Simple General Equilibrium Modeling

Shantayanan Devarajan, Delfin S. Go, Jeffrey D. Lewis, Sherman Robinson, and Pekka Sinko

I Introduction

This chapter describes how to specify, solve, and draw policy lessons from small, two-sector, general equilibrium models of open, developing economies. In the last two decades, changes in the external environment and economic policies have been instrumental in determining the performance of these economies. The relationship between external shocks and policy responses is complex; this chapter provides a starting point for its analysis.

Two-sector models provide a good starting point because of the nature of the external shocks faced by these countries and the policy responses they elicit. These models capture the essential mechanisms by which external shocks and economic policies ripple through the economy. By and large, the shocks have involved the external sector: terms-of-trade shocks, such as the fourfold increase in the price of oil in 1973–74 or the decline in primary commodity prices in the mid-1980s; or cutbacks in foreign capital inflows. The policy responses most commonly proposed (usually by international agencies) have also been targeted at the external sector: (1) depreciating the real exchange rate to adjust to an adverse terms-of-trade shock or to a cutback in foreign borrowing and (2) reducing distorting taxes (some of which are trade taxes) to enhance economic efficiency and make the economy more competitive in world markets.

A "minimalist" model that captures the shocks and policies mentioned should therefore emphasize the external sector of the economy. Moreover, many of the problems—and solutions—are related to the relationship between the external sector and the rest of the economy. The model thus should have at least two productive sectors: one producing tradable goods and the other producing non-tradables. If an economy produces only traded goods, concepts like real devaluation are meaningless. Such a country will not be able to affect its international competitiveness since all of its domestic prices are determined by world prices. If a country produced only nontraded goods, it would have been immune to most of the shocks reverberating around the world economy since 1973. Within the category of tradable goods, it is also useful to distinguish importables and exports. Such a characterization enables us to look at terms-of-trade shocks as well as the impact of policy instruments such as import tariffs and export subsidies.

The minimalist model that incorporates these features, while small, captures a rich array of issues. We can examine the impact of an increase in the price of oil (or other import and/or export prices). In addition, this model enables us to look at the use of trade and fiscal policy instruments: export subsidies, import tariffs, and domestic indirect taxes. The implications of increases or decreases in foreign capital inflows can also be studied within this framework.

While the minimalist model captures, in a stylized manner, features characteristic of developing countries, it also yields policy results that cut against the grain of received wisdom. For example, it is not always appropriate to depreciate the real exchange rate in response to an adverse international terms-of-trade shock; reducing import tariffs may not always stimulate exports; unifying tariff rates need not increase efficiency; and an infusion of foreign capital does not necessarily benefit the nontradable sector (in contrast to the results from "Dutch disease" models).

A major advantage of small models is their simplicity. They make transparent the mechanisms by which an external shock or policy change affects the economy. In addition, the example presented in this chapter can be solved analytically—either graphically or algebraically. It also can be solved numerically by using the most widely available, personal computer- (PC)-based spreadsheet programs; hence, it is not necessary to learn a new, difficult programming language in order to get started. The presentation will introduce the approach used to solve larger, multisector models. Finally, these minimalist two-sector models behave in a similar fashion to more complex multisector models, so we can anticipate some of the results obtained from multisector models, such as those presented in some of the ensuing chapters of this volume.

The plan of the chapter is as follows: In Section II, we present the simplest two-sector models. We specify the equations and discuss some modeling issues. We then analyze the impact of terms-of-trade shocks and changes in...
foreign capital inflows. In Section III, we describe an easy way of implementing the framework and use it to discuss some policy issues. The conclusion, Section IV, draws together the main points of the chapter.

II Two-Sector, Three-Good Model

The basic model refers to one country with two producing sectors and three goods; hence, we call it the “1–2–3 model.” For the time being, we ignore factor markets. The two commodities that the country produces are (1) an export good, E, which is sold to foreigners and is not demanded domestically, and (2) a domestic good, D, which is only sold domestically. The third good is an import, M, which is not produced domestically. There is one consumer who receives all income. The country is small in world markets, facing fixed world prices for exports and imports.

The equation system is presented in Table 6.1. The model has three actors: a producer, a household, and the rest of the world. Equation 6.1 defines the domestic production possibility frontier, which gives the maximum achievable combinations of E and D that the economy can supply. The function is assumed to be concave and will be specified as a constant elasticity of transformation (CET) function with transformation elasticity Ω. The constant X also corresponds to real GDP. The assumption that X is fixed is equivalent to assuming full employment of all primary factor inputs. Equation (6.4) gives the efficient ratio of exports to domestic output (E/D) as a function of relative prices. Equation (6.9) defines the price of the composite commodity and is the cost-function dual to the first-order condition, equation (6.4). The composite good price \( P^* \) corresponds to the GDP deflator.

Equation (6.2) defines a composite commodity made up of D and M which is consumed by the single consumer. In multisector models, we extend this treatment to many sectors, assuming that imports and domestic goods in the same sector are imperfect substitutes, an approach which has come to be called the Armington assumption. Following this treatment, we assume the composite commodity is given by a constant elasticity of substitution (CES) aggregation function of \( M \) and \( D \), with substitution elasticity \( σ \). Consumers maximize utility, which is equivalent to maximizing \( Q \) in this model, and equation (6.5) gives the desired ratio of \( M \) to \( D \) as a function of relative prices. Equation (6.10) defines the price of the composite commodity. It is

the cost-function dual to the first-order conditions underlying equation (6.5). The price \( P^* \) corresponds to an aggregate consumer price or cost-of-living index.

Equation (6.6) determines household income. Equation (6.3) defines household demand for the composite good. Note that all income is spent on the single composite good. Equation (6.3) stands in for the more complex system of expenditure equations found in multisector models and reflects an
important property of all complete expenditure systems: The value of the goods demanded must equal aggregate expenditure.

In Table 6.1, the price equations define relationships among seven prices. There are fixed world prices for $E$ and $M$; domestic prices for $E$ and $M$; the price of the domestic good $D$; and prices for the two composite commodities, $X$ and $Q$. Equations (6.1) and (6.2) are linearly homogeneous, as are the corresponding dual price equations, (6.9) and (6.10). Equations (6.3) to (6.5) are homogeneous of degree zero in prices – doubling all prices, for example, leaves real demand and the desired export and import ratios unchanged. Since only relative prices matter, it is necessary to define a numéraire price; in equation (6.11), this is specified to be the exchange rate $R$.

Equations (6.12), (6.13), and (6.14) define the market-clearing equilibrium conditions. Supply must equal demand for $D$ and $Q$, and the balance of trade constraint must be satisfied. The complete model has fourteen equations and thirteen endogenous variables. The three equilibrium conditions, however, are not all independent. Any one of them can be dropped and the resulting model is fully determined.

To prove that the three equilibrium conditions are not independent, it suffices to show that the model satisfies Walras's Law. Such a model is "closed" in that there are no leakages of funds into or out of the economy. First note the three identities – (6.15), (6.16), and (6.17) – that the model satisfies. The first two arise from the homogeneity assumptions and the third from the fact that, in any system of expenditure equations, the value of purchases must equal total expenditure. Multiplying equations (6.12) and (6.13) by their respective prices, the sum of equations (6.12), (6.13), and (6.14) equals zero as an identity (moving $R$ in equation [6.14] to the left side). Given these identities, simple substitution will show that if equations (6.12) and (6.13) hold, then so must (6.14).

The 1–2–3 model is different from the standard neoclassical trade model with all goods tradable and all tradables perfect substitutes with domestic goods. The standard model, long a staple of trade theory, yields wildly implausible results in empirical applications. Empirical models that reflect these assumptions embody "the law of one price," which states that domestic relative prices of tradables are set by world prices. Such models tend to yield extreme specialization in production and unrealistic swings in domestic relative prices in response to changes in trade policy or world prices. Empirical evidence indicates that changes in the prices of imports and exports are only partially transmitted to the prices of domestic goods. In addition, such models cannot exhibit two-way trade in any sector ("cross-hauling"), which is often observed at fine levels of disaggregation.

Recognizing these problems, Salter (1959) and Swan (1960) specified a two-sector model distinguishing "tradables" (including both imports and exports) and "nontradables." Their approach represented an advance and the papers started an active theoretical literature. However, they had little impact on empirical work. Even in an input–output table with over five hundred sectors, there are very few sectors which are purely non-traded; i.e., with no exports or imports. So defined, non-traded goods are a very small share of GDP; and, in models with ten to thirty sectors, there would be at most only one or two non-traded sectors. Furthermore, the link between domestic and world prices in the Salter–Swan model does not depend on the trade share, only on whether or not the sector is tradable. If a good is tradable, regardless of how small is the trade share, the domestic price will be set by the world price.

The picture is quite different in the 1–2–3 model with imperfect substitutability and transformability. All domestically produced goods that are not exported ($D$ in Table 6.1) are effectively treated as non-tradables (or, better, as "semi-tradables"). The share of non-tradables in GDP now equals 1 minus the export share, which is a very large number, and all sectors are treated symmetrically. In effect, the specification in the 1–2–3 model extends and generalizes the Salter–Swan model, making it empirically relevant.

De Melo and Robinson (1985) show, in a partial equilibrium framework, that the link between domestic and world prices, assuming imperfect substitutability at the sectoral level, depends critically on the trade shares, for both exports and imports, as well as on elasticity values. For given substitution and transformation elasticities, the domestic price is more closely linked to the world price in a given sector the greater are export and import shares. In multisector models, the effect of this specification is a realistic insulation of the domestic price system from changes in world prices. The links are there, but they are not nearly as strong as in the standard neoclassical trade model. Also, the model naturally accommodates two-way trade, since exports, imports, and domestic goods in the same sector are all distinct.

Given that each sector has seven associated prices, the model provides for a lot of product differentiation. The assumption of imperfect substitutability

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4 For the demand equation, one must show that nominal income doubles when all prices double, including the exchange rate. Tracing the elements in equation (6.6), it is easy to demonstrate that nominal income goes up proportionately with prices.

5 In this model equation (6.3) and identity (6.17) are the same. In a multisector model, as noted, identity (iii) is a necessary property of any system of expenditure equations.

6 Empirical problems with this specification have been a thorn in the side of modelers since the early days of linear programming models. For a survey, see Taylor (1975).
on the import side has been widely used in empirical models.\(^7\) Note that it is equally important to specify imperfect transformability on the export side. Without imperfect transformability, the law of one price would still hold for all sectors with exports. In the 1–2–3 model, both import demand and export supply depend on relative prices.\(^8\)

De Melo and Robinson (1989) analyze the properties of this model in some detail and argue that it is a good stylization of most recent single-country, trade-focused, computable general equilibrium (CGE) models. Product differentiation on both the import and export sides is very appealing for applied models, especially at the levels of aggregation typically used. The specification is a faithful extension of the Salter–Swan model and gives rise to normally shaped offer curves. The exchange rate is a well-defined relative price. If the domestic good is chosen as the numéraire commodity, setting \(P^d\) equal to 1, then the exchange rate variable \(R\) corresponds to the real exchange rate of neoclassical trade theory: the relative price of tradables \((E\ and\ M)\) to non-tradables \((D)\). Trade theory models (and our characterization in Table 6.1) often set \(R\) to 1, with \(P^d\) then defining the real exchange rate. For other choices of numéraire, \(R\) is a monotonic function of the real exchange rate.\(^9\)

The 1–2–3 model can also be seen as a simple programming model. This formulation is given in Table 6.2 and is shown graphically in Figure 6.1. The presentation emphasizes the fact that a single-consumer general equilibrium model can be represented by a programming model that maximizes consumer utility, which is equivalent to social welfare.\(^10\) In this model, the shadow prices of the constraint equations correspond to market prices in the CGE model.\(^11\) We will use the graphical apparatus to analyze the impact of two shocks: an increase in foreign capital inflow and a change in the international terms of trade.\(^12\) We will also use this programming-model formulation, including endogenous prices and tax instruments, to derive optimal policy rules under second-best conditions.

The transformation function (equation [6.1] in Table 6.1 and constraint [6.18] in Table 6.2) can be depicted in the fourth (southeast) quadrant of the four-quadrant diagram in Figure 6.1. For any given price ratio \(P^d/P^e\), the point of tangency with the transformation frontier determines the amounts of the domestic and exported good that are produced. Assume, for the moment, that foreign capital inflow \(B\) is zero. Then, constraint 6.19, the balance-of-trade constraint, is a straight line through the origin, as depicted in the first quadrant of Figure 6.1. If we assume for convenience that all world prices are equal to 1, then the slope of the line is 1. For a given level of \(E\) produced, the balance-of-trade constraint determines how much of the imported good the country can buy. Intuitively, with no capital inflows \((B=0)\), the only source of foreign exchange is exports. The second constraint shows the “consumption possibility frontier,” which represents the combinations of the domestic and imported goods that the consumer can buy, given the production technology as reflected in the transformation frontier and the balance of trade constraint. When world prices are equal and trade is balanced, the consumption possibility frontier is the mirror image of the transformation frontier. Equation (6.2) in Table 6.1 defines “absorption,” which

\(^7\) The CES formulation for the import-aggregation function has been criticized on econometric grounds (see Alston et al., 1990, for an example). It is certainly a restrictive form. For example, it constrains the income elasticity of demand for imports to be one in every sector. Rather than completely rejecting approaches that rely on imperfect substitutability, this criticism would seem to suggest that it is time to explore the many alternative functional forms that are available. For example, Hanson, Robinson, and Tokarcik (1993) estimate sectoral import demand functions based on the almost ideal demand system (AIDS) formulation. They find that sectoral expenditure elasticities of import demand are generally much greater than one in the United States, results consistent with estimates from macroeconometric models. Factors other than relative prices appear to affect trade shares, and it is important to study what they might be and how they operate. Alston and Green (1990) also estimated the AIDS import formulation. A related paper is Shieh, Roland-Holst, and Reineri (1993).

\(^8\) Dervis, de Melo, and Robinson (1982) specify a logistic export supply function in place of equation (6.4) in Table 6.1. Their logistic function is locally equivalent to the function that is derived from the CET specification.

\(^9\) Dervis, de Melo, and Robinson (1982), Chapter 6, discuss this relationship in detail.

\(^10\) Ginsburgh and Waelbroeck (1981) discuss, in detail, the general case where a multicountry CGE model can be represented by a programming model maximizing a Negishi social welfare function. See also Ginsburgh and Robinson (1984) for a brief survey of the technique applied to CGE models.

\(^11\) In the programming model, we implicitly choose \(Q^d\) as the numéraire good, with \(P^e=1\). In the graphical analysis, we set \(R=1\).
is maximized in the programming problem. The tangency between the “isoabsorption” (or indifference) curves and the consumption possibility frontier will determine the amount of $D$ and $M$ the consumer will demand, at price ratio $P^d/P^e$. The economy produces at point $P$ and consumes at point $C$.

Now consider what would happen if foreign capital inflow increased from its initial level of zero to some value ($B > 0$). For example, the country gains additional access to world capital markets or receives some foreign aid. Alternatively, there is a primary resource boom in a country where the resource is effectively an enclave, so that the only direct effect is the repatriation of export earnings. In all of these cases, we would expect domestic prices to rise relative to world prices and the tradable sector to contract relative to the non-tradable sector. In short, the country would contract “Dutch disease.” That this is indeed the case can be seen by examining Figure 6.2. The direct effect is to shift the balance of trade line up by $B$. This shift, in turn, will shift the consumption possibility frontier up vertically by the same $B$. The new equilibrium point will depend on the nature of the import aggregation function (the consumer’s utility function). In Figure 6.2, the consumption point moves from $C$ to $C^*$, with increased demand for both $D$ and $M$ and an increase in the price of the domestic good, $P^d$. On the production side, the relative price has shifted in favor of the domestic good and against the export – an appreciation of the real exchange rate.

Will the real exchange rate always appreciate? Consider two polar extremes, which bracket the range of possible equilibria. Suppose the elasticity of substitution between imports and domestic goods is nearly infinite, so that the indifference curves are almost flat. In this case, the new equilibrium will lie directly above the initial one (point $C$), since the two consumption possibility curves are vertically parallel. The amount of $D$ consumed will not change and all the extra foreign exchange will go toward purchasing imports. By contrast, suppose the elasticity of substitution between $M$ and $D$ is zero, so the indifference curves are L-shaped. In this case (assuming...
homotheticity of the utility function), the new equilibrium will lie on a ray radiating from the origin and going through the initial equilibrium. In this new equilibrium, there is more of both $D$ and $M$ consumed, and the price ratio has risen. Since $P_m$ is fixed by hypothesis, $P_d$ must have increased – a real appreciation. The two cases bound the range of possible outcomes. The real exchange rate will appreciate or, in the extreme case, stay unchanged. Production of $D$ will either remain constant or rise and production of $E$, the tradable good in this economy, will either stay constant or decline. The range of intermediate possibilities describes the standard view of the Dutch disease.

Consider now an adverse terms-of-trade shock represented by an increase in the world price of the imported good. The results are shown in Figure 6.3. The direct effect is to move the balance of trade line, although this time it is a clockwise rotation rather than a translation (we assume that initially $B=0$). For the same amount of exports, the country can now buy fewer imports. The consumption possibility frontier is also rotated inward. The new consumption point is shown at $C^*$, with less consumption of both imports and domestic goods. On the production side, the new equilibrium is $P^*$. Exports have increased in order to generate foreign exchange to pay for more expensive imports, and $P_d/P_m$ has also increased to attract resources away for $D$ and into $E$. There has been a real depreciation of the exchange rate.

Will there always be a real depreciation when there is an adverse shock in the international terms of trade? Not necessarily. The characteristics of the new equilibrium depend crucially on the value of $\sigma$, the elasticity of substitution between imports and domestic goods in the import aggregation function.

Consider the extremes of $\sigma=0$ and $\sigma=\infty$. In the first case, as in Figure 6.3, there will be a reduction in the amount of domestic good produced (and consumed) and a depreciation of the real exchange rate. In the second case, however, flat indifference curves will have to be tangent to the new consumption possibility frontier to the left of the old consumption point ($C$), since the rotation flattened the curve. At the new point, output of $D$ rises and the real exchange rate appreciates. When $\sigma=1$, there is no change in either the real exchange rate or the production structure of the economy. The intuition behind this somewhat unusual result is as follows: when the price of imports rises in an economy, there are two effects: an income effect (as the consumer’s real income is now lower) and a substitution effect (as domestic goods now become more attractive). The resulting

Figure 6.3. Change in world prices

equilibrium will depend on which effect dominates. When $\sigma<1$, the income effect dominates. The economy contracts output of the domestic good and expands that of the export commodity. In order to pay for the needed, non-substitutable import, the real exchange rate depreciates. However, when $\sigma>1$, the substitution effect dominates. The response of the economy is to contract exports (and hence also imports) and produce more of the domestic substitute.

For most developing countries, it is likely that $\sigma<1$, so that the standard policy advice to depreciate the real exchange rate in the wake of an adverse terms-of-trade shock is correct. For developed economies, one might well expect substitution elasticities to be high. In this case, the responses to a terms-of-trade shock are a real revaluation, substitution of domestic goods for the more expensive (and non-critical) import, and a contraction in the aggregate volume of trade. In all countries, one would expect substitution elasticities to be higher in the long run. The long-run effect of the real exchange rate will thus differ, and may be of opposite sign, from the short-run effect.

14 We derive the result analytically later.
The relationship between the response of the economy to the terms-of-trade shock and the elasticity of substitution can also be seen by solving the model algebraically. By considering only small changes to the initial equilibrium, we can linearize the model and obtain approximate analytical solutions. We follow this procedure to analyze the impact of a terms-of-trade shock.

Let a "^" above a variable denote its log-differential. That is, \( \dot{z} = d(\ln z) = dz/z \). Log-differentiate equations (6.4), (6.5), and (6.14) in Table 6.1, assuming an exogenous change in the world price of the import. The results are

\[
\hat{E} - \hat{D} = \Omega \cdot \hat{p}^d
\]

\[
\hat{M} - \hat{D} = \sigma (\hat{p}^d - \hat{p}_{w}^m)
\]

\[
\hat{M} + \hat{p}_{w}^m = \hat{E}
\]

Eliminating \( \hat{M}, \hat{D}, \) and \( \hat{E} \) and solving for \( \hat{p}^d \) yields

\[
\hat{p}^d = \frac{\sigma - 1}{\sigma + \Omega} \hat{p}_{w}^m
\]

Thus, whether \( P^d \) increases or decreases in response to a terms-of-trade shock depends on the sign of \( (\sigma - 1) \), confirming the graphical analysis discussed. Figure 6.4 illustrates the impact of a 10 percent import price shock on \( P^d \) under varying trade elasticities, \( 0 < \sigma < 2 \) and \( 0 < \Omega < 2 \). Note that the direction of change in \( P^d \) will determine how the rest of the economy will adjust in this counterfactual experiment. If \( P^d \) falls (the real exchange rate depreciates), exports will rise and production of the domestic good will fall.

Our analysis with the 1–2–3 model has yielded several lessons. First, the bare bones of multisector general equilibrium models are contained in this small model. Second, and perhaps more surprisingly, this two-sector model is able to shed light on some issues of direct concern to developing countries. For example, the appreciation of the real exchange rate from a foreign capital inflow, widely understood intuitively and derived from more complex models, can be portrayed in this simple model. In addition, results from this small model challenge a standard policy dictum: Always depreciate the real exchange rate when there is an adverse terms-of-trade shock. The model shows the conditions under which this policy advice should and should not be followed.

Of course, many aspects of the economy are left out of the small model. In particular, there are no government, factor markets, and intermediate goods; the framework is also static. Devarajan, Lewis, and Robinson (1990) discuss several extensions and modeling issues in a one-period setting; Devarajan and Go (1993) present a dynamic version of the 1–2–3 framework in which producer and consumer decisions are both intra- and intertemporally consistent. All these extensions require that the model be solved numerically. We turn therefore to the numerical implementation of the 1–2–3 model, extending the basic 1–2–3 model to include the government sector in order to look at policy instruments such as taxes.

**III Numerical Implementation**

As a means of evaluating economic policy or external shocks, general equilibrium analysis has several known advantages over the partial approach and its numerical implementation has become increasingly the preferred tool of investigation. So far, however, CGE models are cumbersome to build.
how this real exchange rate adjusts in response to exogenous shocks. In order to apply the framework to a particular country, however, it has to be modified to fit real data and to handle policy issues. For example, the real exchange rate is not an instrument, which the government directly controls. Rather, most governments use taxes and subsidies as well as expenditure policy to adjust their economies. Nor did the previous section touch on the equality of savings and investment, which is important in bringing about macroeconomic balance or equilibrium. Table 6.3 presents an extended version of the 1-2-3 model to include government revenue and expenditure and also savings and investment. We make sure that the modifications introduced will conform to data that are commonly available (see calibration later). In the new setup, four tax instruments are included: an import tariff \( t^m \), an export subsidy \( t^e \), an indirect tax on domestic sales \( t' \), and a direct tax rate \( t' \). In addition, savings and investment are included. The single household saves a fixed fraction of its income. Public savings (budgetary deficit or surplus) is the balance of tax revenue plus foreign grants and government expenditures (all exogenous) such as government consumption and transfers to households. The current account balance, taken to represent foreign savings, is the residual of imports less exports at world prices, adjusted for grants and remittances from abroad. Output is fixed for reasons cited in Section II. Foreign savings is also presently fixed, so that the model is savings-driven; aggregate investment adjusts to aggregate savings. In sum, we have twenty equations and nineteen endogenous variables. By Walras’s Law, however, one of the equations, say the savings–investment identity, is implied by the others and may be dropped.

### III.2 Defining Model Components

Building the 1-2-3 framework in Excel requires the usual modeling steps: (1) declaration of parameters and variables, (2) data entry, (3) assignment of initial values to variables and parameters, and (4) specification of equations. In addition, the model has to be precisely defined as a collection of equations; in some cases, it may require an objective function to be optimized. Finally, the solver is called on to conduct numerical simulations.

A suitable way to arrange the 1-2-3 model in an Excel worksheet is to assign separate columns or blocks for parameters, variables, and equations.

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17 See Prest (1962) and Chellish and Chand (1974) for a discussion of such an approach.
18 Microsoft Excel and Windows are trademarks of Microsoft Corporation.
19 The discussion of Excel procedures is compatible with version 5 or later, such as version 5 under Windows 3.1 or version 7 under Windows 95. We also include in the notes, where applicable, how to implement the same procedures in the previous version of Excel.
20 In the alternative investment-driven closure, aggregate investment is fixed and savings adjust through foreign savings (endogenous). For a discussion of alternative macroclosures, see the original work of Sea (1965) or the surveys by Rattso (1982) and Robinson (1989).
Table 6.3. The 1–2–3 model with government and investment

<table>
<thead>
<tr>
<th>Real Flows</th>
<th>Nominal Flows</th>
<th>Accounting Identities</th>
<th>Endogenous Variables</th>
<th>Exogenous Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6.21) ( \bar{X} = G(E,D; \bar{Y}) )</td>
<td>(6.26) ( T = p \cdot R \cdot pw^* \cdot M + t^r \cdot P \cdot Q^e + t^s \cdot Y )</td>
<td>(6.41) ( P \cdot \bar{X} = P \cdot E + P \cdot D^e )</td>
<td>( P^w ): World price of import good</td>
<td>( pw^* ): World price of export good</td>
</tr>
<tr>
<td>(6.22) ( Q^e = P(M,D^e; y) )</td>
<td></td>
<td>(6.42) ( P \cdot Q^e = P^m \cdot M + P \cdot D^e )</td>
<td>( t^r ): Tariff rate</td>
<td>( t^s ): Sales/excise/value-added tax rate</td>
</tr>
<tr>
<td>(6.23) ( Q^e = C + Z + G )</td>
<td></td>
<td></td>
<td>( t^f ): Direct tax rate</td>
<td>( t^g ): Government transfers</td>
</tr>
<tr>
<td>(6.24) ( E/D^e = g(p)^m )</td>
<td>(6.27) ( Y = P \cdot X + t^r \cdot P \cdot M + n \cdot R )</td>
<td></td>
<td>( t^r ): Government transfers</td>
<td>( X ): Foreign transfers to government</td>
</tr>
<tr>
<td>(6.25) ( M/D^e = f(p^m, p^f) )</td>
<td>(6.28) ( S = t^s \cdot Y + R - B + S^e )</td>
<td>(6.40) ( T = P^e \cdot E - t^r \cdot M - t^f \cdot R - S^e = 0 )</td>
<td>( P^m ): Average savings rate</td>
<td>( X ): Aggregate output</td>
</tr>
</tbody>
</table>

Separate columns are assigned for the base year and simulation values of variables. Labels and explanations for parameters, variables, and equations are easily provided in the adjacent left column to improve readability. We also assign a block for the dataset with both initial and calibrated values displayed. Thus, we are able to arrange all necessary ingredients conveniently on a single worksheet.

### III.3 Variables and Parameters

Table 6.4 is an example of how to organize the parameters and variables in an Excel-based model. We separate out from the rest of the exogenous variables the parameters related to the trade elasticities; the trade elasticities are generally defined at the outset of an experiment, and parameters such as the share and scale values of the CES and CET functions are calibrated just once for both the base case and the current simulation (see the calibration section later). Column A provides a brief description of each parameter and column B lists the corresponding numerical value. The exogenous variables (described in column C) specify the external or policy shocks introduced in a particular experiment – their magnitudes are defined in column D while their base-year values are presented in column E. Likewise, the endogenous variables are listed in columns F to I. New values are computed for the endogenous variables during a simulation and entered in column H as Current. Column I, Curl/Base, provides simple indices of change of the endogenous variables.

A useful feature in Excel is the capability to define names for various model parts. This is done by using the Name command and Define option under the Insert menu. The cell in B6 of Table 6.4, for example, can be called by its parameter name, \( st \); hence, we can refer to parameters, variables, or equations by using their defined or algebraic names instead of cell locations. By doing this, we make the model specifications easier to read and mistakes easier to detect. To keep track of these names, it is advisable to write them out in explanation cells adjacent to the corresponding parameters, variables, and equations. In the example shown in Table 6.4, we write a short description and put in parentheses the Excel label or name. Base year and current values of variables are distinguished by using the normal convention – in the case of export good \( E \), for example, the base year level is labeled as \( E0 \) while \( E \) is retained for the simulated level.

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21 Prior to version 5 of Excel, this is done by using the Define Name command in the Formula menu.
### 3.4 Equations

The organization of the equations of our model is illustrated in Table 6.5. The equations are numbered and listed (in column J of Table 6.5) in the same order as Table 6.3. Column K of Table 6.5 lists the equation descriptions and the Excel names in parentheses. The corresponding mathematical expressions are entered in column L. In the normal mode the formulas are hidden in the background and only the current numerical values are evident. The formulas are easily displayed by using the Options command on the Tools menu, selecting (or clicking) the View tab, and choosing Formulas in the Window Options box.\(^{22}\)

In a spreadsheet like Excel, a formula is typically entered into a cell by writing out just the right-hand side of an equation as shown in Table 6.5. To complete the equation, each of these mathematical expressions has to be matched and set equal to a variable defined as indicated earlier (see the Solver section).

The complicated expressions in column L of Table 6.5 require some explanations. Equations (6.21) and (6.22), called CTEQ and ARMG in Excel, are the right-hand expressions of the CET and Armington (CES) functions in the 1–2–3 model, which usually take the following algebraic form:

$$Y = A\left[\delta \cdot X^p + (1-\delta) \cdot X^q\right]^{|1/p|}$$

where the CES substitution elasticity \(\sigma\) and CET transformation elasticity \(\Omega\) are given by \(\sigma = 1/(1-\rho); -\infty < \rho < +1\) in the CES case and \(\Omega = 1/(\rho - 1); 1 < \rho < \infty\) in the CET case. In the Excel implementation, the share parameter \(\delta\) is labeled as \(bt\) or \(bg\), the exponent \(\rho\) as \(rt\) or \(rg\), and the elasticities as \(st\) or \(sq\). Equation (6.24), EDRAT, is the right-hand side of the export supply function or the first-order condition of the CET function:

$$\frac{E}{D} = \left[\frac{(1-\delta) \cdot P^p}{\delta \cdot P^q}\right]$$

while equation (6.25) (MDRAT) in Table 6.5 is the corresponding case (import demand function)

$$\frac{M}{D} = \left[\frac{\delta \cdot P^q}{(1-\delta) \cdot P^m}\right]$$

---

22 In earlier versions of Excel, the equations are easily unveiled by pulling down the Options menu and selecting Formula among the Display options.
The dual price equations, equations (6.33) \((PXEQ)\) and (6.34) \((PQEQ)\), can take the following form:

\[
P = A^{-1} \left[ \delta^{1/(1-\rho)} P_1^{\rho/(1-\rho)} + (1-\delta)^{1/(1-\rho)} P_2^{\rho/(1-\rho)} \right]^{-1/\rho}
\]

\[
P^* = \frac{P^m \cdot M + P^d \cdot D}{Q}
\]

However, in practice, it is often convenient to replace the dual price equations with the expenditure identities, invoking Euler's theorem for linearly homogeneous functions:

\[
P^* = \frac{P^* \cdot E + P^d \cdot D}{X}
\]

In the 1-2-3 model, the dual price equations embody the same information as the CET export transformation and CES import aggregation functions. In some applications, it is convenient to include the dual price equations, but drop the CET and CES functions.

### III.5 Calibration

Another convenient feature of the 1-2-3 framework is its modest data requirements. Data from national income, fiscal, and balance-of-payments accounts, those normally released by national governments, are sufficient. To carry out the model, we used the 1991 data for Sri Lanka (Table 6.6). The original data were measured in billions of rupees. In the calibration, all data were scaled and indexed with respect to output, which is set to 1.00 in the base year (note columns P and T).

Tables 6.7 and 6.8 show the calibration of parameters and variables. The values of the parameters and variables are linked to the data in Table 6.6 so that model calibration is automatically done whenever the elasticities or base year data are changed. In Table 6.7, the calibration of the exponents, \(\tau\)
Table 6.8. Calibration of variables in the Excel-based 1-2-3 model

<table>
<thead>
<tr>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exogenous Variables</td>
<td>Base Year</td>
<td>Current</td>
<td>Endogenous Variables</td>
<td>Base Year</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>World Price of Imports (wI)</td>
<td>= Pr0/E0/(1+i+tm0)</td>
<td>= wI0</td>
<td>Export Good (E)</td>
</tr>
<tr>
<td>3</td>
<td>World Price of Exports (wO)</td>
<td>= Pr0*(1+i+tm0)/E0</td>
<td>= wO0</td>
<td>Import Good (M)</td>
</tr>
<tr>
<td>4</td>
<td>Import Tariffs (tm)</td>
<td>= 0.12/0.15</td>
<td>= tm0</td>
<td>Supply of Domestic Good (Dg)</td>
</tr>
<tr>
<td>5</td>
<td>Export Duties (se)</td>
<td>= 0.20/0.14</td>
<td>= se0</td>
<td>Supply of Composite Good (Gc)</td>
</tr>
<tr>
<td>6</td>
<td>Indirect Taxes (ts)</td>
<td>= 0.18/Gs0</td>
<td>= ts0</td>
<td>Demand of Domestic Good (Dd)</td>
</tr>
<tr>
<td>7</td>
<td>Direct Taxes (ty)</td>
<td>= SUM(P21,P23)/Y0</td>
<td>= ty0</td>
<td>Tax Revenue (TAX)</td>
</tr>
<tr>
<td>8</td>
<td>Savings rate (s)</td>
<td>= Y0*P0 -s0/0.1</td>
<td>= s00</td>
<td>Total Income (Y)</td>
</tr>
<tr>
<td>9</td>
<td>Govt. Consumption (Gc)</td>
<td>= 0.12/21 + 0.10/Pr0</td>
<td>= G00</td>
<td>Aggregate Savings (S)</td>
</tr>
<tr>
<td>10</td>
<td>Govt. Transfer (tr)</td>
<td>= T11<em>1 + T12/T13</em>Pr0</td>
<td>= tr0</td>
<td>Consumption (Cn)</td>
</tr>
<tr>
<td>11</td>
<td>Foreign Grants (fr)</td>
<td>= 0.11<em>22</em>Pr0</td>
<td>= fr0</td>
<td>Import Price (Pr0)</td>
</tr>
<tr>
<td>12</td>
<td>Net Priv Remittances (ns)</td>
<td>= SUM(T11,T12)/E0</td>
<td>= ns0</td>
<td>Export Price (Pe)</td>
</tr>
<tr>
<td>13</td>
<td>Export Saving (B)</td>
<td>= wo0<em>Pr0 - wo0</em>E0 - f00 - re0/E0</td>
<td>= B00</td>
<td>Sales Price (P)</td>
</tr>
<tr>
<td>14</td>
<td>Output (X)</td>
<td>= wo0<em>Pr0 - wo0</em>E0 - f00 - re0/E0</td>
<td>= X00</td>
<td>Price of Supply (Ppg)</td>
</tr>
<tr>
<td>15</td>
<td>Price of Output (Ppx)</td>
<td>= 1</td>
<td></td>
<td>Price of Dom. Good (Pd)</td>
</tr>
<tr>
<td>16</td>
<td>Exchange Rate (Ee)</td>
<td>= 1</td>
<td></td>
<td>Exchange Rate (Ee)</td>
</tr>
<tr>
<td>17</td>
<td>Investment (I)</td>
<td>= P12/Pr0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Government Savings (Gg)</td>
<td>= T30 - 0.1<em>Pr0 - 0.1</em>Pr0 + re0*E0</td>
<td>= Gg0</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Wales Law (Wz)</td>
<td>= 0.12<em>Pr0 - 0.10</em>Pr0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
solves the model as an optimization or programming problem. In the Set Target Cell space, at the top of the dialog box, the name of the variable that is being maximized (max option) or minimized (min option) in the objective function may be entered. We select the consumption variable \( CN \) in this case, but this has no effect in a CGE application since there will be as many variables and equations. The space may also be left empty. The "optimal" solution is found by Changing Cells, where all the endogenous variables in the model are entered using their names or cell locations, and Subject to the Constraints, where all equations and non-negativity constraints of the model are listed. The Add option in the dialog box allows us to specify the equations and constraints one at a time. For example, the line highlighted in Figure 6.5 matches the mathematical expression of the Armington function to total supply \( \text{ARMG} = Q \), which corresponds to the first equation of our model when arranged alphabetically.

The Options command in the Solver Parameters menu controls the solution process. The Options command lets one adjust the maximum iteration time and tolerance level as well as choose the appropriate search method. In the model, we used the Newton solution algorithm, which proved out to be robust and fast. Average time for solving simulations with a 486/33 PC was around 10 seconds.

The model is run by choosing the Solve command. The solver starts iterating and the number of trial solutions appears in the lower-left part of the worksheet. Once a solution that satisfies all the constraints has been found, the Solver stops and displays a dialog box to show the results. A variety of ways for reporting the outputs are possible. One can now choose between displaying the solution values on the worksheet and restoring the original values (initial guesses) of variables. Also, one may choose the option that produces both the original values and solution values. If there is no shock and the model is correctly calibrated, one should find a solution where all the variables equal their base-year values within the fixed tolerance.26 For example, 0.33, the base-year value of \( EO \) (export good) in cell G6 in Table 6.4, is entered as the initial guess or current value for the variable \( E \) in cell H6. It is important to enter some feasible initial guesses for current values of variables before starting the solver. An empty cell is interpreted as zero, which is frequently an infeasible value for a variable.

### III.7 Simulations

To test the model, we conduct two experiments. The first is a trivial case—we double the nominal exchange rate, which is our numéraire. This is done by changing the right-hand side of equation 6.35 from 1.0 to 2.0 as shown in cell L22 in Table 6.5. After the experiment is run, the results are shown as the current values of the variables in column H of Table 6.4. As expected, all prices and incomes double while all quantities remain the same.

Next, we look at one important tax policy issue in developing countries—the fiscal/revenue implications of a tariff reform. Tariffs are a significant source of public revenue in many developing countries. In Sri Lanka, about 28 percent of tax revenue came from import duties in 1991. Therefore, the potential revenue losses of a tariff reduction in any attempted trade liberalization has to be offset by other revenue sources so as to prevent the balance of external payments from deteriorating.27 In the experiment, we set the tariff collection rate to 0.05 (down from 0.13 in the base year) and ask by how much the domestic indirect taxes need to be raised to maintain the current account deficit from deteriorating, while keeping the same level of productive investment in the economy. To do this, we simply replace investment \( Z \) with the sales tax \( ts \) in the variable list and run the 1-2-3e model again. To attain the preceding policy objective, we find that sales and excise taxes need to be raised by about 33 percent (from the current rate of 0.08 to

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26 A good way of testing the model is to maximize and minimize the objective variable, which should produce identical solutions in a general equilibrium framework.

27 Greenaway and Milner (1991) and Mitra (1992) discuss the substitution of the domestic and trade taxes in greater detail.
This chapter shows how two-sector models can be used to derive policy shocks. In particular, we derive the assumptions underlying the conventional policy recommendation of exchange rate depreciation in response to adverse shocks of a foreign capital inflow and terms-of-trade shock may be analyzed. In small, open economies function. They are useful for qualitative analysis. However, policymakers are also concerned with the magnitude of the response to their initiatives. Furthermore, they require models that incorporate the more distinctive structural and institutional features of their economies. The lessons drawn from this chapter will facilitate the interpretation of results from more complex models, since these are essentially multisectoral analogues of the small models developed here.

### References


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We also implemented the model by using a popular spreadsheet software, Excel, and by using widely available data. While Excel is not suitable for all types of tax or CGE models and certainly other programs, such as GAMS, offer greater capability and indexing ease (e.g., over sectors or time), it is simple to use and a great way to get started. Add-in programs also extend its potential in new directions; for example, it is possible to add the element of uncertainty over critical parameters (e.g., trade elasticities) or exogenous shocks (e.g., the collapse of an export market like the CMEA trade) by performing risk analysis and Monte Carlo simulations.  

The models in this chapter present a stylized picture of how developing economies function. They are useful for qualitative analysis. However, policymakers are also concerned with the magnitude of the response to their initiatives. Furthermore, they require models that incorporate the more distinctive structural and institutional features of their economies. The lessons drawn from this chapter will facilitate the interpretation of results from more complex models, since these are essentially multisectoral analogues of the small models developed here.

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0.11 in cells G25 and H25, respectively, in Table 6.9. This figure of course depends on, among other factors, the degree of substitution possibilities between imports and domestic goods. Because of the “automatic” calibration embedded in the worksheet, it would be straightforward to test the sensitivity of the results on alternate values of critical parameters by just entering new estimates to the corresponding cells.

### IV Conclusion

This chapter shows how two-sector models can be used to derive policy lessons about adjustment in developing countries. Starting from a small, one-country, two-sector, three-good (1–2–3) model, we show how the effects of a foreign capital inflow and terms-of-trade shock may be analyzed. In particular, we derive the assumptions underlying the conventional policy recommendation of exchange rate depreciation in response to adverse shocks.


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Edited by

JOSEPH F. FRANCOIS
Eraumus University,
World Trade Organization,
and the Centre for
Economic Policy Research

KENNETH A. REINERT
Kalamazoo College

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