CHAPTER II

The Laws of Impact: Wallis, Wren, and Huygens

In the year 1668, 24 years after the publication of Descartes' Principia Philosophiae, the correct laws of impact, already discovered through private researches, were brought together and publically made available through the efforts of the Royal Society of London. The principle of $m|v|$ in collisions was corrected to the conservation of mv by Wallis, Wren, and Huygens and the principle of the conservation of mv^2 in elastic impact was stated by Huygens. The discovery of these laws of impact will be treated in detail since confusion was later wrought by participants in the controversy over living force who said that either the mv equation was correct or that the mv^2 relationship was true, but not both.

The problem of the collision of bodies was brought to the attention of the Royal Society in October 1666, by the following experiment:

> An experiment was tried of the propagation of motion by a contrivance whereby two balls of the same wood, and of equal bigness, were so suspended, that one of them being let fall from a certain hight against the other, the other was impelled upwards to near the same hight, from which the first was let fall, the first becoming then almost quiescent, and the other returning, impelled the first upward again to almost the

> hight it had fallen from before, itself becoming then in a manner motionless, till after some returns they both vibrated together. It was ordered that this experiment be prosecuted and others of that kind thought upon.[1]

On January 16, 1667,

> It was mentioned by Mr. Oldenburg that the council had thought fit, that the experiments for making out a theory of the laws of motion formerly begun by Dr. Wren, Dr. Croune, and Mr. Hooke... should be prosecuted. The Society thereupon desired Dr. Wren to give in those experiments of motion devised by himself; but he alledging that the account of them was at Oxford, Dr. Croune and Mr. Hooke were desired to bring in theirs.[2]

Then on October 22, 1668, Hooke suggested that the experiments on the laws of motion be continued, but the president, Brouncker, remarked that Huygens and Wren had already found a theory sufficient to explain the phenomena involved. Accordingly it was decided that the secretary, Oldenburg, should invite these two to communicate their results to the Royal Society.[3]

Much of the November 12 session was devoted to a discussion of the problem of impact, in the course of which it was decided to issue the same invitation to John Wallis.

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[1]Thomas Birch, <u>The History of the Royal Society of London for improving of Natural Knowledge from its First Rise, in Which the Most Considerable of Those Papers Communicated to the Society Which Have Hitherto Not Been Published Are Inserted in Their Proper Order As a Supplement to the Philosophical Transactions</u>, London, 1756, <u>2</u>, 116-117(October 17,1666).

[2]Birch, <u>History of the Royal Society</u>, <u>2</u>, 140.

[3]Birch, <u>History of the Royal Society</u>, <u>2</u>, 315.

The experiment of the communcation of motion was tried by a contrivance, whereby three balls of the same wood, and of near bigness, were so suspended, that either of the two extremes being let fall from a certain hight against the intermediate ball, the other extreme was impelled upwards to near the same hight, from whence the first was let fall, that in the middle moving but very little; of which the president conceived this to be the reason, that the intermediate ball when struck by one of the lateral ones, found the resistance of the lateral ball; but the other lateral ball met with no resistance but that of the air.

Mr. Hooke was ordered to think upon other experiments for making out this hypothesis about motion, which is, that no motion dies, nor is any motion produced anew.

Sir Robert Moray moved, that bodies might be provided of several degrees of hardness, and of the same matter and weight, as steel bodies, and the like to see whether the harder they are, the more they will rebound.

Others moved that bodies might be provided that had no springiness, or but little, to see, how much that quantity contributed to the rebounding.

It was also moved, that since the society was upon the disquisition of the nature, principles, and laws of motion, all authors, who had written on that subject and delivered their hypotheses concerning it, might be consulted and examined, and an account of their opinions brought in, to see, what had already been done in this matter. Whereupon Mr. Collins was desired to persue such authors, and particularly DesCartes, Borelli, and Marcus Marci; and Mr. Oldenburg was desired to write to Dr. Wallis that he should take a share of this work.

Mr. Oldenburg read a letter from Monsr. Huygens, dated at Paris November 13, 1668, N. S. in answer to what he had lately written to him by order of the society, desiring him, that if he did not yet think fit to print what he had discovered on the subject of motion, he would impart to them his theory of it, together with such experiments as he grounded his theory upon. ~~together with such experiments as he grounded his theory upon.~~ Mnsr. Huygen's answer was, that he was ready to communicate to the society those rules and theorems, which he had found out in all species of motion, not doubting the society would secure to him the honor of that discovery by giving it place in their Register-book, as coming from him. And as he desired to know what part of motion the

> society would have him treat first, the secretary was ordered to acquaint him that the society left that to his discretion, not doubting but that he had treated the subject methodically; and therefore would begin with such particulars as were simplest and clearest, giving light to what should follow. His letter was ordered to be entered in the Letter-Book.[z] ([z]Vol. ii, 321)[4]

The three manuscripts were received and read within a short time, Wallis' on November 26, 1668, Wren's on December, 17, and Huygens on January 7, 1669.

In his paper, "A Summary Account of the General Laws of Motion", John Wallis gave the laws of motion for collisions in which two hard bodies suffering no distortion adhere and move as one after the impact.[5]

In three rules he describes three cases of "hard body" (dura) collisions. His hard inflexible bodies obey the laws of inelastic impact. In this paper Wallis wrote: "If the bodies were perfectly hard there would be no rebounding and therefore it was necessary to have recourse to elasticity. If the bodies are elastic, yielding to the stroke and then restoring themselves by an equal force, the bodies instead of moving together may recede from one another and that more or less in proportion to the restoring force....

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[4]Birch, History of the Royal Society, 2, 320.

[5]John Wallis, "A Summary Account of the General Laws of Motion, by way of a Letter Written by Him to the Publisher and Communicated to the Royal Society, November 26, 1668," Phil.Trans., 3 (1669) 864-866.

That all rebounding comes from springynesse is my opinion."[6]

He introduces these rules with some preliminary considerations of the proportions holding between force (vis), V, weight (pondus), P, and speed (celeritas), C. He did not distinguish between mass and weight.

> If an agent A produce the effect E, then the agent 2A will produce the effect 2E, 3A will produce 3E etc. and universally, mA will produce the effect mE putting m for the "exponent" /rationis exponens/ of any ratio. If the power of force, V, can move a weight, P, with a speed, C, then the force, mV, will either move the weight, P, with a speed, mC, or the weight, mP, with the same speed or lastly any weight with such a speed that the product of the weight and speed is mPC.[7]

In case (1) a weight, P, moved with speed C by force, V = PC, directly collides with weight, mP, at rest and free to move. After the impact the two will be moved together by

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[6] Wallis, Ibid., 866. Translation is by Joseph Frederick Scott, The Mathematical Work of John Wallis, (1616-1703), London, 1938, 104. On the meaning of hard body for Descartes, Wallis, Wren, and Huygens see this dissertation, Ch. I, p. 24 , note 18. In his treatise DeMoto, "in discussing impact Wallis observes that bodies fall into three classes. Bodies which suffer no distortion upon impact are termed absolutely hard. Those which are endowed with the faculty of altering their shape upon impact and immediately recovering it, are said to be elastic whilst the third class, soft bodies suffer distortion and do not recover their shape. Here a part of the force of percussion is used up in deforming the body." J. F. Scott, Ibid., 102.

[7] Translation taken from J. F. Scott, op. cit., 103. The "exponent of the ratio" is the quotient obtained by dividing the antecedent by the consequent. For example 6 is the exponent of the ratio that 30 has to 5; and m is the exponent of the ratio that mE has to E.

the force, PC, but the greater weight $(1 + m)P$ will now cause the joint speed to be diminished to $\frac{1}{1+m}C$. This "impetus" of the two bodies together will be made up of the final "impetus" of body A i.e. $\frac{1}{1+m}PC$ and that of body B, $\frac{1}{1+m}mPC$ (i.e., PC).

In case (2) a weight P, moved with speed C by force $V = PC$, directly collides with a weight mP, moving more slowly with speed $nC < C$ and force $mnV = mnPC$. The two together will be moved by a force $(1 + mn)PC$ composed of the two weights $(1 + m)P$ and the speed $\frac{1 + mn}{1 + m}C$. The resultant force of motion, $(1 + mn)PC$, will be composed of the two forces: $\left(\frac{1 + mn}{1 + m}\right)PC$ of body A and $\left(\frac{1 + mn}{1 + m}\right)mPC$ of body B(i.e., $(1 + mn)PC$).

In case (3) a weight, P, moved with speed C by force, $V = PC$, collides with weight, mP, moving in the contrary direction with speed, $-nC$, and force $-mnV = -mnPC$. The final force of the two weights together will be $(1-mn)PC$. composed of the combined weights $(1+m)P$ and the speed $\frac{1-mn}{1+m}C$. This final force will be made up of the two forces, $\frac{1-mn}{1+m}PC$ of body A and $\frac{1-mn}{1+m}mPC$ of body B i.e., $(1-mn)PC$. Thus the direction is taken into account as shown by the use of the positive and negative signs, the correct law for the conservation of momentum for such bodies being thereby given.

Wallis also found that for oblique collisions the "impetus" of the body striking is to that of its direct "impetus" "as the radius to the secant of the angle of obliquity".

Elastic impact was not discussed in this short presentation but Wallis added this subject to his treatment of percussion in his Mechanica sive de Motu, (London, 1669-1671). Here an elastic body was defined as one which has in impact sufficient force of restitution to recover its shape after compression.[8]

The propositions given here are as follows:

> If a heavy body impinge upon a solid [immovable] obstacle, and if either one or the other be elastic, the body will rebound with the same speed and in the same right line. [This assumes equality of deforming and restitutive forces.] Prop. I.
>
> If the two striking bodies are unequal, but have their velocities inversely proportional to their masses (so that their momenta are equal) each will rebound with the same speed that it approached, and in the same right line. (Prop. 3)
>
> If a heavy body impinge directly upon an equally heavy body at rest, the moving body will be brought to rest and the other will move forward with the velocity possessed by the striking body. (Prop. 5)
>
> And generally; If a heavy body moving with any speed whatever impinge upon a heavy body at rest, and if one or the other be elastic, the final speed of the striking body will be to its speed before impact as the difference of the masses to their sum, and will be forward, backward, or stationary according as the mass of the striking body is greater than, less than, or equal to that of the body at rest.

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[8] John Wallis, Mechanica sive de Motu, 2 vols., London, 1669-1671, 1, 660-682.

> The stationary body however will acquire a speed which bears to the original speed of the striking body the ratio of double the mass of the striking body to the sum of the masses. (Prop. 8)[9]

The summary of these laws in modern language is given in a quote from J. F. Scott:

> When two bodies impinge the sum of their momenta along the line of impact, is the same as before, and (for bodies perfectly elastic) the relative velocity after impact is the same as that before impact, though reversed in direction. For if M and m are the masses, U and O (zero) their speeds before impact, V and v their speeds after impact, we have the principle already enunciated.

$$MV + mv = MU + 0$$

and $$V - v = -U$$

whence $$V = \frac{MU - mU}{M + m}$$

and therefore $$\frac{V}{U} = \frac{M - m}{M + m}$$

i.e. $\frac{\text{Speed of striking body after impact}}{\text{Speed before impact}} = \frac{\text{Difference of masses}}{\text{Sum of the masses [of striking body and struck body]}}$

Again from the above relations

$$v = \frac{MU + MU}{M + m}$$

whence $$\frac{v}{U} = \frac{2M}{M + m}$$

i.e. $\frac{\text{Speed acquired by stationary body}}{\text{Original Speed of striking body}} = \frac{\text{Double the mass of the striking body}}{\text{Sum of the Masses [of striking body and struck body]}}$

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[9] Wallis, Mechanica, 660-669. Translation and summary taken from J. F. Scott, p. 122, 123.

Christopher Wren's paper was read to the Royal Society on December 17, 1668.[10] In it he developed a rule comparable to the conservation of momentum for elastic bodies in terms of the body's "proper velocity" (propriās velocitates). Like Wallis he did not distinguish between mass and weight. If a body has a "proper velocity", its velocity is inversely proportional to its weight. Bodies R and S (where R and S refer to their weights) moving with their proper velocities, conserve these velocities in collision. The bodies are thought of as being restored to equilibrium after collision in the manner of a scale oscillating over its center of gravity.

If the bodies have "improper velocities", (improprias velocitates) that is, they are not inversely proportional to their masses, the collision is comparable to a scale over two centers of equality equally distant from the center of gravity; a couple is produced.

For bodies with "improper velocities", if before the collision the velocity of body R is greater than its "proper velocity" by a certain amount and that of S is less than its "proper velocity" by the same amount, then after the collision this amount is added to the proper velocity of S and subtracted from that of R, to give the final velocities.

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[10] Christopher Wren, "Lex Natūrae de Collisione Corporum" Phil. Trans., 3 (1669)867-868. (Theory concerning the same subject; imparted to the Royal Society December 17, last, though entertained by the author divers years ago, and verified by many experiments...")

There are three cases for equal [i.e. equal weights] bodies moving "improperly", either in the same or opposite directions, and there are ten cases for unequal bodies [unequal weights] moving "improperly", of which five are conversions (of 6,7,8,9, and 10). The cases are shown in the diagram, p. 46.

a represents the center of gravity or fulcrum of the scale.

o and e are the centers of equality, equally distant from the center of gravity, a.

R and S are equal bodies in cases 1-3, and unequal with R the greater in cases 6-10.

Z is the higher of the velocities of the bodies R and S.

Re is the velocity of the body R before impact.

Se is the velocity of body S before impact.

oR is the velocity of the body R after impact.

oS is the velocity of the body S after impact.

The relationships are:

(1) R + S : S : : Z : Ra

R + S : R : : Z : Sa

(2) Re - 2Ra = oR

2Sa $\pm$ Se = oS [11]

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[11] If the initial velocities are referred to center of equality o, the following relations obtain:

So is the velocity of body S before impact.
Ro is the velocity of body R before impact.
eS is the velocity of body S after impact.
eR is the velocity of body R after impact.

Thus,

So - 2Sa = eS
2Ra + Ro = eR

Dr. Christopher Wrens

Theory concerning the same Subject; imparted to the R. Society Decemb. 17. *last, though entertain'd by the Author divers years ago, and verifi'd by many Experiments, made by Himself and that other excellent Mathematician* M. Rook *before the said Society, as is attested by many Worthy Members of that Illustrious Body.*

Lex Naturæ de Collisione Corporum.

Velocitates Corporum propriæ & maxime Naturales sunt ad Corpora reciproce proportionales.

Lex Naturæ.

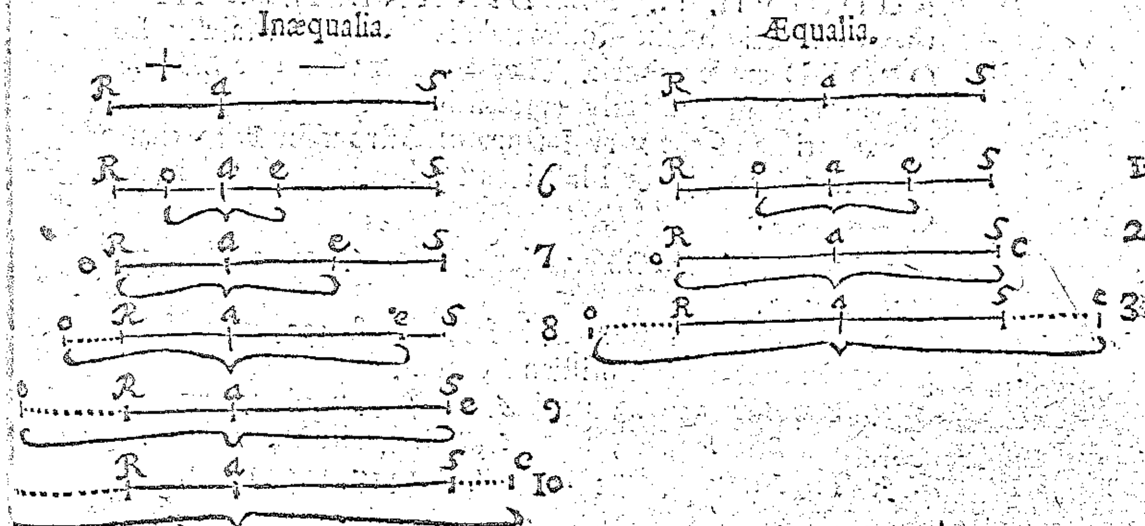
Itaque Corpora R. S. *habentia proprias Velocitates, etiam post Impulsum retinent proprias.*

Et Corpora R. S. *improprias Velocitates habentia ex Impulsu restituuntur ad Æquilibrium; hoc est, Quantum* R *superat, &* S *deficit à propria Velocitate ante Impulsum, tantum ex Impulsu abstrahitur ab* R *& additur ipsi* S *& e contra.*

Quare Collisio Corporum proprias Velocitates habentium æquipollet Libræ oscillanti super Centrum Gravitatis.

Et Collisio Corporum improprias Velocitates habentium æquipollet Libræ super bina Centra æqualiter huic inde à Centro Gravitatis distantia: Libræ vero Jugum, ubi opus est, producitur.

Itaque Corporum æqualium impropriè moventium tres sunt casus. Corporum vero inæqualium impropriè moventium (sive ad contrarias sive ad easdem partes) decem sunt omnino Casus, quorum quinque oriuntur ex Conversione.



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When expressed in modern notation this is equivalent to the conservation of momentum equation:

$$M_R V_{Ro} \pm M_S V_{So} = M_S V_{eS} + M_R V_{eR}$$

where M_R and M_S are the masses of bodies R and S; V_{Ro} and V_{So} the initial velocities; and V_{eS} and V_{eR} the final velocities.

The paper submitted by Huygens to the Royal Society in 1669 was summarized without proofs and printed in both the Philosophical Transactions and in the Journal de Scavans (March) of the same year. Both these papers were excerpted from a work largely complete by 1656 but not published until 1703 after Huygens death, De Motu Corporum ex Percussione. Here the 1669 article in the Journal de Scavans will be discussed followed by a more complete discussion of the relevant material from De Motu Corporum ex Percussione.

The article was a letter from Huygens to the editor containing the "Rules of Motion in the Impact of Bodies." These rules, Huygens stated, were founded on good demonstrations and accorded perfectly with experience.[12] Here Huygens formulated the following rules for bodies of the

[12] Christian Huygens, "Extrait d'une lettre de M. Huygens a l'Auteur du journal" containing the "Regles du mouvement dans la rencontre des corps, "Mar. 18 issue of Journal de Scavans (1669) 19-24. See Huygens, Oeuvres La Haye, 1929, 16, 179-181.

same material, whose sizes are to be estimated by their weights. For Huygens, hard bodies behave as do elastic bodies.

1. When a hard body directly encounters another hard body equal to it and at rest, it gives all its motion to it, itself remaining motionless after collision.

2. But if this other equal body is also in motion and when it moves in the same straight line, they reciprocally exchange their motions.

3. A body somewhat smaller than it and having somewhat less velocity, in encountering another greater and at rest will give some of its motion to it.

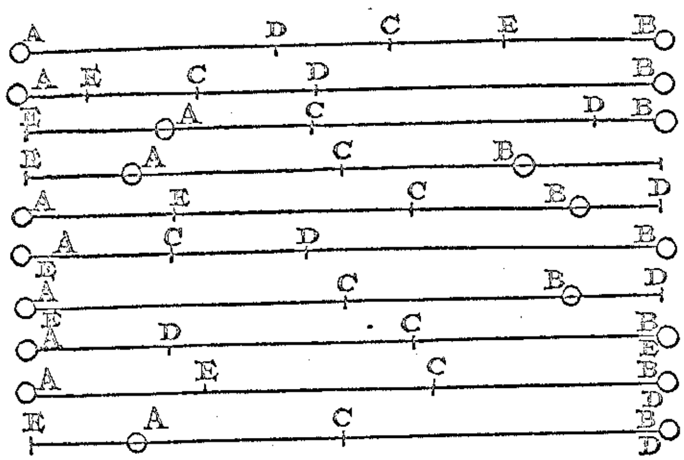
4. The general rule for determining the motion which hard bodies acquire by direct impact is: Let there be bodies A and B of which A moves with the velocity AD (see figure, p.49) and B moves toward it with velocity BD, or is at rest at the point D, and having found on the line AB the point C the center of gravity of the bodies A and B, it is necessary to take CE = CD. EA will be the velocity of body A after impact and EB that of body B. If point E falls at A or B the bodies A or B will be reduced to rest.

5. The quantity of motion that two bodies have can be increased or decreased by their impact, but there will always remain the same quantity toward the same side by subtracting the contrary quantity of motion. / This rule

Letter published in Journal de Sçavans (1669). From Huygens, Oeuvres, 16.

3. Un corps, quelque petit qu'il ſoit, & quelque peu de viteſſe qu'il ait, en rencontrant un autre plus grand qui ſoit en repos, luy donnera quelque mouvement [1]).

4. La regle generale pour determiner le mouvement qu'acquierent les corps durs par leur rencontre directe, eſt telle [2]).



Soient les corps A & B, deſquels A ſoit meu avec la vîteſſe AD, & que B aille à ſa rencontre ou bien vers le meſme coſté avec la viteſſe BD, ou que meſmes il ſoit en repos, le point D en ce cas étant le même, que B, ayant trouvé dans la ligne AB le point C centre de gravité des corps AB, il faut prendre CE égale à CD, & l'on aura EA pour la vîteſſe du corps A apres la rencontre, & EB pour celle du corps B, & l'une & l'autre vers le coſté que montre l'ordre des points EA, EB: Que s'il arrive que le point E tombe en A ou en B, les corps A ou B ſeront reduits au Repos.

5. La quantité du mouvement qu'ont deux corps, ſe peut augmenter ou diminuer par leur rencontre [3]); mais il y reſte toûjours la meſme quantité vers le meſme coſté, en ſouſtrayant la quantité du mouvement contraire [4]).

6. La ſomme des produits faits de la grandeur de chaque corps dur, multiplié par le quarré de ſa vîteſſe, eſt toûjours la meſme devant & apres leur rencontre [5]).

7. Un corps dur qui eſt en repos, recevra plus de mouvement d'un autre corps dur plus grand ou moindre que luy, par l'interpoſition d'un tiers de grandeur moyenne, que s'il en eſtoit frappé immediatement: Et ſi ce corps interpoſé eſt moyen proportionnel entre les deux autres, il fera le plus d'impreſſion ſur celuy qui eſt en repos [6]).

Ie conſidere en tout cecy des corps d'une meſme matiere, ou bien j'entends que leur grandeur ſoit eſtimée par le poids.

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[1]) Comparez (p. 39) la Prop. III.
[2]) On retrouve cette règle au Traité „De Motu" dans l'alinéa qui commence en bas de la p. 65.
[3]) Comparez (p. 49) la Prop. VI.
[4]) Consultez à propos de ce Théorème les p. 24—25.
[5]) Comparez (p. 73) la Prop. XI.

like Wallis' corrected Descartes' conservation of the absolute quantity of motion./

6. The sum of the products of the size of each hard body multiplied by the square of its velocity is always the same before and after impact.[13]

7. A hard body at rest will receive more motion than another hard body greater or less than it by the interposition of a third of medium size that it will hit directly; and if this body interposed is the mean proportional between the other two, it will make a greater impression on the one which is at rest.

Finally there is a law of nature which seems to be general for all bodies hard or soft whether they collide directly or obliquely: This is that the common center of gravity of two or three (or such as one wishes) bodies, always advances equally toward the same side in a straight line before and after impact.

These rules were the same as those which were published in the Philosophical Transactions of 1669 (see copies, Phil. Trans., 4). Huygens had also sent to Oldenburg (Jan. 5, 1669) the demonstrations of the first four of these seven

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[13]There is thus no question as to the priority of the mv^2 law between Huygens and Leibniz. For a discussion of this see Erwin Hiebert, Historical Roots of the Principle of Conservation of Energy, Madison, 1962, 73.

dire satagerent. *Factum hinc, ut selectus ille Virorum præstantissimorum Trias, post paucarum septimanarum spatium, Theorias suas, eleganter compendifactas, tantum non certatim transmitterent*, Regiæque Societatis *super iis sententiam exquirerent.* *Primus omnium D.* Wallisius, *sua de* Motibus æstimandis Principia, *literis d.* 15. Novemb. 1668. *datis, ejusdemque mensis die* 29. *traditis et prælectis, communicavit. Mox eum excepit D.* Christophorus Wren, *qui* Naturæ Legem de Collisione Corporum, *proximo mense* Decembri, *ejusque die* 17. *eidem* Societati *publice exhiberi curavit: quæ in mandatis mox dedit (præ-habito tamen utriusque hujus Authoris consensu) ut ad commodiorem horum Scriptorum communicationem, discussionemque diffusiorem, res tota typis mandaretur.*

Hæc dum apud Nos geruntur, Ecce adfert Nobis tabellarius d. 4. Ianuarij *insequentis* (St. Angl.) *Dn.* Hugenii *literas, ejusdem Mensis d.* 5. (*at* st. nov.) *exaratas, ejusque Scripti, de* Motu Corporum ex mutuo impulsu, *priores Regulas quatuor, una cum demonstrationibus, continentes. Habebam ego in promptu Theoriæ* Wrennianæ *Apographum, idque actutum eodem plane die, sic favente Tabellione publico, D.* Hugenio, *redhostimenti vice, remittebam, dilata interim literarum* Hugenianarum *(quibus tale quid includi, ob molem, et antegressum Authoris promissum, suspicabar) resignatione, donec ferret occasio Nobilissimum et Sapientissimum Regiæ Societatis Præsidem, Dn. Vice-Comitem Brouncker, compellandi. Quo facto, amborumque Regulis in modo dicta* Societate *collatis, mirus confestim in utroque consensus effulsit; id quod insignem in nobis lubentiam pariebat, utrumque hoc Scriptum prælo nostro committendi. Nihil hic nobis deerat a parte* Hugenii, *quam ejus consensus; absque quo fas nequaquam judicabamus, ipsius Inventum, maxime cum illud haud integrum eo tempore nobis dedisset, in lucem emittere. Curæ interim nobis erat, scriptum Ipsius publicis Regiæ Societatis monumentis inferendi; simul & Authori d.* 11. Januar. *solennes pro cordata illa communicatione gratias reponendi; additâ dehinc (die scil.* 4. Februarij) *sollicitâ commonefactione, ut suam hanc Theoriam vel* Parisiis (*quod proclive erat factu in* Eruditorum, *ut vocant,* Diario) *vel hic* Londini *in* Adversariis Philosophicis, *inprimendam curaret, vel saltem permitteret. Quibus expeditis litteris paulo*

paulo post secundas accepimus ab Hugenio, *scripti* Wrenniani *de hoc argumento recte traditi mentionem facientes, nil tamen quicquam de suimet scripti Editione, vel* Parisiis *vel* Londini *paranda, commemorantes.*

Unde liquere omnino autumem, ipsum sibi defuisse Hugenium *in illa publicatione maturanda; quin imo occasionem dedisse procrastinando, ut laudatus Dn.* Wren, *pro ingenii sui sagacitate geminam omnino Theoriam eruens, in gloriæ, huic Speculationi debitæ, partem jure veniret; cum extra omne sit dubium, neutrum horum Theoriæ illius quicquam, priusquam Scripta eorum simul comparerent, resciviße ab altero, sed utrumque, propriâ ingenii fæcunditate, pulchellam hanc sobolem enixum fuisse.*

Solvit equidem Hugenius, *ante aliquot jam annos,* Londini *cum ageret, illos de* Motu *Casus qui ipsi tunc proponebantur; luculento sane argumento, eum jam tum exploratas habuisse Regulas, quarum id evidentiâ præstaret. At non affirmabit ipse, cuiquam se* Anglorum *suæ Theoriæ quicquam aperuisse; quin fateri tenetur, se ab eorum nonnullis ad communicationem ejus sollicitatum, nec tamen unquam, nisi nuperrime, ad id faciendum pertractum fuisse.*

His itaque veritati et Iustitiæ litatis, ipsas jam Hugenii *Regulas sermone* Latino, *in ampliorem Eruditorum usum, sic donamus.*

Regulæ de Motu Corporum ex mutuo impulsu.

1. *Si Corpori quiescenti duro aliud æquale Corpus durum occurrat, post contactum hoc quidem quiescet, quiescenti vero acquiretur eadem quæ fuit in Impellente celeritas.*

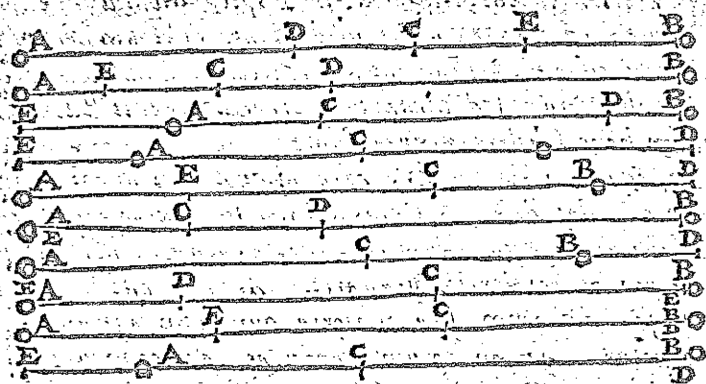
2. *At si alterum illud Corpus æquale etiam moveatur, feraturque in eadem linea recta, post contactum permutatis invicem celeritatibus ferentur.*

3. *Corpus quamlibet magnum à corpore quamlibet exiguo et qualicunque celeritate impacto movetur.*

4. *Regula generalis determinandi motum, quem corpora dura per occursum suum directum acquirunt, hæc est:*

Sint Corpora A et B, quorum A moveatur celeritate A D, B vero ipsi occurrat, vel in eandem partem moveatur celeritate B D.

B D, vel denique quiescat, hoc est, cadat in hoc casu punctum in B. Divisâ lineâ A B in C, (centro gravitatis Corporum A B.) sumatur C E æqualis C D. Dico, E A habebit celeritatem corporis A post occursum; E B vero, corporis B, et utrumque in eam partem, quam demonstrat Ordo punctorum E A, E B. Quod si E incidat in punctum A vel B, ad quietem redigentur corpora A vel B.



5. *Quantitas motus duorum Corporum augeri minuive potest per eorum occursum; at semper ibi remanet eadem quantitas versus eandem partem, ablatâ inde quantitate motus contrarii.*

6. *Summa Productorum factorum à mole cujuslibet corporis duri, ducta in Quadratum suæ Celeritatis, eadem semper est antè et post occursum eorum.*

7. *Corpus durum quiescens, accipiet plus motus ab alio corpore duro, se majori minorive, per alicujus tertii, quod medie fuerit quantitatis, interpositionem, quam si percussum ab eo fuisset immediatè. Et si corpus illud interpositum, fuerit medium proportionale inter duo reliqua, fortiùs aget in quiescens.*

Considerat Author in his omnibus (ut ipse ait) Corpora ejusdem materiæ, sive id vult, ut eorum moles æstimetur ex pondere.

Cæterùm subjungit, notasse se miram quandam Naturæ legem, quam demonstrare se posse affirmat in corporibus Sphæricis, *quæque generalis ipsi videtur in reliquis omnibus sive duris sive mollibus, sive directe sive obliquè sibi occurrentibus*, viz. *Centrum commune Gravitatis duorum, trium, vel quotlibet Corporum, æqualiter semper promoveri versus eandem partem in linea recta, ante et post occursum.*

An

An Account

Concerning the Resolution *of* Equations in Numbers; *imparted by Mr.* John Collins.

This Account should have been annex'd to what was discoursed of Monsieur *Slusius* his *Mesolabe* in the precedent Tract, if then we had found room for it. For, the Reader having there understood, how farr the *Geometrick* part of *Algebra* is advanc'd by that excellent person, 'twas likely, he would be inquisitive to hear somewhat concerning the *Exegesis Numerosa*, or the Resolution of Æquations in *Numbers*. For whose satisfaction herein, we shall here insert the Account then omitted, being part of a narrative, formerly made by M. *John Collins* touching some late Improvements of *Algeqra* in *England*, upon the occasion of its being alledged, that none at all were made since *Des Cartes*.

1. It hath been observ'd by divers of this Nation, that in any Æquation, howsoever affected, if you give a Root, and find the Absolute number or Resolvend (which *Vieta* calls *Homogeneum Comparationis*) and again give more Roots and find more Resolvends, that if these Roots or rather rank of Roots be assum'd in *Arithmetical* progression, the Resolvends, as to their first, second or third differences, *&c.* imitate the Laws of the pure Powers of an Arithmetical progression of the same degree, that the higest Power or first term of the Equation is of. e. g. In this Equation $aaa - 3aa + 4a = N$,

If *a* be =		Then N. or the Absolutes or Resolvends will be found to be	1. *dif*	2. *dif*	3. *dif.*
	10	740	218	48	6
	9	522	170	42	6
	8	352	128	36	
	7	224	92		
	6	132			

52

To wit the 3d. differences of those *Absolutes* are equal, as, in the Cubes of an Arithmetical Progression.

2. To find, what habitude those *differences* have to the *Coefficient*, of the Equation, 'ist best to begin from an Unit.

3. In any Arithmetical Progression, if you multiply Numbers

propositions. After having received them Oldenburg sent him a copy of Wren's theory which had been presented to the society 15 days before (Dec. 17, 1668) and which conformed completely to his own rules. These four demonstrations of Huygens which were not published in the *Philosophical Transactions* were far more complete in detail than Wren's.[14] Thus in the *Journal de Scavans* and the *Philosophical Transactions* Huygens had set down the essential rules for the treatment of elastic impact problems: (1) the conservation of momentum (rule 5) and (2) the conservation of mv^2 (rule 6). The conservation of relative velocities is found in proposition IV of his *De Motu Corporum ex Percussione* to be discussed subsequently. To this treatment by Huygens, Leibniz added the algebraic expressions for each of these conservation principles and showed their interdependence. Jean Bernoulli followed Leibniz's discussion very closely in his paper submitted for the prize of the French Academy in the years 1724 and 1726.[15]

In the posthumous work, *De Motu Corporum ex Percussione*, published in 1703, but essentially completed about 1656, Huygens had developed a more detailed theory of percussion.[16]

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[14] For the demonstrations see, Huygens, *Oeuvres*, La Haye, 1895 6,336-343. For Huygens letter see *Phil.Trans.*,4 (1669)925-928.

[15] See this dissertation, Ch. III, p. 90 , and Ch.VIII p. 288 .

[16] Christian Huygens, "De Motu Corporum ex Percussione," *Oeuvres*, La Haye, 1929, 16, 30-91.

Three hypotheses precede the development of the propositions:

1. /The principle of inertia7: Any body once set in motion if nothing opposes it, continues to move with the same velocity and in a straight line.

2. Whatever is the cause that hard bodies rebound upon mutual contact, we suppose that two equal hard bodies of the same velocity colliding directly rebound with the same velocity with which they approached. /Again hard bodies behave here as elastic bodies.7

3. /The relativity of motion7: The "motion of bodies" and "equal or unequal velocities" should be understood relative to other bodies which are suppoed at rest although both may be subject to other motions common to both of them. Consequently when two bodies collide even if they are also subject to some other uniform motion, they will act on each other with respect to an observer having this same common motion, as if this common uniform motion were absent.

Thus when someone transported by a boat which advances with a uniform motion causes two equal balls with equal velocities to collide, with respect to himself they will rebound with equal velocities as if the collision took place in a boat at rest or on the land. (See copy of title page, p. 55.)

The first proposition is:

When a body at rest is met by another equal to it,

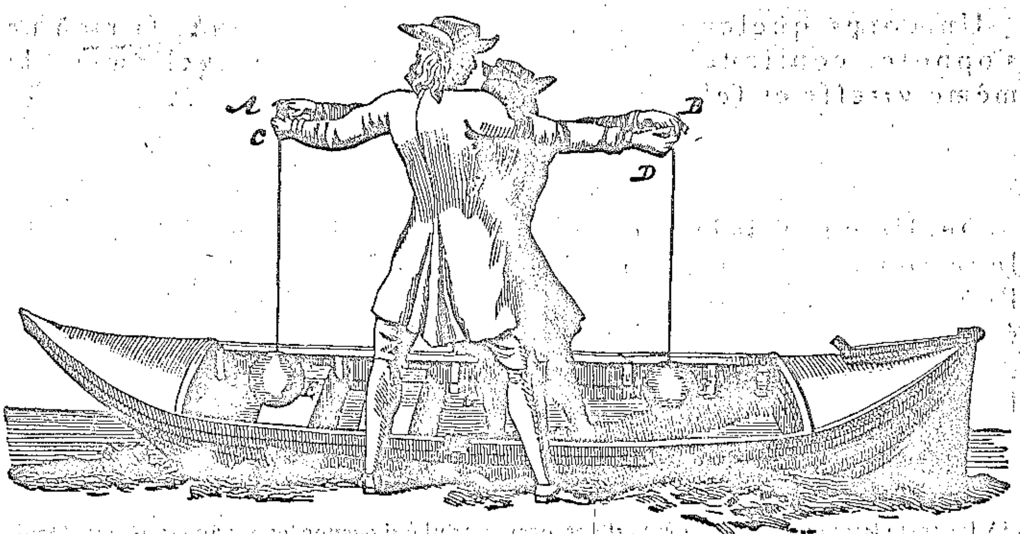
CHRISTIANUS HUGENIUS

DE

MOTU CORPORUM

EX

PERCUSSIONE.



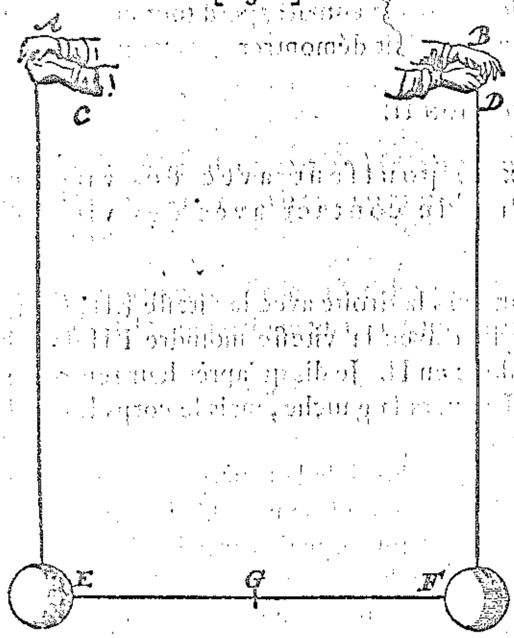
after collision the colliding body will be at rest but the one at rest will acquire the velocity of the other.

The proof is as follows: Imagine that a boat near the river bank is carried by a current and close enough that a navigator can touch the hands of a companion on the bank. The navigator holds in his hands A and B (see Huygens' fig. 1, p. 57) two equal balls E and F suspended from two strings and of which the distance EF is divided into two equal parts by the point G. By an equal motion of his hands with respect to himself, he can impart an equal velocity to the two balls so that they rebound with equal velocities with respect to the boat. But the navigator is carried at the same time toward the left with velocity GE, that is, with the same velocity with which his left hand is transported toward the right.

It is thus clear that hand A of the navigator with respect to the bank and to the companion remains motionless, but that hand B with respect to the same companion is moved with velocity FE double that of GE, that is with FG. Thus if the companion on the bank takes with hand C the hand A of the navigator and with it the end of the string holding ball E and with hand D takes hand B and ball F from the navigator, it appears that ball F hits ball E at rest with velocity FE with respect to the bank. Thus after the collision with respect to the companion the ball F rests motionless and the other E moves toward the left with a

Teneat vero vector manibus suis A & B [Fig. 1] [1]) duo corpora æqualia ex filis suspensa E, F, quorum distantia EF bifariam divisa sit puncto G: motuque æquali manus ad occursum mutuum promovens, sui nempe & navigii respectu, etiam globulos E, F æquali celeritate inter se collidi faciet, quos itaque necesse est & æquali ce|leritate * a contactu mutuo resilire ejusdem vectoris & navigii respectu: Navigium autem [2]) ponatur interim ferri sinistram versus celeritate GE, eâdem nempe quâ manus sinistra A delata fuit dextram versus.

[Fig. 1.]



p. 371.

* Hyp. II.

Patet itaque vectoris manum A, respectu ripæ & socii in illa consistentis, immotam stetisse; manum vero B, respectu ejusdem socii motam fuisse celeritate FE, duplâ ipsius GE vel FG. Quamobrem si socius in ripâ stans prehendisse ponatur manu suâ C manum vectoris A, cumque eâ caput fili globum E sustinentis [3]); alterâ vero manu D manum vectoris B, quæ sustinet funiculum e quo pendet F [4]); apparet dum vector globulos E, F, æquali celeritate concurrere facit, suo & navigii respectu, simul socium in ripâ stantem globulo E quiescenti impegisse globulum F motum celeritate FE, respectu ripæ & sui ipsius. Et constat quidem, vectori globulos suos [5]), uti dictum est, moventi, nihil officere quod socius in ripâ stans manus ejus & filorum capita apprehenderit, cum tantum comitetur earum motum, nec ei ullum impedimentum afferat. Eâdem ratione nec socio in ripâ stanti globulumque F versus immotum E deferenti, quidquam obstat, quod vector manibus suis manus conjunctas habeat, siquidem manus A & C utraque respectu ripæ & socii quiescunt, duæ vero D & B moventur eâdem celeritate FE. Quia autem uti dictum fuit globuli E, F, post mutuum contactum, æquali celeritate resiliunt, respectu vectoris & navigii; globulus nempe E celeritate GE, & globulus F celeritate GF, ipsumque interim navigium pergit sinistram versus celeritate GE seu FG [6]), sequitur, respectu ripæ & socii in illâ stantis, globulum F post impulsum restare immotum,

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[4]) De même Huygens remplaça „BF" par „e quo pendet F".
[5]) Le mot „suos" manque dans le Manuscrit. Comparez la note 2.
[6]) Dans le Manuscrit Huygens a intercalé après coup les mots „seu FG".

velocity FE with which it pushed the ball F toward E. This was to be demonstrated.

Proposition II is the case in which equal bodies collide with unequal velocities and reciprocally exchange them after collision. This is proved in a similar manner by the device of the boat in motion.

Two hypotheses precede the propositions where the masses are unequal. Hypothesis IV states: When a greater body hits a smaller one at rest it gives it some of its own motion and itself loses some.

Hypothesis V states that if one of the colliding bodies conserves its motion after collision, the other will do likewise.

Proceeding to Proposition IV, Huygens shows that the relative velocities of the bodies before and after collision are the same.

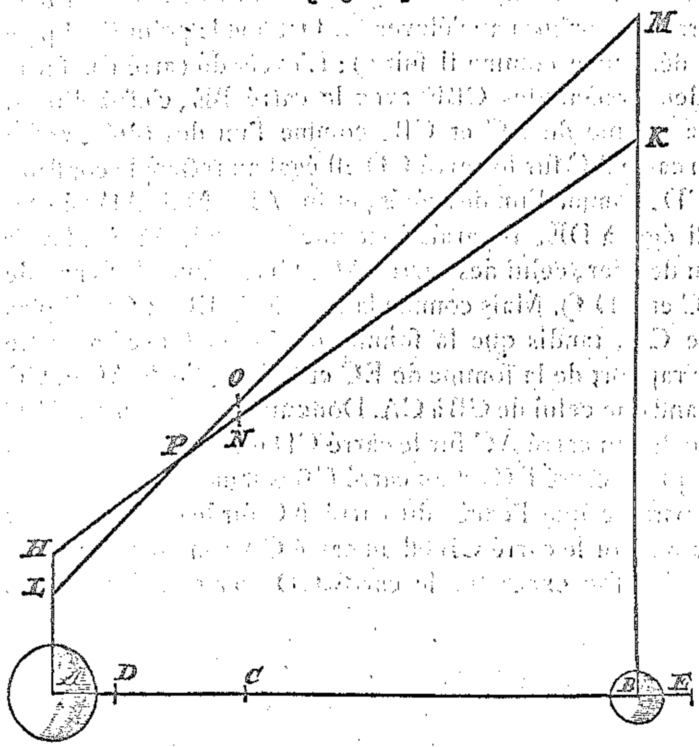
Proposition VI refutes Descartes' conservation of the product of the mass and its absolute velocity, $m|v|$ correcting it to the proper law: In two colliding bodies, the quantity of motion taken for the two together is not always conserved after collision, but can be increased or decreased.

Proposition VIII uses the mv^2 principle later known as "living force" relating the heights of fall to the velocities acquired. Since the development and use of this relationship is of particular interest to the controversy its use in Proposition VIII will be examined in some detail. The principle of the living force of a body and its relation to that body's distance of fall was used by Leibniz in the demonstrations which initiated the controversy. It was the basis for the experimental work done by Poleni (1718) and 's Gravesande (1722). It was the fundamental argument of all those who supported the Leibnizian position in the dispute.

Proposition VIII states: When two bodies of which the velocities are inversely proportional to their sizes meet from opposite sides, each will rebound with the same velocity with which it approached.

Let there be two bodies which collide and of which the first is greater than the second and of which the size of A is to the size of B as the velocity BC of body B is to the velocity AC of body A. (See Huygens' fig. 11, p. 60). It is necessary to prove that after collision each returns with the velocity with which it approached, A with velocity CA and B with velocity CB. If A is reflected with velocity CA, then B must be reflected with CB since by proposition IV the relative velocity is always the same before and after impact. If body A does not return with velocity CA but with a lesser velocity CD, then B will rebound with a velocity CE greater than that with which it arrived, such that DE=AB. Suppose that body A has acquired its first velocity corporis B, quæ ſit BC, ad celeritatem corporis A, quæ ſit AC. oſtendendum eſt, poſt contactum mutuum, utrumque eâdem quâ venit celeritate reverti, nempe A, celeritate CA, B vero, celeritate CB: conſtat autem, ſi A reflectatur celeritate CA, etiam B reflecti celeritate CB, quia alioqui non eadem eſſet mutuo reſpectu celeritas recedendi, quæ fuit appropinquandi *. Si igitur corpus A non revertitur celeritate CA, reſiliat primò, ſi fieri poteſt, celeritate minori CD; ergo B reſiliet celeritate CE, majori quam quâ advenerat, ita ut DE, ſit æqualis AB *. Ponamus corpus A acquiſiviſſe celeritatem priorem AC, quâ tendebat ad occurſum, cadendo ex altitudine HA, ut nimirum poſtquam deſcenderit usque in A, motum perpendicularem mutaverit in horizontalem cujus celeritas AC; corpus autem B acquiſiviſſe ſimiliter celeritatem BC, cadendo ex altitudine KB [1]); ſunt igitur hæ altitudines in celeritatum ratione duplicatâ, hoc eſt, ſicut quadratum AC ad quadratum CB, ita HA ad KB. Quod ſi deinde, poſt occurſum, corpora A & B motus ſuos Horizontales, quorum celeritates metiuntur CD, CE, convertant in motus perpendiculares ſurſum; conſtat corpus A perventurum ad altitudinem AL, ita ut ſit AL ad AH,

[Fig. 11.]



* Prop. IV.

* Prop. IV.

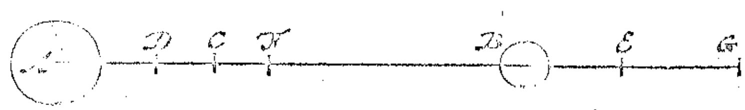
p. 382. ſicut quadratum | CD, ad quadratum CA. Quando enim hujusmodi rationem habet AL ad AH, certum eſt corpori decidenti [2]) ex altitudine LA, acquiri velocitatem CD; unde & viciſſim, velocitatem habens CD, attingere poterit altitudinem AL, per ea quæ ſuperius poſita fuere [3]); corpus autem B convertendo celeritatem CE in motum perpendicularem ſurſum, perveniet ad altitudinem BM, ut ſit MB ad KB ſicut quadratum CE ad quadratum CB. Jungantur HK, LM quæ neceſſario ſe mutuo ſecabunt, puta in P [4]); & dividantur utraque ſimiliter in N & O, ut, ſicut magnitudo B ad A, hoc eſt, ſicut AC ad CB, ita ſit HN ad NK, itemque LO ad OM.

AC with which it moves toward the point of contact by falling from height HA in such a manner that after having descended to A it changes its vertical motion to a horizontal motion with velocity AC. Similiarly body B has acquired its velocity by falling from height KB. These heights are in the "double ratio" (ratione duplicata) of the velocities, that is, the square (quadratum) AC is to the square CB as HA is to KB. Now if after collision the bodies A and B change their horizontal motions of which the velocities are measured by CD and CE into perpendicular motions upward, then body A will arrive at height AL such that AL is to AH as the square CD is to the square CA. Because if such is the ratio of AL to AH it is certain that a body falling from height LA will acquire velocity CD and reciprocally with velocity CD it can attain height AL. But body B in changing its velocity into a vertical motion upwards will reach height BM such that MB is to KB as the square CE is to the square CB. Join HK, LM and let each of these segments be divided into the same ratio at N and O such that AC is to CB as HN is to NK and LO is to OM. Then when the center of gravity (gravitatis centrum) of body A is situated at H and the center of gravity of body B is at K, their combined center of gravity will be at point N. But after they fall from H and from K and after their contact are elevated to L and M, their combined center of gravity will be at O. However this cannot happen because the point O is higher than point N and it is a certain axiom in mechanics that by a motion of bodies resulting from their "gravity" their common center of gravity cannot be raised. A geometrical proof then follows to show that point O is indeed higher than point N. Then the case for the velocity of A, i.e. CD being greater than CA is proved similarly. The case for body A remaining at rest and B only being reflected with velocity AB is also treated in a similar fashion. The series of cases is completed by treating that for body A continuing to move in the same direction with velocity $CF < AC$ hitting body B so that the excess FG of velocity CF equals AB. (See Huygens' fig. 14, p. 63). Huygens shows that body C cannot continue to move in the same direction after collision. Therefore to conclude Proposition VIII it must follow from the impossibility of all the above cases that body A returns after collision with the same velocity with which it approached and B does likewise. As was shown this conclusion was reached through the use of the principle later known as "living force", relating the height of fall of a body to the velocity acquired in its free fall.

In Proposition IX rules are given in geometric form for finding the velocities after collision of two unequal elastic bodies, one of both being in motion and the velocity of each being given. (See Huygens' fig. 15, p. 64). This is a similar treatment and the same result which Huygens

BM fera la hauteur à laquelle pourra monter le corps B s'il change le mouvement horizontal par lequel il eſt entraîné avec la viteſſe AB en un mouvement perpendiculaire en haut; mais le corps A, puiſque après le choc il a été dit être ſans mouvement, reſtera dans la droite AB. Si donc on tire MA et ſi on la diviſe en O de ſorte que AO ſoit à OM comme AC à CB, O ſera le point jusqu'auquel montera le centre de la gravité compoſée des deux corps. Mais les corps étant ſitués en H et en K, d'où l'on ſuppoſe qu'ils ſont deſcendus, avaient leur centre de gravité commun en N qui diviſe pareillement la droite HK dans le rapport de AC à CB; donc ſi l'on montre comme auparavant que le point O eſt plus élevé que le point N la démonſtration ſera réduite à la même abſurdité que plus haut. Or, cela peut être montré comme il ſuit. Puisque le carré AB eſt au carré BC comme, en longueur, MB eſt à BK, on aura, par partage, que l'excès du carré AB ſur le carré BC eſt au carré BC comme MK à KB; mais comme le carré BC eſt au carré CA ainſi KB eſt auſſi à HA, car ceci a été ſuppoſé comme dans le premier cas; donc, par égalité, l'excès du carré AB ſur le carré BC ſera au carré CA comme MK eſt à HA; mais le rapport du dit excès au carré CA eſt certainement plus grand que celui de la droite BC à CA [1]): donc auſſi le rapport de MK à HA, c'eſt-à-dire de MP à PA, ſera plus grand que le rapport de BC à CA, c'eſt-à-dire de MO à OA. Et, par compoſition, le rapport de MA à AP ſera plus grand que celui de MA à AO; d'où il réſulte que le point O tombe par rapport au point d'interſection P au côté qui eſt vers M. Or, M eſt plus haut que K; donc, puisque ON eſt néceſſairement parallèle à MK, le point O ſera auſſi plus haut que N: ce qui reſtait à démontrer.

[Fig. 14.]

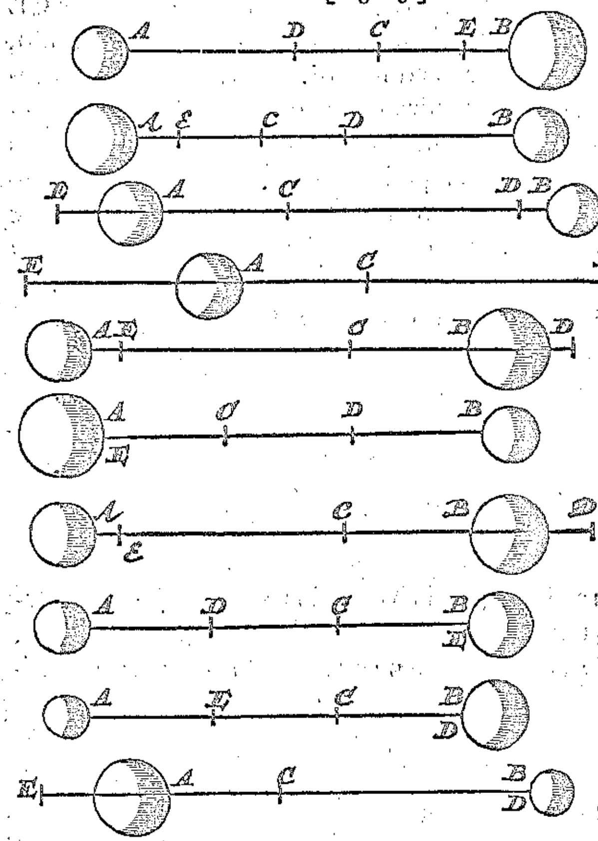


Si, enfin on diſait que le corps A après le choc continuera de ſe mouvoir vers le même côté avec la viteſſe CF [Fig. 14], celle-ci certes ne ſera pas plus grande que la viteſſe AC avec laquelle il ſe mouvait avant le choc: or, le corps B devra dans ce cas pré- [illegible] le corps A avec la [illegible], dont l'excès FG ſur la viteſſe CF [illegible] lieu ſera établi comme il

1) Puisqu'on a $\frac{(p+q)^2-q^2}{p^2} > \frac{q}{p}$.

mais EB celle du corps B, et cela dans la direction indiquée par l'ordre des points E A, E B[1]). Alors que le point E tombe en A, le corps A sera réduit au repos: mais si E tombe en B, le corps B sera en repos.

[Fig. 15.]



En effet, lorsque nous aurons montré que ces événements se passent ainsi dans un navire qui est emporté avec une vitesse uniforme, il sera certain qu'ils arriveront de la même façon pour celui qui se trouve à terre. Figurons nous donc que le navire se meuve le long de la rive d'un fleuve et que dans ce navire un passager porte de ses mains F, G deux boules A, B, suspendues à des fils, lesquelles, en les mouvant avec des vitesses AD & BD, savoir par rapport à lui-même et au navire, il fasse se rencontrer au point D; mais posons que le navire s'avance avec la vitesse DC dans la direction indiquée par l'ordre des points D, C. Il arrivera donc que, par rapport à la rive et au spectateur qui s'y trouve, la boule A se meut avec la vitesse AC vers la droite, puisque par rapport au navire elle avait la vitesse AD. Mais la boule B ayant dans le bateau la vitesse BD, aura par rapport à la rive la vitesse BC vers la gauche. Si donc le spectateur qui se trouve sur la rive prend de ses mains H K les mains F G du passager, et avec elles les têtes des fils qui soutiennent les boules A B, il paraît que tandis que le passager, par rapport à lui-même, les meut avec les vitesses AD, BD, en même temps celui qui est sur la rive les meut, par rapport à lui-même et à la rive, avec les vitesses AC, BC. Or, puisque ces vitesses sont en proportion réciproque des grandeurs des corps A et B, il faut que ces corps, par rapport au même spectateur, rejaillissent de leur contact avec les mêmes vitesses CA et CB, ainsi qu'il a été demontré plus haut[3]).

published in the Philosophical Transactions of the Royal Society (1669) and the Journal de Scavans (1669). It is the same result arrived at by Christopher Wren.

Of final interest is proposition XI giving the mv^2 law for impact: In the case of two colliding bodies, the sum of their sizes [masses] multiplied by the squares of their velocities will be found equal before and after collision. The proof uses solid geometry. It equates the sum of the two cubes constructed by means of lines representing the size (grandeur) of bodies A and B on the squares formed from the initial velocities, with the cubes constructed on the squares formed from their final velocities.[17]

Huygens also used the principle of living forces in his derivation of the law for the compound pendulum in the Horologium Oscillatorium (1673) where he employed the concept that the center of gravity cannot rise to a height higher than its initial position.

In asserting this rule of motion for colliding bodies involving mv^2, Huygens did not apply it beyond the relative motion of bodies. It was Leibniz's contribution to interpret this quantity as a measure of an absolute, rather than a relative quantity, existing in the universe as force. He saw its nature as a conserved "substantial" quantity which

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[17] Oeuvres, 16, 73.

could not be created or destroyed.[18]

Conclusion

The work of Wallis, Wren and Huygens, when taken as a unit correctly described the rules of motion for the collision of elastic and inelastic bodies. They corrected Descartes' erroneous law of the conservation of quantity of motion $m|v|$ to the conservation of mv.

In the paper he presented to the Royal Society, Wallis (1668) gave the law of conservation of momentum, $m\vec{v}$, for hard ≡ inelastic bodies. In his Mechanica (1669-1671) he gave the laws for elastic impact. Wren's paper, read to the Royal Society in 1668, gave the law of momentum conservation for elastic bodies. Huygens in the Philosophical Transactions (1669) and the Journal de Scavans (1669) also corrected Descartes' quantity of motion and stated the law of conservation of mv^2. His hard bodies behaved as

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[18] For a discussion of this comparison between Huygens and Leibniz, see Martial Gueroult, Dynamique et metaphysique Leibniziennes, Paris, 1934, 101, 105-106. Gueroult cites Huygens, Oeuvres, 10, 614.

For Huygens' reaction to Leibniz's 1686 paper (see Ch. III, this disertation) see Oeuvres, 19, 162-165. Here Huygens concludes: "Ainsi M. Leibniz n' a pas raison d' imputer une telle erreur a des Cartes, ... Et que pour cela il ait suppose que la mesme force motrice soit conservée dans la nature; et que cette force motrice fust equivalente avec la quantité de mouvement; qui sont 2 choses qu'on ne trouve point que des Cartes ait avancées."

elastic bodies. He gave a more detailed discussion of impact in his De Motu Corporum ex Percussione, largely completed by 1656, but published posthumously in 1703.

The problem of impact became a central issue during the controversy over living force which was initiated by Leibniz in 1686 (See Ch. III). Those participants who accepted mv as the measure of force solved all impact problems, elastic and inelastic, by use of conservation of momentum alone [Eg., 's Gravesande (1719) and Mazière (1726)] In fact this was the general method of solution accepted in physics textbooks until the nineteenth century (see appendix). Those participants however who championed mv^2 as the measure of force used only the principle of living forces and mainly restricted themselves to straightforward well-known cases of impact. They were unable to completely resolve the problem of the loss of mv^2 in inelastic impact. Leibniz and Jean Bernoulli used both mv and mv^2 in collision problems while insisting on mv^2 as the measure of force. Leibniz accounted for the loss of mv^2 in inelastic impact by stating that it was transferred to and remained in the "small parts of matter" as "when men change large money into small."

Not until the controversy over living force had died out and mv^2 had been accepted as a general conservation principle, sometime in the early nineteenth century, were problems of elastic impact solved by the simultaneous use of the two conservation equations, momentum and kinetic energy.