retarded by degrees by these little bands placed at equal intervals in its path. Each one of these bands would offer a resistance equal to that of a body of mass 1 moving with velocity 1. As a body with some initial quantity of motion brushes past these strips it loses mv. Mairan calculated the mv lost by the body in successive instants by the number of bent strips, and he also calculated the number passed by a body in uniform motion in the same time. He then measured the total mv of the body by the difference or total number of strips *not* lifted.[73] He concluded that the portions of matter *not* displaced in retarded motion—the elastic bands *not* lifted, or bent, the objects *not* flattened, and in

69 *Ibid.*
70 *Ibid.*, sec. 43.
71 *Ibid.*, sec. 41.
72 *Ibid.*, secs. 40–41.
73 *Ibid.*

general the obstacles *not* overcome which would be under uniform motion—are proportional to the forces or simple velocities. The obstacles not overcome represent the effect of a contrary force exercising itself against the original force. The sum of the contrary efforts is equal to the total force of the body. Thus by considering the resistance of these various obstacles to be equivalent to momentum he reinterpreted the problem of moving bodies and springs. He analyzed events which did not occur rather than those which did; he appealed to nature, not as it was, but as it was not.

Mairan's elimination of force was one of the unsuccessful attempts to rescue a Cartesian mechanics based on matter in motion. It illustrated the great lengths to which Cartesians were willing to go in order to save their science. The inadequacy of Cartesianism in the face of Newtonian and Leibnizian mechanics was nowhere more apparent.

In the foregoing analysis I have tried to indicate in what way the Cartesian natural philosophy influenced the mechanics of contributors to the publications of the Paris Academy of Sciences during the 1720s. To the extent to which their mechanical points were valid the results became united with Newtonian or Leibnizian mechanics. Thus Mazière and Louville used aspects of the Cartesian worldview to make Newtonian points; Bernoulli and Camus strengthened Leibnizian concepts. However, Crousaz and Mairan, working within the traditional Cartesian framework, unsuccessfully attempted to retain a Cartesian kinematics. In this manner the Cartesian worldview declined in mechanics, for it could not produce a unique and adequate physics of the terrestrial world.