AGENDA

I. Instantaneous and Average Power in the Wind
II. Extracting Energy from the Wind
III. Hydrogen and Fuel Cells – How do they work
IV. Thermodynamics of Fuel Cells

I. Instantaneous and Average Power in the Wind

What is essentially taking place in a wind turbine?
The base holds up the wind turbine to a place that there is wind. The wind pushes on a turbine, which captures the energy by rotating. In the gear box the gear turns the big and slow rotation of the wind turbine into a small and fast rotation needed by the generator. The rotating gears will cause the generator to rotate, creating electricity.

a. Instantaneous Power in the Wind:

How much power is in the wind?

Consider a ‘piece’ of air with mass m, moving at a speed v, across an area A (area swept by the turbine rotor). Its Kinetic Energy is given by the equation

\[ K.E. = \frac{1}{2}mv^2 \]

Power is energy per unit of time. The power P represented by this mass of air moving at velocity v though area A is:

\[ P = \frac{\text{Energy}}{\text{time}} = \frac{1}{2} \times \frac{\text{mass}}{\text{time}} \times v^2 \]

The mass flow rate through the area A is a product of air density \( \rho \) and volume (which is wind speed \( v \) * cross-sectional area A).

\[ \text{mass flow rate} = \frac{\text{mass passing through A}}{\text{time}} = \rho Av \]

where \( \rho = \text{air density} = 1.225 \text{ kg/m}^3 \) at 15°C and 1 atm

Combining these two relationships gives the instantaneous power in the wind, P

\[ P = \frac{1}{2} \rho Av^3 \]

Note from this equation that the power in the wind increases as the cube of wind speed.
Doubling wind speed increases the power eight fold. Also note that power goes up as the swept
area increases. Since Area is $\pi r^2$ then a doubling of the rotor blade diameter increases available power by a factor of four.

b. Average Power in the Wind

Problem:
Suppose the wind blows for 10 hours at 8 m/s and 10 hours at 4 m/s. What would be the total energy and average power per square meter of area over those 20 hours?

Solution:
Applying wind power formula to each regime:

\[
\text{Energy per area} = \frac{1}{2} \rho v^3 \left(\frac{W}{m^2}\right) \times \Delta t (hr)
\]

\[
\text{Energy (10hr per 8 m/s)} = 0.5 \times 1.225 \times 8^3 \times 10 = 3136 \left(\frac{Wh}{m^2}\right)
\]

\[
\text{Energy (10hr per 4 m/s)} = 0.5 \times 1.225 \times 4^3 \times 10 = 392 \left(\frac{Wh}{m^2}\right)
\]

Total Energy = 3136 + 392 = 3528 \left(\frac{Wh}{m^2}\right)

Notice how insignificant the energy contributed by the low wind speed of 4 m/s winds is.

\[
\text{Average power over those 20 hours} = \frac{3528 \left(\frac{Wh}{m^2}\right)}{20h} = 176.4 \left(\frac{W}{m^2}\right)
\]

Suppose we had simply plugged the average wind speed of 6 m/s into the equation. What would we have gotten for average power?

\[
\text{Average power} = \frac{1}{2} \rho v^3 = 0.5 \times 1.225 \times 6^3 = 132.3 \left(\frac{W}{m^2}\right)
\]

Our 132.3 W/m² estimate, which incorrectly uses the average wind speed is 25% lower than the correct answer of 176.4 W/m². – BE CAREFUL!!!

The previous example shows us the non-linear relationship between wind speed and power and reminds us that we need to be cautious in estimating average power in winds that have variable speeds. The previous example had only two speeds in its distribution. However, in reality, wind speeds at a site can vary over a wide range. We thus need some distribution of wind speeds at a site if we want to estimate the average power or total energy that a wind turbine will produce.

Instead of the discrete wind speed histogram (like the 2 wind speed histogram in the previous example), the distribution of wind speeds is often represented as a continuous function, called the probability density function (p.d.f.). The area under the p.d.f. curve is equal to unity, and the area under the curve between any two wind speeds is the probability that the wind is between those two speeds.
The distribution of wind speeds is often assumed to follow what is known as a Weibull probability density function, described by a complicated equation. Note, that the bulk of the energy in the wind comes at speeds above the mean (average) wind speed at the site. The equation describing the distribution is:

\[ f(v) = \frac{k}{\lambda} \left(\frac{v}{\lambda}\right)^{(k-1)} \times \exp\left(-\frac{v}{\lambda}\right)^k \]

Where the “shape” parameter \( k > 1 \),
the “scale” parameter \( \lambda > 0 \)
the wind speed \( v \geq 0 \)

The “shape” parameter, \( b \) affects the spread of the distribution. For \( b \) close to 1, the distribution is broad, indicating a wide range of wind speeds. For \( b > 2 \) the distribution becomes sharper indicating more consistent wind speeds. Most wind sites with appropriate wind distributions for wind turbine installations have shape parameters approximately equal to 2.

A weibull distribution with shape parameter equal to 2 is called a Rayleigh distribution, and is a common assumption for the wind speed distribution if the true distribution is unknown.

A wind regime that has Rayleigh statistics has the nice property that \( a(v^3) = 1.91 \times (v_{avg})^3 \). Then average power is given by:

\[ P_{avg} = 1.91 \times \frac{1}{2} \times \rho \times A \times v_{avg}^3 \]

The variability of wind speeds is why they are often grouped in Classes, where Class 2 winds are called marginal (5.6 – 6.4 m/s) and Class 7 winds are called superb (greater than 8.8 m/s). Most wind turbines installed today are in Class 4 and Class 5 sites.
II. Extracting Energy from the Wind - Power Curve, Capacity Factor and Efficiency of Wind Turbines

Based on the above theory and operating constraints, what can we say about the relationship between Power Output and Wind Speed?

Wind turbine CF’s are affected by both the wind regime and the particular wind turbine’s power curve. The power curve is a graph of power output as a function of wind speed.

- **Cut-in Wind Speed** – Below this speed the turbine is not turned on because the power that would be generated is not enough to offset the generator losses.
- Above the cut-in speed the power output climbs rapidly as the cube of wind speed
- **Rated wind Speed** – At this speed the generator is delivering as much power as it can. Above this wind speed the pitch of the turbines is adjusted to shed some of the wind to keep from overpowering the generator.
- **Cut-out wind speed** – Above this wind speed the winds are just too high and too dangerous so the turbine shuts down (by turning the entire rotor in the direction of the wind, or turning its blades).

![Power Curve Diagram](image)

The annual energy production in the wind is given by:

\[
\text{Annual energy production} = P_R \times 8760\text{hrs} \times \text{Capacity factor}
\]

Where \( P_R \) is the rated power of a turbine

An empirical analysis of the relationship between capacity factor and average wind speeds with a Rayleigh distribution for various turbines provided the following:

\[
\text{Capacity factor} = 0.87 \, v_{avg} \, - \frac{P_R}{D^2}
\]
Where $v_{\text{avg}}$ is the average wind speed (m/s), $P_R$ is the rated power of a turbine (kW), and $D$ is the rotor diameter of the turbine (m).

**Problem:**
Consider a 2MW wind turbine, with a 80m rotor diameter, in Rayleigh winds with average wind speed of 7m/s. What is its capacity factor and annual electricity production?

### What is the Theoretical Efficiency Limit (Betz Limit) of a Wind Turbine?

There is a limit to how much of the power from the wind can be extracted by a rotor or wind turbine. The rotor efficiency depends on the number of blades and the tip-speed-ratio (the speed at which the outer tip of the blade is rotating divided by the wind speed).

**The theoretical maximum rotor efficiency is called the Betz limit, and is 59.3%.

The law can be simply explained by considering that if all of the energy coming from wind movement into the turbine were converted into useful energy then the wind speed afterwards would be zero. But, if the wind stopped moving at the exit of the turbine, then no more fresh wind could get in - it would be blocked. In order to keep the wind moving through the turbine, to keep extracting energy, there has to be some wind movement on the other side with energy left in it.

$$P_{\text{avg}} = \eta_{\text{turbine}} \times A_{\text{rotor}} (m^2) \times P_{\text{wind}} (W/m^2)$$

Although turbine efficiencies vary depending on the wind regime, in good winds they tend to operate with an overall efficiency of somewhere between 25 – 35% efficiency.

### III. H₂ and Fuel Cells – how it works

Fuel cells (FC) make electricity through a chemical reaction between H₂ and O₂. H₂ is supplied to the negative electrode of an FC, where a catalyst (e.g., platinum) strips electrons from the H atoms. To generate electricity, the electrons move from the FC’s negative to positive electrode. Rather than circling around like the electrons, the H atoms that have shed their electrons become H ions (H⁺) and travel straight through a polymer electrolyte membrane (PEM) to reach the positive electrode. On the positive side, with the help of a catalyst, the H⁺ electrons, and oxygen (from air) re-combine to form water (see figure). A single FC generates ~0.6V, so hundreds of cells are stacked in series connection to raise the voltage to 12V, 24V, 48V, etc.
IV. Thermodynamics of Fuel Cells

- **Entropy (S)** – *Simple definition* - a thermodynamic quantity representing the amount of energy in a system that is no longer available for doing mechanical work without adding additional work to the system. *Advanced definition* – Entropy is defined by quantifying the probability of every possible microscopic state that the constituents of a macroscopic system can occupy: i.e. \( S = -k_B \sum p_i \ln(p_i) \)

- **Enthalpy (H)** – thermodynamic state function that is the sum of both internal energy (U) and the product of pressure and volume, P·V so that H = U + P·V. H is a measure of the energy required to form a substance out of its constituent parts. It is useful to define changes in energy for processes that occur at near constant pressure like combustion and it quantifies the amount of energy in a system capable of doing mechanical work.

- **Gibbs Free Energy (G)** – A thermodynamic state function that defines the criteria for a process to proceed spontaneously. It is defined as the difference of enthalpy and entropy x temp.

\[
\Delta G = \Delta H - T \cdot \Delta S
\]

A natural process occurs spontaneously if and only if the associated change \( \Delta G < 0 \). A system reaches equilibrium when \( \Delta G = 0 \), and no spontaneous process will occur if \( \Delta G > 0 \).

The ideal efficiency of a fuel cell is defined as the ratio of \( \Delta G \) to \( \Delta H \).

Fuel cell efficiency, \( \eta_{FC} = \frac{\Delta G}{\Delta H} \).
Example

For the reaction: $\text{H}_2 + \frac{1}{2} \text{O}_2 \rightarrow \text{H}_2\text{O}$ (let’s assume we are dealing with just 1 mol of H$_2$), find $\Delta G$ and the efficiency of fuel cell.

\[
\Delta G = \Delta H - T\Delta S
\]

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<th>$\text{H}_2$</th>
<th>$\text{O}_2$</th>
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