6. Carolyn Merchant
Sonderdruck

AKTEN DES II. INTERNATIONALEN LEIBNIZ-KONGRESSES HANNOVER, 17. — 22. JULI 1972

BAND II

WISSENSCHAFTSTHEORIE UND WISSENSCHAFTSGESCHICHTE

FRANZ STEINER VERLAG GMBH · WIESBADEN
1974

Leibniz' Concept of Force: Physics and Metaphysics

by CAROLYN ILTIS (SAN FRANCISCO)

A scientific system can be conceived as a reorganization of information around a central core concept. Such a scientific system is illustrated by Leibnizian dynamics which gives a primary ontological status to the older concept of "force", defined as *vis viva*, $\underline{mv^2}$. I shall first analyze the elements of Leibniz' process of reorganization of the mechanical philosophy around the idea of "force". These foundations are philosophical and ontological. Secondly, I shall discuss the specific mechanical phenomena which are organized by the *vis viva* principle showing the limitations in Leibniz' use of the principle.

A scientific system is often described as a culmination of various strands of thought which become united in the work of one man. For example, seventeenth century mechanics is sometimes seen as a union of the Democritean tradition with the Pythagorean[1]. Or a system may be viewed as the addition of a new concept to those of an older system. Thus Newton is said to have added the concepts of force and space to those of matter and motion which constituted the mechanical philosophy of the 17th century[2]. Again, a new ordering of nature is seen as a "transposition in the mind of the scientist", "putting on a different kind of thinking cap", or picking up the other end of the stick, a vague metaphorical expression for the creative process of looking at a phenomenon from a new or opposed viewpoint[3]. Or perhaps a new system is formed by "seeing" the smoother pebbles amidst the maze of irregular chips along the vast shoreline[4].

These viewpoints do not adequately describe the process which takes place in the formation of scientific systems of thought. Yet in some of the above descriptions there is the suggestion that a transforming element is present in the formation of such systems. A reorganization is a transformation[5]. But more specifically it is a transformation which takes an older concept present in men's thought for decades or even centuries, redefines it, and endows it with a central and primary status. The concept then carries the role of an organizing principle around which phenomena can be ordered and explained. In Leibnizian dynamics, this process took place around the concept of "force" defined to mean the mathematical measure mv^2 which Leibniz called *vis viva*. It has often been said that Leibniz generalized and extended the *vis viva* principle of Huygens and was the first to show its philosophical import as the foundation of a new science[6]. The ideas "generalization" and "extension" fail to explain the process by which dynamics was created. The creation of a science of dynamics took place when force was given a central role in the mechanistic explanation of nature reconstructing the old view of matter in motion.

This process occurred early in Leibniz' series of writings on the *vis viva* concept, that is, by 1686, but fairly late in the intellectual development which represented the results of his early criticism and questioning. According to Leibniz' own statements this culmination in a new system of thought took place by 1686 when he was forty years old[7].

We can examine the events of 1686 with the idea that a transformation of the mechanical philosophy took place based on a new metaphysics which constituted a new ontological foundation for natural science. It is useful to recall that one year later in 1687 Newton's own reorganization of mechanics appeared, likewise based on the core concept "force"[8]. But Newton's "force" was a *vis impressa*, the *vis mortua* of Leibniz and carried the connotation of an external force rather than the inherent inner force of *vis viva*. Furthermore, although Newton gave the concept of force a central and primary status in explaining the phenomena of gravitation and terrestrial mechanics, the Principia did not present an ontological reorientation of the mechanical philosophy[9]. This was published in Newton's later investigations of the foundations of the new system presented mainly in the queries to the Optics[10]. For Newton, therefore, a phenomenological reorganization formed the problem of the Principia, although a concern with its ontological foundations is evident throughout his scientific thought. Leibniz began with an ontological and phenomenological reorientation and then proceeded to illustrate the new system by analyzing specific mechanical phenomena, while concurrently developing a more sophisticated philosophical analysis of "force".

In 1686 Leibniz presented two important works, simultaneously mechanical and philosophical, in which he publically announced his concept "force". In the Discours de Métaphysique he wrote:

This force is something different from size, figure and motion, and from this

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we conclude that not everything which is conceived in a body consists solely in extension and its modifications, as our moderns have persuaded themselves . . . And although all particular phenomena of nature can be explained mathematically or mechanically by those who understand them, it becomes more and more apparent that the general principles of corporeal nature and of mechanics themselves are nevertheless metaphysical rather than geometrical and pertain to certain forms or indivisible natures as the causes of what appears . . . .[11]

The Brief Demonstration Concerning a Notable Error of Descartes which was more specifically mechanical, announced:

There is thus a big difference between motive force and quantity of motion . . . . It seems that force is rather to be estimated from the quantity of the effect it can produce; for example, from the height to which it can elevate a heavy body of a given magnitude and kind but not from the velocity which it can impress upon the body.[12]

Leibniz had learned both Cartesian mechanics and Huygensian kinematics while in Paris in the years 1672–1676[13]. By 1669 he was familiar with the critique of Descartes' principle of the conservation of quantity of motion which had been made by Wallis, Wren, and Huygens. Huygens had employed the principle of mv^2 in manuscripts as early as 1656 and used its conservation in his 1669 Rules of Motion. But in Paris, Leibniz learned other essential elements of vis viva conservation from his contact with Huygens. The technique of converting a horizontal motion into a vertical motion of which the height is proportional to the square of the velocity, as well as the use of the principle of the impossibility of perpetual motion were both utilized by Huygens and taken over by Leibniz[14]. The relationship between the height and the square of the velocity were, of course, Galilean,the mechanical principle of the conservation of mv^2 was historically, therefore, an older concept.

This mechanical principle, merely a part of Huygens' kinematics, became central to Leibniz' dynamics when transformed by ontological considerations. Leibniz' dissatisfaction with Descartes' philosophy had begun in Paris and included disagreements over his mechanics[15]. Conservation of an absolute quantity of motion mv was blatantly false. If something was to be conserved in nature it must be another absolute quantity, mv^2. This was the mathematical manifestation of a single underlying substance, a real and unifying basis for nature. It served as an organizing principle around which to interpret the phenomena of elastic collisions, pendulum motion, and free fall. Leibniz was convinced a priori, on metaphysical grounds that "force" was conserved in nature. He successfully argued in the Brief Demonstration of 1686 that mv^2 not mv was the mathematical measure of this "force", He was convinced that the phenomenal world could be explained on a strictly mechanistic basis. But as I shall show in a number of examples, he did not successfully establish the conservation of mv^2 in particular mechanical cases because he lacked an adequate idea of the importance of specifying closed conservative systems for these cases. This stemmed from his failure to make explicit the mechanical interactions involved in a given situation. In several instances he argues along the lines: let us suppose that the total 'force' (mv^2) of the first body can be transferred to the second, the first remaining at rest in its place . In many instances this total transfer of force is not mechanically possible unless additional unspecified mechanisms are employed.

I think that Leibniz was led to neglect these mechanical interactions because of his view that point centers of force, his simple substances, do not interact on a metaphysical level, yet conserve the total absolute "force" in the universe during the unfolding of their monadic lives. The world of "well founded" phenomena operated mechanistically, but the real world of substances operated according to final causes. Since "force" was conserved metaphysically the specific mechanisms for its conservation were perhaps unimportant to Leibniz. But they are important to mechanics and therefore it is of interest to point out these limitations in Leibniz' application of his vis viva principle[16]. These limitations were not encountered in his analysis of elastic collisions but rather in problems involving free fall, inclined planes, and pendulum motion.

The Brief Demonstration of 1686 for example purports to supply conservation arguments. Its full title reads: A Brief Demonstration of a Notable error of Descartes and others Concerning a Natural Law According to which God is Said to Conserve the Same Quantity of Motion; A Law which They Also Misuse in Mechanics[17]. Leibniz also mentioned conservation three times in his introductory paragraph: 1. "It is reasonable that the sum of motive force should be conserved in nature", 2. "This led Descartes who held motive force and conservation to be equivalent to assert that God conserves (conservari) the same quantity of motion in the world."[18]

But this demonstration of Descartes' error is not a demonstration that his measure, mv ist not conserved, as the quotes imply. In the example Leibniz chooses, two bodies fall to the ground side by side never touching. There is no mechanical interaction on the basis of which any kind of mechanical conservation statement could be made. Leibniz did successfully establish mv^2 as a measure of force but did not empirically demonstrate its con-

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In the Discourse on Metaphysics of the same year, Leibniz presented the same demonstration with the same implications concerning conservation[19].

During the years 1689–1691 Leibniz carried on a controversy with the Cartesian Denis Papin. The argument centered around a mechanical example suggested by Leibniz specifically for the purpose of demonstrating conservation of vis viva. A small weight was allowed to descend along an inclined plane where it met a larger weight at rest in the horizontal plane. All the force of the first body was transferred to the second body and the first body remained at rest in its place. By the use of levers and inclined planes Leibniz argued that the perpetual motion could arise if force were measured by mv rather than mv^2.[20] Since no new force had been contributed or absorbed by other agents Leibniz concluded "against the Cartesians that quantity of motion should not always be conserved".

Papin shrewdly replied that he would concede the argument if Leibniz could devise a method by which all the force of the first body could be transferred to the second body[21].

In this example which was conceived to demonstrate conservation, Leibniz was equally vague about the mechanical methods for the transfer of "force". A collision between the two bodies would not result in a transfer of all the vis viva since rebound would occur. A device such as an ideal spring would have to be utilized to store and then release the vis viva. This would conserve vis viva but not momentum or quantity of motion. Leibniz himself suggested two methods for accomplishing this transfer claiming other demonstrations had been left with a friend in Florence. However none of them were physically feasible.

The same example, designed to demonstrate conservation and containing the same vagueness concerning the transfer of force can be found in a text of 1692 written in the formal structure of definitions, axioms and postulates[22].

A third example of Leibniz' lack of attention to mechanical details appeared in Specimen dynamicum in 1695[23]. Here instead of discussing conservation he argued from an equivalent principle, equality of total cause and effect. A simple pendulum descends from a certain height. At the vertical an equipollent body is substituted for it. What velocity will the equipollent body have so that it will not rise to a height so great as to achieve perpetual motion? If mv is the measure of "force" perpetual motion will arise.

The mechanism for carrying out this thought experiment under idealized conditions is subject to the same difficulties as the former experiment. A collision will not accomplish the required total transfer; an external force such as an ideal spring could store the vis viva but would violate mv conservation.

In all these examples Leibniz' object is to show that mv^2 is a more adequate conservation principle than quantity of motion. Although on philosophical grounds he succeeded well, on mechanical grounds his examples created confusion and opened unanswered questions concerning the details of closed conservation systems. Nevertheless his creative reorganization of mechanics around the vis viva principle opened possibilities for its further extension and application to new problems.

Notes

1 Richard Westfall, Force in Newton's Physics, New York, 1971, p. 148. Alexandre Koyré, Newtonian Studies, Chicago, 1965, p. 12.
2 Westfall, ibid., p. 323, Koyré, ibid., pp. 12–13.
3 Herbert Butterfield, Origins of Modern Science, 1300–1800, New York, 1957, pp. 1,5.
4 John Herivel, Newton's Achievement in Dynamics, in: Texas Quarterly, 1967, 10:117.
5 Herivel, op. cit. p. 117: "If one recalls that Newton's attention may originally have been drawn to the notion of deviation by his study of Descartes' treatment of centrifugal force . . . it would . . . serve only to emphasize the transforming power of his genius." Westfall, op. cit. (note 1): "Vis viva was not as far removed from the mechanical philosophy as Leibniz perhaps thought. What it raised to a metaphysical status and gave a new quantitative statement, was a direct descendant of the concept of force prevalent throughout the seventeenth century."
6 René Dugas, Mechanics in the Seventeenth Century, Neuchâtel, Switzerland, 1958, p. 453. M. Gueroult, Dynamique et Métaphysique Leibniziennes, Paris, 1934, pp. 82–97.
7 John Theodore Merz, Leibniz, Edinburgh and London, 1884, p. 1.
8 Isaac Newton, Principia Mathematica, lst edition, London, 1687.
9 Pierre Costabel. Newton's and Leibniz's Dynamics, trans. J.M. Briggs, Jr. in Texas Quarterly, 1967, 10: 121: "In this 'method of philosophy' (i. e., that of the Principia) huic philosophandi modo, the question of knowing if force is real or not, if it is a primary notion or not, is not formally posed nor does it emerge. The question is one of 'deriving' a way of arguing from mechanical phenomena translated into mathematical terms and in which the 'demonstrative' scheme should be applicable throughout. The mechanics of Newton is truly a Mechanica rationalis, but it is not a dynamics in the sense of Leibniz."
10 Newton's concern with the ontological foundations of force has been discussed J. E. McGuire, Force, Active Principles, and Newton's Invisible Re-

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alm, in: Ambix, 15:154–208, See also: Alan Gabbey, Force and Inertia in Seventeenth - Century Dynamics, in: Studies in History and Philosophy of Science (1971), 2:11: "If it is true therefore to say with Costabel, that Principia is not Leibnizian dynamics, that it does not offer an ontology of force, it would be a mistake to assume that the same holds for the whole of Newton's thought on natural philosophy. This is not to say that we can recognize in the Queries and Manuscripts the Leibnizian themes I have been discussing, or that Newton gives there a metaphysical foundation for force with anything like the same assurance or philosophical sophistication that we find in Leibniz or Descartes. Newton's characteristic caution disinclined him from proffering a physics tied to an intricate metaphysical schema, in lieu of empirically grounded and experimentally verifiable knowledge. Nonetheless, he did make some attempt to work out an ontology of force which significantly broadens the functionalist perspectives of the Principia."

11 Leibniz, Discourse on Metaphysics, in: Philosophical Papers and Letters, edited by Leroy E. Loemker, Chicago, 1956, 1:484–485.
12 Leibniz, A Brief Demonstration of a Notable Error of Descartes and Others Concerning a Natural Law, Loemker, ibid., p. 457.
13 Merz, op. cit. (note 7), pp. 41, 44, 57, 66. Gueroult, op. cit. (Note 6), pp. 87–90.
14 Gueroult, ibid., pp. 91, 93–94.
15 Merz, op. cit. (note 7) pp. 66–67.
16 Some of these examples are analyzed in greater detail in Carolyn Iltis, Leibniz and the Vis Viva Controversy, in: Isis (1970) 62:21–35.
17 Leibniz, Brief Demonstration, Loemker, op. cit. (note 12), p. 455.
18 Ibid. p. 456. Italics added.
19 Leibniz, Discourse on Metaphysics, Loemker, op. cit., I, pp. 482, 483. "Our new philosophers commonly make use of the famous rule that God always conserves the same quantity of motion in the universe . . . . Now it is reasonable that the same force should be conserved in the universe . . . . So these mathematicians have thought that what can be said of force can also be said of the quantity of motion."
20 Italics added. Leibniz, De Causa gravitatis et defensio sententiae suae de veris naturae legibus contra Cartesianos, in: Acta Eruditorum, May 1690, pp. 228–239.
21 Denis Papin, Mechanicorum de viribus motricibus sententja, asserta adversus cl. G. G. L. objectiones, in: Acta Eruditorum, Jan., 1691, pp. 6–13.
22 Pierre Costabel, Leibniz et la dynamique, Les textes de 1692, (Paris, 1960), pp. 97–106.
23 Leibniz, Specimen Dynamicum, Loemker, op. cit. (note 11), 2, p. 727.

Verzeichnis der Abkürzungen

A	=	Leibniz: Sämtliche Schriften und Briefe. Hrsg. von der Preußischen (später: Deutschen) Akademie der Wissenschaften zu Berlin. Darmstadt (später: Leipzig, zuletzt: Berlin) 1923 ff.
C	=	Opuscules et fragments inédits de Leibniz. Extraits des manuscrits . . . par Louis Couturat. Paris 1903 (Neudruck: Hildesheim 1961).
Dutens	=	Leibniz: Opera omnia, nunc primum collecta . . . studio Ludovici Dutens. T. 1–6. Genevae 1768.
E	=	Leibniz: Opera philosophica quae extant latina, gallica, germanica omnia. Ed. Joannes Eduardus Erdmann. 1.2. Berolini 1839–40 (Neudruck: Aalen 1959).
El	=	Leibnitiana. Elementa philosophiae arcanae de summa rerum. Ed. I. Jagodinskij. Kasan 1913.
GM	=	Leibnizens mathematische Schriften. Hrsg. von C.I. Gerhardt. Bd. 1–7. Berlin (später: Halle) 1849–1863 (Neudruck: Hildesheim 1962).
GP	=	Die philosophischen Schriften von Gottfried Wilhelm Leibniz. Hrsg. von C.I. Gerhardt. Bd. 1–7. Berlin 1875–1890 (Neudruck: Hildesheim 1960–1961).
Grua	=	Leibniz: Textes inédits . . . Publiés et annotés par Gaston Grua. 1.2. Paris 1948.
Klopp	=	Die Werke von Leibniz gemäß seinem handschriftlichen Nachlasse in der Königlichen Bibliothek zu Hannover. Hrsg. von Onno Klopp. Erste Reihe: Historisch-politische und staatswissenschaftliche Schriften. Bd. 1–11. Hannover 1864–1884.
LH	=	Niedersächsische Landesbibliothek Hannover: Leibniz-Handschriften.
Loemker	=	Leibniz: Philosophical Papers and Letters. A Selection. Transl. and ed. by Leroy E. Loemker. vol. 1–2. Chicago 1956.
Mollat	=	Mittheilungen aus Leibnizens ungedruckten Schriften. Von Georg Mollat. Neue Bearbeitung. Leipzig 1893.
NE	=	Nouveaux Essais sur l'entendement humain.
Pertz	=	Leibnizens gesammelte Werke. Aus den Handschriften der Königlichen Bibliothek zu Hannover hrsg. von Georg Heinrich Pertz. Erste Folge: Geschichte. Bd. 1–4. Hannover 1843–1847 (Neudruck: Hildesheim 1966).