Hotelling Under Pressure

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Hotelling’s Rule for non-renewable resource extraction

- Choose quantity in each time period to maximize the present value of the resource (or a “cake-eating problem”)

\[
\max_{q(t)} \int_{0}^{T} U [q(t)] e^{-rt} dt
\]
Hotelling’s Rule for non-renewable resource extraction

- Choose quantity in each time period to maximize the present value of the resource (or a “cake-eating problem”)

\[
\max_{q(t)} \int_0^T U[q(t)] e^{-rt} dt
\]

- Resource price increases at interest rate

\[ p(t) = p_0 e^{-rt} \]
Hotelling (1931)

Hotelling’s Rule for non-renewable resource extraction

- Choose quantity in each time period to maximize the present value of the resource (or a “cake-eating problem”)

$$\max_{q(t)} \int_0^T U[q(t)] e^{-rt} dt$$

- Resource price increases at interest rate

$$p(t) = p_0 e^{-rt}$$

- Empirical evidence generally does not support the Hotelling’s Rule
Texas oil industry over 1990-2007

- Observed patterns of oil production and prices are not consistent with Hotelling’s Rule
- Constraints exist on well-level oil production
Preview of Results

Texas oil industry over 1990-2007

- Observed patterns of oil production and prices are not consistent with Hotelling’s Rule
- Constraints exist on well-level oil production

Model of oil well drilling and oil production

- Hotelling model recast as a well-drilling investment problem ("keg-tapping problem," not a “cake-eating problem”)
- Production from drilled wells is insensitive to oil prices
- Drilling of new wells and drilling rig rental prices respond strongly to oil price shocks
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Oil production and well drilling

- Texas Railroad Commission, 1990-2007
- Date and location of every well drilled
- Monthly crude oil production by lease

Oil prices

- West Texas Intermediate crude oil delivered in Cushing, Oklahoma
- Front-month futures price
- Longer-term futures prices
Data

Oil production and well drilling
► Texas Railroad Commission, 1990-2007
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Oil prices
► West Texas Intermediate crude oil delivered in Cushing, Oklahoma
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► Longer-term futures prices
Oil Prices and Production from Existing Wells
Oil Prices and Production from Existing Wells

Production from wells that never shut in

Front month price

Production from intermittent wells

Production from wells that are never shut in (bbl/d)
Oil Price and Well Drilling

![Graph showing oil price and well drilling activity over time. The x-axis represents years from January 1990 to January 2008, and the y-axis represents the number of wells drilled per month. The graph includes two lines: one representing drilling activity and the other representing front month price.]
Oil Price and Well Drilling
Characteristics of Oil Industry Cost Structure

- Rate of production from a well is physically constrained, and the constraint asymptotically declines to zero.
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- Marginal cost of production is small relative to oil prices
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- Fixed costs of operating a producing well are non-zero; there may also be costs for restarting a shut-in well, but not too large to be overcome
Characteristics of Oil Industry Cost Structure

- Rate of production from a well is physically constrained, and the constraint asymptotically declines to zero

- Marginal cost of production is small relative to oil prices

- Fixed costs of operating a producing well are non-zero; there may also be costs for restarting a shut-in well, but not too large to be overcome

- Drilling rigs are fixed in the short-run; higher prices are required to attract more rigs, leading to an upward-sloping supply curve
Ruling Out Possible Explanations

Leasing agreements require non-zero production

- Multiple-well leases show the same results
Ruling Out Possible Explanations

Leasing agreements require non-zero production
  ▶ Multiple-well leases show the same results

Races-to-oil induced by open-access externalities
  ▶ Fields controlled by a single operator show the same results
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Well-specific production quotas
   ▶ Production quotas are not binding
Ruling Out Possible Explanations

Leasing agreements require non-zero production
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Races-to-oil induced by open-access externalities
- Fields controlled by a single operator show the same results

Well-specific production quotas
- Production quotas are not binding

Producer myopia or misaligned price expectations
- Producers respond to high futures prices by stockpiling drilled oil
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Planner’s Problem

\[
\max_{F(t), a(t)} \int_0^\infty e^{-rt} [U(F(t)) - D(a(t))] \, dt
\]

subject to \(0 \leq F(t) \leq K(t)\)

\(a(t) \geq 0\)

\[\dot{R}(t) = -a(t), \quad R_0 \text{ given}\]

\[\dot{K}(t) = a(t)X - \lambda F(t), \quad K_0 \text{ given}\]

where \(F(t) = \text{rate of oil flow}\)

\(a(t) = \text{rate at which new wells are drilled}\)

\(K(t) = \text{constraint on oil flow}\)

\(R(t) = \text{measure of wells that remain untapped}\)

\(U(\cdot) = \text{instantaneous utility function}\)

\(D(\cdot) = \text{cost of drilling wells}\)

\(X = \text{maximum flow from a new well}\)

\(\lambda = \text{scaling constant}\)
Solution to Planner’s Problem

Current-value Hamiltonian

\[ H = U(F(t)) - D(a(t)) + \theta(t)[a(t)X - \lambda F(t)] + \gamma(t)[-a(t)] + \phi(t)[K(t) - F(t)] \]

where \( \theta(t) = \text{co-state variable on } K(t) \)
\( \gamma(t) = \text{co-state variable on } R(t) \)
\( \phi(t) = \text{shadow value of the oil flow constraint} \)
Solution to Planner’s Problem

Current-value Hamiltonian

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\[ + \gamma(t)[-a(t)] + \phi(t)[K(t) - F(t)] \]

where \( \theta(t) = \) co-state variable on \( K(t) \)
\( \gamma(t) = \) co-state variable on \( R(t) \)
\( \phi(t) = \) shadow value of the oil flow constraint

Selected necessary conditions

\[ F(t) \geq 0, \quad U'(F(t)) - \lambda \theta(t) - \phi(t) \leq 0, \text{ c.s.} \]
\[ a(t) \geq 0, \quad \theta(t)X - d(a(t)) - \gamma(t) \leq 0, \text{ c.s.} \]
\[ \dot{\theta}(t) = -\phi(t) + r\theta(t) \]
\[ \dot{\gamma}(t) = r\gamma(t) \]
Solution to Planner’s Problem

Current-value Hamiltonian

\[ H = U(F(t)) - D(a(t)) + \theta(t)[a(t)X - \lambda F(t)] \\
+ \gamma(t)[-a(t)] + \phi(t)[K(t) - F(t)] \]

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Selected necessary conditions

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\[ a(t) \geq 0, \quad \theta(t)X - d(a(t)) - \gamma(t) \leq 0, \text{ c.s.} \]
\[ \dot{\theta}(t) = -\phi(t) + r \theta(t) \]
\[ \dot{\gamma}(t) = r \gamma(t) \]

Also a competitive equilibrium outcome

\[ U'(F(t)) = p(t) \]
Implications for Oil Production

\[ F(t) \geq 0, U'(F(t)) - \lambda \theta(t) - \phi(t) \leq 0, \text{c.s.} \]
Implications for Oil Production

\[ F(t) \geq 0, \ U'(F(t)) - \lambda \theta(t) - \phi(t) \leq 0, \text{ c.s.} \]

Interpretation of terms

- \( \theta(t) \) is the present discounted shadow value of capacity
  \[ \theta(t) \geq \int_{t}^{\infty} U'(F(\tau)) e^{-(r+\lambda)(\tau-t)} d\tau \]

- \( \lambda \theta(t) \) is the opportunity cost of increased production
Implications for Oil Production

\[ F(t) \geq 0, \quad U'(F(t)) - \lambda \theta(t) - \phi(t) \leq 0, \text{ c.s.} \]

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Implications

- If oil prices are expected to rise slower than \( r \),
  \[ U'(F(t)) > \lambda \theta(t) \]

- If oil prices are expected to rise faster than \( r \) forever,
  \[ U'(F(t)) = \lambda \theta(t) \]

- If oil prices are expected to temporarily rise faster than \( r \), firms want to defer production but cannot due to capacity constraint
Implications for Oil Production
Implications for Oil Production

\[ F(t) \geq 0, \quad U'(F(t)) - \lambda \theta(t) - \phi(t) \leq 0, \text{ c.s.} \]
\[ \dot{\theta}(t) = -\phi(t) + r\theta(t) \]
Implications for Oil Production

\[ F(t) \geq 0, \ U'(F(t)) - \lambda \theta(t) - \phi(t) \leq 0, \text{ c.s.} \]
\[ \dot{\theta}(t) = -\phi(t) + r\theta(t) \]

If oil flow constraint is not binding

\[ U'(F(t)) = \lambda \theta(t) \]
\[ \dot{\theta}(t) = r\theta(t) \]

- \( \theta(t) \) and \( U'(F(t)) \) both increase at \( r \)
- Oil price increases at \( r \)
Implications for Oil Production

\[ F(t) \geq 0, \ U'(F(t)) - \lambda \theta(t) - \phi(t) \leq 0, \text{ c.s.} \]
\[ \dot{\theta}(t) = -\phi(t) + r\theta(t) \]

If oil flow constraint is not binding

\[ U'(F(t)) = \lambda \theta(t) \]
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- \( \theta(t) \) and \( U'(F(t)) \) both increase at \( r \)
- Oil price increases at \( r \)

When production is unconstrained, this model gives Hotelling’s Rule
Implications for Oil Well Drilling

\[ a(t) \geq 0, \theta(t)X - d(a(t)) - \gamma(t) \leq 0, \text{ c.s.} \]
\[ \dot{\gamma}(t) = r\gamma(t) \]
Implications for Oil Well Drilling

\[ a(t) \geq 0, \theta(t)X - d(a(t)) - \gamma(t) \leq 0, \text{ c.s.} \]
\[ \dot{\gamma}(t) = r\gamma(t) \]

Interpretation of terms

- \( \gamma(t) \) is the shadow value of the marginal undrilled well
- \( \theta(t)X \) is the value of capacity created by drilling a new well
- \( d(a(t)) \) is the marginal cost of drilling a new well
Implications for Oil Well Drilling

\[ a(t) \geq 0, \theta(t)X - d(a(t)) - \gamma(t) \leq 0, \text{ c.s.} \]
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- \( \gamma(t) \) is the shadow value of the marginal undrilled well
- \( \theta(t)X \) is the value of capacity created by drilling a new well
- \( d(a(t)) \) is the marginal cost of drilling a new well

Implications

- When well drilling occurs
  \[ \theta(t)X - d(a(t)) = \gamma(t) = \gamma_0 e^{rt} \]
- Returns to well drilling increase at \( r \)
Implications for Oil Well Drilling

\[ a(t) \geq 0, \theta(t)X - d(a(t)) - \gamma(t) \leq 0, \text{c.s.} \]
\[ \dot{\gamma}(t) = r\gamma(t) \]

Interpretation of terms

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Implications

▶ When well drilling occurs

\[ \theta(t)X - d(a(t)) = \gamma(t) = \gamma_0 e^{rt} \]

▶ Returns to well drilling increase at \( r \)

When drilling occurs, oil well drilling (but not necessarily oil production) is governed by Hotelling’s Rule
Implications for Oil Production and Well Drilling

If drilling occurs and production is constrained

\[
U'(F(t)) - \left[ \frac{(r + \lambda)d(a(t))}{X} - \frac{d'(a(t))\dot{a}(t)}{X} \right] = \frac{\lambda \gamma_0}{X} e^{rt}
\]
Implications for Oil Production and Well Drilling

If drilling occurs and production is constrained

\[ U'(F(t)) - \left[ \frac{(r + \lambda)d(a(t))}{X} - d'(a(t))\dot{a}(t) \right] = \frac{\lambda \gamma_0}{X} e^{rt} \]

If drilling costs are affine rather than convex

\[ U'(F(t)) - \frac{(r + \lambda)d(a(t))}{X} = \frac{\lambda \gamma_0}{X} e^{rt} \]

- Standard Hotelling’s Rule for barrel-by-barrel extraction
- Assumptions required to get this result are unrealistic
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Oil Well Drilling with Exogenous Oil Prices

The graph illustrates the drilling rate (wells/year) over time (years) under different oil price scenarios:

- **High oil price** curve: shows a higher drilling rate initially, which decreases sharply over time.
- **Low oil price** curve: displays a lower drilling rate that decreases more gradually over time.

The x-axis represents time in years, ranging from 0 to 80, while the y-axis represents the drilling rate in wells/year, ranging from 0 to 15.
Oil Production with Exogenous Oil Prices

![Graph showing oil production with high and low oil prices over time.](image-url)
Phase Diagram with Endogenous Oil Prices
Phase Diagram with Endogenous Oil Prices

\[ a(t) \]

\[ \dot{a}(t > 0) = 0 \]
\[ \dot{a}(t = 0) = 0 \]

\[ \dot{K}(t) = 0 \]

Region I

Region II

Region III

Region IV

\[ 0 \]
Equilibrium Paths
Equilibrium Paths

- Marginal discounted revenue from drilling ($\theta X$)
- Marginal profit per well increases at $r$ until drilling stops
- Marginal cost of drilling
Equilibrium Model with Demand Shocks
Conclusions

Empirical evidence from the Texas oil industry does not support Hotelling’s Rule

- Oil production is geologically constrained
- Oil production always occurs at capacity and does not respond to oil prices
- Oil well drilling responds to oil prices
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New model of exhaustible resource extraction

- Production from existing wells declines asymptotically and does not respond to oil prices
- Drilling of new wells and drilling rig rental rates strongly co-vary with oil prices
- Local oil-producing regions exhibit production peaks
- Expected future oil prices can be backwardated after positive demand shocks and can rise faster than the interest rate after negative demand shocks