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Section V.

ности механизмов. Соответствующие работы проводятся в СССР, Германии, США. В СССР на базе развития идей Ассура заканчивается формирование советской школы теории механизмов.

Характерной чертой развития науки о машинах были совещания, сначала национальные, а затем и международные. В определенной степени они характеризуют зрелость науки и ее логическую определенность. Первое совещание по чистой и прикладной теории механизмов было созвано научным советом Общества немецких инженеров в 1926 г. Интересно, что на нем присутствовало всего 25 человек. В СССР совещания по отдельным вопросам машиноведения начались еще в 30-х гг., но первое совещание по основным проблемам теории механизмов было созвано в 1954 г. Почти одновременно, в 1953 г. было созвано первое совещание по теории механизмов в США.

Последним периодом, которого следует коснуться в настоящем сообщении, является период современной научно-технической революции. В теории механизмов для начала этого периода характерно расширение исследований динамики машин в реальных условиях их работы, что повлекло за собой создание методики эксперимента; вторым направлением исследований является изучение машин автоматического действия и соответствующих приводных механизмов - кулачковых, гидравлических, пневматических и вибрационных. Третьим направлением можно считать новые методы синтеза механизмов с использованием электронно-вычислительных машин. Наконец, развитие теории механизмов вообще, и теории пространственных механизмов в частности, характеризуется использованием очень обширного математического аппарата, включая новейшие направления математики.

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BERNOULLI'S SPRINGS AND REPERCUSSIONS IN THE VIS VIVA CONTROVERSY

The early phase of the *vis viva* controversy 1686-1716 which surrounded the lifetime of Leibniz was motivated the philosophical considerations concerning the nature of force. A second phase of the controversy in the 1720's concerned itself with the applications of the *vis viva* and momentum principles to three main physical problems. These were collision problems involving hard, elastic, semi-elastic and soft bodies, free fall problems, and compressed spring problems. The problem of the *vis viva* and momentum associated with elastic springs in the 1720's was initially motivated by the question of whether nature was composed of hard or elastic atoms. But it moved toward a discussion of the application of the *vis viva* and momentum concepts applied to expanding springs. The problem of the *vis viva* of elastic springs represents an extension of the *vis viva* principle from the original free fall and collision problems discussed by Leibniz.

In 1722 William's Gravesande considered the action of a spring composed of bent elastic bands.[1] If the unbending of the 1st elastic band imparts 1 unit of velocity to a body at rest against the spring, two additional bands must slacken to add a second unit of velocity to the first and 3 more must slacken to increase the velocity to 3. Thus these three springs do not add 3 units of velocity to the body but only 1 unit. The "force" that the body receives therefore is related to the number of springs which unbend in equal units of time. By a geometrical argument drawn from the areas of iscoseles right triangles whose sides represent equal units of velocity, Gravesande demonstrated that the "force" the body receives is proportional to the square of the velocity. "Force" should therefore be measured by mv^2.

Two years later in 1724 the Swiss mathematician Jean Bernoulli submitted a paper for a contest sponsored by the Academie des Sciences, Paris, which further developed the *vis viva* argument for elastic springs in the mathematical notation of the infinitesimal calculus.[2] Since Bernoulli's paper was based on the argument that nature was composed of elastic rather than hard particles, the paper was disqualified but in 1726 received honorable mention in a contest on elastic bodies. Bernoulli associated the action of elastic springs with the action of elastic particles of matter.

Bernoulli followed Leibniz in championing the mv^2 argument and in using Leibniz's notation for infinitesimals but considered his argument drawn from expanding springs to be superior to Leibniz's initial free fall demonstration of 1686. He argued that a compressed spring has a certain pressure or dead force. This force turns out to be equivalent to the impressed force of Newtonian mechanics. Bernoulli compared springs having equal elastic constants but composed of an unequal number of units expanding against "equal" bodies. As the spring expands, the dead force is converted to living force, mv^2, and imparted to a body set in motion by the spring's expansion. As the moving body accelerates the increment of velocity dv in an increment of time, dt, depends on the pressure or force of the spring: $dv = pdt$. The velocity at any instant is $v = \frac{dx}{dt}$ hence the increment of time dt is $\frac{dx}{v}$. By substitution Bernoulli arrived at the well known result $vdv = pdx$ the integral of which he wrote as $\frac{1}{2} v^2 = \int pdx$. By use of proportions Bernoulli compared the *vires vivae*, mv^2, imparted to accelerated bodies with the pressures exerted by compressed springs of different numbers of elastic units.

Bernoulli's "Discours" containing the *vis viva* argument drawn from compressed springs was printed in 1727. In the next two years three papers written in response to the spring demonstration appeared in the Memoirs of the French Academy. These contributions were made by Abbe Charles Etienne Camus (1728), Jacques Eugene de Louville (1729) and Jean Jacques de Mairan (1728).

Although both 's Gravesande and Bernoulli had related the law of acceleration of bodies moved by expanding springs to that of free fall, Charles Etienne Camus further developed this relationship.[3] His 1728 paper was entitled "On Accelerated Motion due to Springs and the Forces Residing in Moving Bodies." Although stimulated by Bernoulli's spring demonstration Camus fell back on the older method of geometry and proportions rather than employing the newer notation of the calculus.

Camus compared the resistance of a compressed spring geometrical curves representing the variation in the "force" (i.e. potential energy) over distance in compressing a spring.[4] He argued that since the distribution of resistance along the curves representing the ascent and the compression of the spring are the same, a body m which rises along the curve will the same velocity and in the same time compress the spring. A second body, m, compressing a similar spring (i.e. having the same coefficient of elasticity) will meet resistance in the same ration as body m meets in compressing the first spring.[5]

The laws of accelerated motion for bodies descending along the curves will be the same as those for bodies pushed by expending springs. Camus derived four relations for expanding springs in the form of proportions.[6]

Their modern equivalents in the for of equations are:

1. $Ft^2 = ms$,
2. $Fs = mv^2$,
3. $Ft = mv$,
4. $s = vt$.

Because the original formules were written as proportions the 1/2 does not appear in equations 1 and 2. Having derived relations for expanding springs in terms of both the mv and mv^2 principles Camus did not procede to treat both with equal validity. Following Bernoulli he emphasized dead force in which there is pressure but no traversal of space and living force in which the obstacles which the units of the spring present to moving bodies are overcome in compression.[7]

The problem of defining the impulse of an expanding spring was taken up in a paper by Jacques Eugene de Louville (1729) entitled "On the Theory of Varying Motions , i.e., Continually Accelerated or Retarded, with a Method of Estimating the Force of Bodies in Motion".[8] Newton had written his second law in the form, "the change of motion is proportional to the motive force impressed" thus describing the instantaneous changes of motion in elastic impacts where $F = \Delta mv$. In the action of continuous non-contact forces as in gravitation, Newton indicated that the velocity generated in a given time should be employed. The prob-

3. Abbé Charles Etienn Camus. "Du Mouvement accéléré par des ressorts, et les forces qui résident dans les corps en mouvement", Mémoires de l'Academie Royale des Sciences, Paris, 1728, p.159.

4. Ibid., p.159.

5. Ibid., p.160.

6. Ibid., p.169.

7. Ibid., p.190.

8. M. le Chevalier Jacques Eugène de Louville. "Sur la théorie des mouvements varies, c'est-à-dire, qui sont continuellment accélérés ou continuellement retardés; avec la man-ière d'estimer la force des corps en mouvement", Mém. Acad. Sci., Paris, 1729, p.154.

9. Ibid., p.167.

10. Ibid., p.172.

11. Ibid., p.178.

12. Jean Jacques de Mairan. "Dissertation sur l'estimation et la mesure des forces motrices des corps", Mém. Acad. Sci., Paris, 1728, p .1-2,3,8.

13. Ibid., sec.3.

14. Ibid., sec.41.

lem of applying mv to the expansion of springs where the force is continuous and non-constant over a finite time interval was taken up by Louville in his response to Bernoulli's "Discours". Althoug Louville used equations rather than proportions in some of his calculations he did not use the notation of the calculus to express the variable force of the spring.

Louville defined force instantanée to be equivalent to Bernoulli's dead force.[9] He then argued that force actuelle which is measured by mv is the proper measure of the "force" of motion rather than Bernoulli's force vive, mv^2.

In the expansion of a compressed spring the force of each impulsion is communicated in an instant to a body in contact with the spring. These instantaneous forces are equivalent to Leibniz' and Bernoulli's dead forces and also to Newton's impressed forces. Over the time of the spring's expansion these forces become force actuelle, the product of the force of each impulsion by the number of impulsions the moving body receives in equal times.[10] In modern notation we would say that total number or sum of the elements of instantaneous force in equal units of time is $\sum_{i=1}^{n} F_i \Delta t$ or the integral of the impulses $\int_{t_1}^{t_2} F dt$. Louville thus used the concept of a summation over equal time intervals without using the integral notation. He stated that the force lost by the spring in accelerating the moving body is equal to the force gained by the body, i.e. mv.[11] He considered himself to have refuted Bernoulli's demonstration that a body acquires an mv^2 when accelerated by an expanding spring.

The third paper in the series of French Academy responses to Bernoulli's "Discours" was that of the secretary Jean Jacques de Mairan in 1728: "Dissertation on the Estimation and Measure of the Moving Forces of Bodies".[12] Hailed by Academy as having settled the controversy, the paper was primarily an attempt to reduce cases of accelerated and retarded motion to cases of uniform motion where Descartes' measure of force, mv was valid.

Mairan argued that the momentum of a moving body could be retarded by degrees by little elastic bands placed at equal intervals in its path.[13] Each one of these bands would offer a resistance equal to that of a body of mass 1 moving with velocity 1. As a body with some initial quantity of motion brushes past these strips it loses _mv_. Mairan calculated the _mv_ lost by the body in successive instants by the number of bent strips. He also calculated the number passed by a body in uniform motion in the same time. He then measured the total _mv_ of the body by the difference or total number of strips _not_ lifted.[14] He concluded that the portions of matter _not_ displaced in retarded motion, the elastic bands _not_ lifted, or bent, the objects _not_ flattened and in general the obstacles _not_ overcome which would be under uniform motion are proportional to the forces or simple velocities. The obstacles not overcome represent the effect of a contrary force exercising itself against the original force. The sum of the contrary efforts is equal to the total force of the body. It was Mairan's use of uniform motion in estimating force for cases of retarded motion which inspired Madame du Châtelet in 1740 to write a long criticism of Mairan in her defense of the Leibnizian position in dynamics. Compressed spring arguments appeared in the 1740's in contributions of Du Châtelet, Mairan, Abbé Deidier, James Jurin and Jean d'Alembert. Spring problems therefore were an important vehicle by which the meaning and limits of the _vis viva_ principle were clarified.

Footnotes

1. William 'sGravesande. "Essay d'une nouvelle theorie sur le choc des corps", Journal literaire, 1722, _12_, 23.

2. Jean Bernoulli, "Discours sur les liox de la communication du mouvement" in the Recueil des pieces qui a remporté les prix de l'Academie Royale des Sciences, 1727, _2_, p.39.